# Numerical Analysis of Artificial Neural Network and Volterra-based Nonlinear Equalizers for Coherent Optical OFDM

**Elias Giacoumidis[1,2], Jinlong Wei[3], Mutsam A. Jarajreh[4], Son T. Le[1], Paul A. Haigh[5], Jan Bohata[6], Andreas Perentos[1], Sofien Mhatli[7], Mohammad Ghanbarisabagh[8], Ivan Aldaya[9], and Nick J. Doran[1]**

[1]Aston Institute of Photonic Technologies (AIPT), Aston University, Birmingham B4 7ET, UK
[2]Centre for Ultrahigh bandwidth Devices for Optical Systems (CUDOS)
School of Physics, University of Sydney, NSW 2006, Australia
[3]ADVA Optical Networking SE, Campus Martinsried
Fraunhoferstrasse 9a, Martinsried/Munich 82152, Germany
[4]Engineering Department, School of Computing
Fahad Bin Sultan University, Tabuk, Kingdom of Saudi Arabia
[5]High Performance Networking (HPN) Group
Faculty of Engineering, University of Bristol, Bristol BS8 1UB, UK
[6]Faculty of Electrical Engineering, Czech Technical University in Prague, Czech Republic
[7]SERCOM-Lab, EPT Université de Carthage, La Marsa, Tunis 2078, Tunisia
[8]Faculty of Engineering, Islamic Azad University North Tehran Branch, Tehran, Iran
[9]Department of Electrical and Computational Engineering
Instituto Tecnológico y de Estudios Superiores de Monterrey, 64800, Mexico

**Abstract**— One major drawback of coherent optical orthogonal frequency-division multiplexing (CO-OFDM) that hitherto remains unsolved is its vulnerability to nonlinear fiber effects due to its high peak-to-average power ratio. Several digital signal processing techniques have been investigated for the compensation of fiber nonlinearities, e.g., digital back-propagation, nonlinear pre- and post-compensation and nonlinear equalizers (NLEs) based on the inverse Volterra-series transfer function (IVSTF). Alternatively, nonlinearities can be mitigated using nonlinear decision classifiers such as artificial neural networks (ANNs) based on a multilayer perceptron. In this paper, ANN-NLE is presented for a 16 QAM CO-OFDM system. The capability of the proposed approach to compensate the fiber nonlinearities is numerically demonstrated for up to 100-Gb/s and over 1000 km and compared to the benchmark IVSTF-NLE. Results show that in terms of $Q$-factor, for 100-Gb/s at 1000 km of transmission, ANN-NLE outperforms linear equalization and IVSTF-NLE by 3.2 dB and 1 dB, respectively.

## 1. INTRODUCTION

CO-OFDM is a high spectral-efficient modulation format able to virtually eliminate inter-symbol interference (ISI) caused by chromatic dispersion (CD) and polarization mode dispersion (PMD) [1]. One major drawback of CO-OFDM that hitherto remains unsolved is its vulnerability to nonlinear fiber effects due to its high peak-to-average power ratio (PAPR). Several digital signal processing (DSP) techniques have been investigated for the compensation of nonlinearities, e.g., digital back-propagation (DBP) [2], nonlinear pre- and post-compensation [3] and DSP nonlinear equalizers (NLEs) based on the inverse Volterra-series transfer function (IVSTF) [4]. The main disadvantage of DBP is the extensive use of the fast Fourier transform (FFT), which results in high DSP computational load. Pre- and post-compensation algorithms are complex to implement and present a marginal performance enhancement ($< 0.5$ dB in $Q$-factor [3]). IVSTF-based equalization has been considered as an effective method for combating fiber nonlinearities with a reported $Q$-factor enhancement of 1 dB in 256-Gb/s polarization-division multiplexed 16 quadrature amplitude modulation (16 QAM) transmissions [4]. Alternatively, nonlinearities can be mitigated using nonlinear decision classifiers such as artificial neural networks (ANNs) based on a multilayer perceptron (MLP). MLP-ANNs form a complex map with nonlinear decision boundaries between its input and output spaces, which helps in inverting the effects of nonlinear distortion. ANNs have shown promising results in wireless communications applications to reduce the effect of the nonlinear distortion of amplifiers in various system configurations including OFDM [5–7].

A novel ANN-NLE is presented for a 16 QAM CO-OFDM system in this paper, which is an extension of our previous work reported in [8, 9]. The capability of the proposed approach to compensate the fiber nonlinearities is numerically demonstrated at 100-Gb/s over a standard single-mode fiber (SSMF) link of 100 km and compared to the benchmark IVSTF-NLE. Results reveal that in terms of $Q$-factor, for 100-Gb/s at 1000 km of transmission, ANN-NLE outperforms linear equalization and IVSTF-NLE by 3.2 dB and 1 dB, respectively.

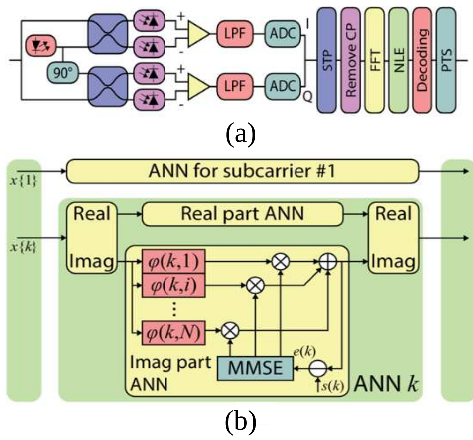



Figure 1: (a) Block diagram of the CO-OFDM receiver equipped with the proposed ANN-NLE. (b) Receiver 16 QAM CO-OFDM block diagram showing the equalization schematic diagram of the ANN sub-neural network. LPF: low-pass filter, ADC: analogue-to-digital converter, STP/PTS: serial-to-parallel/parallel-to-serial, CP: cyclic prefix, FFT: fast Fourier transform, NLE: nonlinear equalizer, MMSE: minimum mean-square error.

## 2. ARTIFICIAL NEURAL NETWORK NONLINEAR EQUALIZER

The schematic diagram of the proposed ANN-based NLE for the 16 QAM CO-OFDM receiver is depicted in Figure 1(a), where $s(k)$ is the training vector, i.e., the pre-known subcarrier set transmitted during the training stage. Since the CO-OFDM signal consists of $k$ subcarriers, ANN-NLE is comprised of $k$ sub-neural networks, with each sub-network being associated to each subcarrier. The received symbols for every subcarrier $x(k)$ are fed to NLE neurons where they are multiplied with the weight value for a given OFDM subcarrier and neuron $w(k,i)$, and afterwards, the outputs of the different neurons are summed. In the training stage, the well-known minimum mean-square error (MMSE) algorithm, which is standard in the new feed-forward (FFD) networks, is used to determine the error signal and update the weights. The weights are iteratively updated until the desired error value is reached, thus indicating the optimum match between the sub-network output and the transmitted (undistorted) OFDM subcarrier symbols. The error signal is given as:

$$e(k) = s(k) - \hat{s}(k) \tag{1}$$

where $\hat{s}(k)$ is calculated in terms of a nonlinear activation function $\varphi(k,i)$, performing the NLE and $w(k,i)$, which is given by:

$$\hat{s}(k) = \sum\nolimits_{i=1}^{16} w(k,i)\varphi(k,i)s(k). \tag{2}$$

The nonlinear activation function is application dependent and it is mostly required to be a differentiable function. For the proposed ANN-NLE a sigmoid function is used, which can satisfy a conflicting relationship between the boundedness and the differentiability of a complex function. This is called the "split" complex activation function, where two conventional real-valued activation functions process the in-phase and quadrature components.

It is important to mention that the number of neurons in every neural sub-net is equal to the number of the signal modulation format level, which in the case of 16 QAM is 16. In this work, the ANN-NLE is based on the FFD network that uses the Riedmiller's resilient back-propagation (RR-BP) algorithm [10]. The training function updates the weights and the bias values according to the RR-BP algorithm, which is computationally more efficient than other training algorithms, and it performs an approximation to the global minimization achieved by the steepest descent [11]. Hence, as mentioned, RR-BP minimizes the difference between the ANN output and the desired output, i.e., the target output. This is achieved in real-time splitting the complex OFDM data into two real-valued data collections; the real $\hat{s}_r(k)$ and imaginary $\hat{s}_i(k)$ parts are fed separately into two ANN sub-networks and the outputs are recombined as following:

$$\hat{s}_{Final} = \hat{s}_r(k) + j \cdot \hat{s}_i(k) \tag{3}$$

For our numerical investigations, the employed transfer functions for the hidden layer are differentiable and similar to the hyperbolic tangent function, as suggested in [11]. For the output layer, the linear function "purelin" is used. The block identified as MMSE in Figure 1(b), represents the subsystem that implements the RR-BP algorithm used to find the weights that minimize the error vector (the vector whose $k$th component is $e(k)$):

$$E(n) = \|S(n) - \hat{S}(n)\|^2 \tag{4}$$

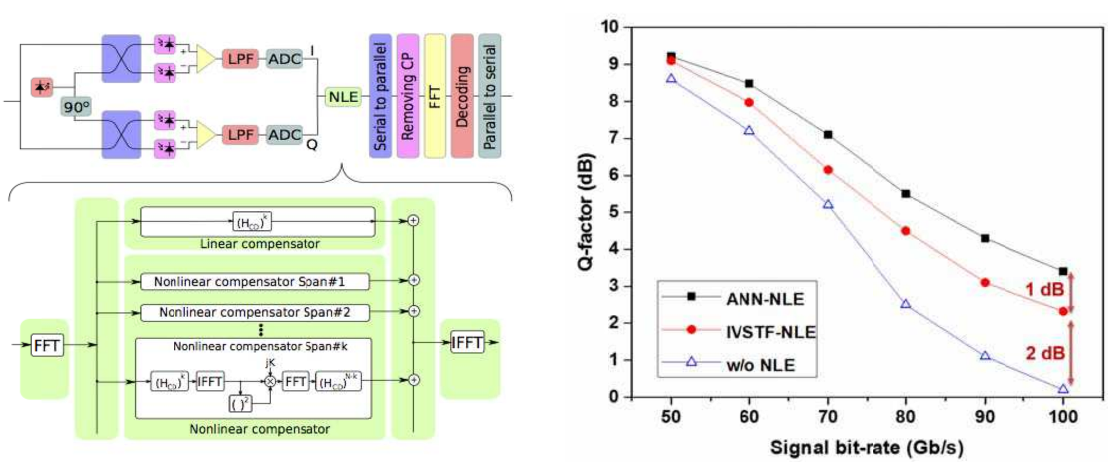
where $S(n)$ and $\hat{S}(n)$ are the desired and calculated output vectors, respectively. The weights are updated according to the following 5 steps by applying gradient descent on the cost function $E(n)$ in order to reach a minimum:

- *Step 1*: Initialize the weights and thresholds to small random numbers.
- *Step 2*: Present the input vector, $X(n)$, and the desired output vector $S(n)$.
- *Step 3*: Calculate $\hat{S}(n)$ from the $X(n)$ and compute the error vector $E(n)$ using (4).
- *Step 4*: Adapt weights based on:

$$w(k,i)_{n+1} = w(k,i)_n - \tilde{n}\left(\partial E(n)/\partial w(k,i)'_n\right) \tag{5}$$

  where $w(k,i)_n$ is the weight of the $i$th neuron of the $k$th sub-neural network ($i$th symbol of the $k$th subcarrier) at the $n$th iteration, and $\tilde{n}$ is the learning rate parameter. It is worth mentioning that when $\tilde{n}$ is very small, the algorithm will take long time to converge; whereas, when $\tilde{n}$ is too large the system may run into an unstable state.
- *Step 5*: If $E(n)$ is above the threshold, go to *step 2.*

The schematic block diagram of the benchmark IVSTF equalizer is depicted in Figure 2(a), which is similar to that reported in [8, 9] to account for single-polarization CO-OFDM. Compared to ANN, the IVSTF-NLE is placed just after the ADCs to reduce DSP complexity by means of reducing the number of FFT/IFFT blocks. The IVSTF-NLE inherits some of the features of the hybrid time-and-frequency domain implementation, such as non-frequency aliasing and simple




(a) (b)

Figure 2: (a) 16 QAM CO-OFDM receiver block diagram showing the equalization schematic diagram of the benchmark inverse Volterra-series transfer function (IVSTF) [4, 8, 9]. LPF: low-pass filter; ADC: analog-to-digital converter; CP: cyclic prefix; (I) FFT: (inverse) fast-Fourier transform; NLE: nonlinear equalizer; $H_{CD}$: linear system chromatic dispersion. (b) $Q$-factor in terms of the signal bit-rate (up to 100-Gb/s) for a 1000 km link without using NLE, using IVSTF-based NLE and the proposed ANN-NLE.

implementation. From Figure 2(a), it can be clearly identified that CD, i.e., $(H_{CD})^k$, and the fiber nonlinearity are combated by the linear and nonlinear compensator tool, respectively. It should be mentioned that for purposes of reduced complexity and processing time, very high order Volterra kernels have not been considered, thus offering ∼50% reduced computational complexity compared to the single-step/span DBP [8]. Additionally, it should be noted that ANN-NLE for the current CO-OFDM configuration is less complex than IVSTF-NLE and $>$ 50% less complex than single-step/span DBP [9].

The proposed ANN and benchmark IVSTF equalization schemes were validated by carrying out numerical simulations in a Matlab/VPI-transmission-Maker co-simulation environment (electrical domain in Matlab and optical domain in VPI-*version 9*). For the IVSTF-NLE, we have calculated up to 3rd order Volterra kernels [4, 8, 9]. A CO-OFDM system with ideal homodyne reception was considered using 16 QAM subcarrier modulation and signal bit-rates ranging from 50-Gb/s up to 100-Gb/s, before adding the different transmission overheads, i.e., the cyclic prefix (CP) and training symbols for channel estimation. A 64-point IFFT/FFT pair was used to reduce the complexity of the ANN-NLE, and 1000 OFDM symbols were generated. A transmission of up to 1000 km (10 homogeneous spans ×100 km) was considered. A large CP overhead of 25% was added to virtually eliminate all ISI caused by CD and PMD, which also relaxes the synchronization requirements of the CO-OFDM demodulator. The rest of transceiver parameters are similar to [9].

## 3. RESULTS

The nonlinearity compensation capability was assessed based on $Q$-factor, which was estimated from the bit-error-rate (BER) obtained by error counting after hard-decision decoding. The $Q$-factor is related to the BER value by: $Q = 20\log_{10}[\sqrt{2}erfc^{-1}(2BER)]$. For a Gray-coded 16 QAM modulation, a $10^{-3}$ (the commonly adopted threshold for forward error correction [FEC] codes) results in a required $Q$-factor of 9.8 dB. Figure 2(b) shows the $Q$-factor vs. the signal bit-rate for up to 100-Gb/s 16 QAM CO-OFDM using ANN-NLE, IVSTF-NLE, or linear equalization (the launched optical power was set to −6 dBm, which was the optimum level for linear equalization). It is shown that ANN-NLE can improve the $Q$-factor compared to linear equalization by ∼3.2 dB and by ∼1 dB compared to IVSTF-NLE at 1000 km of transmission. It seems that self-phase modulation and inter-subcarrier four-wave mixing crosstalk effects can be combated very well with the introduction of the ANN-based nonlinear activation function. The work presented here, shows improved nonlinearity compensation capability using ANN-NLE at higher signal bit-rate (3.2 dB at 100-Gb/s) compared to [9], where 3 dB improvement between ANN-NLE and linear equalization (i.e., without [w/o] NLE) was estimated at 80-Gb/s of CO-OFDM transmission. Future work will consider the performance of such NLEs in high-capacity wavelength-division multiplexing long-haul systems for inter-channel nonlinearities mitigation.

## 4. CONCLUSION

A novel low-complexity ANN-based NLE has been proposed for CO-OFDM systems. ANN-NLE proved to be a robust nonlinearity DSP technique for up to 100-Gb/s 16 QAM CO-OFDM systems. For 100-Gb/s transmission at 1000 km of uncompensated link, ANN-NLE outperforms in terms of $Q$-factor, linear equalization and IVSTF-NLE by ∼3.2 dB and ∼1 dB, respectively. This work should trigger the implementation of nonlinear ANN-based equalizers in next generation high-capacity core networks.

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