Physical Meaning of an Induction Machine Dynamic Model

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Abstract— The presented paper summarizes the approaches to dynamic description of an induction machine and presents them from the view point of the modern theory of dynamic systems. Great impact has the selection of the particular state variables. It decides about the mutual dependency of calculated values. Despite of the fact that it is still a correct description of the same system, it influences the design of control structures. Only the matrix form underlines clearly the particular differences and shows what combinations of state variables are eligible to be employed in hierarchical control structures.

1. INTRODUCTION

The dynamic model of an induction machine is well known, but it covers still its secrets. Depending on the author different decisions are made, what state variables will be used. However, this happens often without any justification, just according the trained technical experience — gut instinct. Thus, let us investigate here, how the set of equation changes in accordance with selected set of variables.

2. Γ-SHAPED TWO-PHASE DYNAMIC MODEL

Since both the T-shaped model and the Γ-shaped model are designed to describe the same physical object, we can find many similarities. However, the Γ-shaped two-phase model simplifies significantly the description and thus it will be used throughout this article. Consequently, the analysis bellow will be based upon the Γ-equivalent circuit (see Figure 1).

It is the second mostly employed model for induction machines. As shown in [1] it can be used as full equivalent to the most widespread T-shaped dynamic model. From Figure 1, corresponding

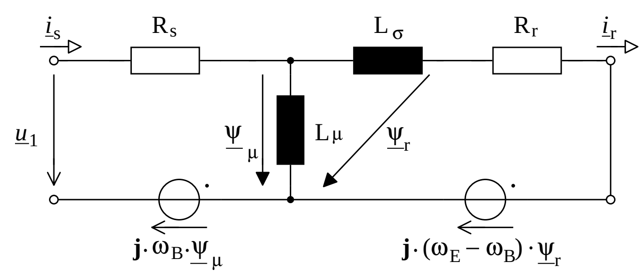

Figure 1: Γ-shaped two-phase induction machine equivalent circuit.

Table 1: Description of the symbols used in the two-phase Γ-shaped equivalent circuit.

Symbol	Unit	Dimension	Description
$\underline{u}_1$	[V]	[kg·m^2·s^{-3}·A^{-1}]	supplying voltage space-vector
$\underline{\psi}_\mu$	[Wb]	[kg·m^2·s^{-2}·A^{-1}]	stator flux space-vector
$\underline{\psi}_\mathrm{r}$	[Wb]	[kg·m^2·s^{-2}·A^{-1}]	rotor flux space-vector
$\underline{i}_\mathrm{s}$	[A]	[A]	stator current space-vector
$\underline{i}_\mathrm{r}$	[A]	[A]	rotor current space-vector
ω_B	[rad·s^{-1}]	[s^{-1}]	angular speed of the reference frame
ω_E	[rad·s^{-1}]	[s^{-1}]	electric angular speed of the rotor
$\mathrm{R_s}$	[Ω]	[kg·m^2·s^{-3}·A^{-2}]	stator resistance
$\mathrm{R_r}$	[Ω]	[kg·m^2·s^{-3}·A^{-2}]	rotor resistance
L_σ	[H]	[kg·m^2·s^{-2}·A^{-2}]	total leakage inductance
L_μ	[H]	[kg·m^2·s^{-2}·A^{-2}]	magnetizing inductance

Equations (1) and (2) can be derived.

$$\dot{\underline{\psi}}_{\mu} - \underline{u}_1 + \mathrm{R_s}\cdot\underline{i}_{\mathrm{s}} + \mathbf{j}\cdot\omega_{\mathrm{B}}\cdot\underline{\psi}_{\mu} = 0 \tag{1}$$

$$-\dot{\underline{\psi}}_{\mathrm{r}} + \mathrm{R_r}\cdot\underline{i}_{\mathrm{r}} + \mathbf{j}\cdot(\omega - \omega_{\mathrm{B}})\cdot\underline{\psi}_{\mathrm{r}} = 0 \tag{2}$$

Each and every of the particular variables and parameters appearing in Figure 1 are listed with its description Table 1. Among all of them $\underline{\psi}_{\mu}$, $\underline{\psi}_{\mathrm{r}}$, $\underline{i}_{\mathrm{s}}$ and $\underline{i}_{\mathrm{r}}$ can be regarded as state variables (ω_{E} is considered to be an input variable in this context, even though it is also a variable related to energy accumulating elements).

3. DEGREE OF FREEDOM IN SELECTION OF STATE-VARIABLE COMBINATION

The selection of the state variables can be defined as a combination, i.e., in particular, selecting two members from a grouping of four (stator and rotor current, stator and rotor flux), such that the order of selection does not matter. Thus there exist $n_{\mathrm{svc}} = 6$ variants.

$$n = 4 \tag{3}$$

$$k = 2 \tag{4}$$

$$n_{\mathrm{svc}} = \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{4!}{2!(4-2)!} = 6 \tag{5}$$

The combination $\underline{\psi}_{\mu}$ with $\underline{\psi}_{\mathrm{r}}$ selected in (1), (2) can be named as Flux-Flux system description. The complete list of the combination is to be found in the Table 2.

Table 2: Description of state-variable combinations.

State-Variable Combination	Combination Name
$\underline{\psi}_{\mu}$, $\underline{\psi}_{\mathrm{r}}$	flux-flux system
$\underline{i}_{\mathrm{s}}$, $\underline{i}_{\mathrm{r}}$	current-current system
$\underline{i}_{\mathrm{s}}$, $\underline{\psi}_{\mathrm{r}}$	current-flux system
$\underline{\psi}_{\mu}$, $\underline{i}_{\mathrm{r}}$	flux-current system
$\underline{i}_{\mathrm{s}}$, $\underline{\psi}_{\mu}$	stator system
$\underline{i}_{\mathrm{r}}$, $\underline{\psi}_{\mathrm{r}}$	rotor system

4. MATRIX STATE-SPACE DESCRIPTION

Independently on the particular system, its description can be always reformulated into the following matrix form.

$$\dot{\underline{\mathbf{x}}} = \underline{\mathbf{A}}\cdot\underline{\mathbf{x}} + \underline{\mathbf{B}}\cdot\underline{\mathbf{w}} \tag{6}$$

$$\underline{\mathbf{y}} = \underline{\mathbf{C}}\cdot\underline{\mathbf{x}} + \underline{\mathbf{D}}\cdot\underline{\mathbf{w}} \tag{7}$$

Using the form introduced in (6) and (7) standardizes the description form and simplifies conversion of equation set among different coordinate systems. Let us call the vector build from space-vectors (e.g., $\underline{\mathbf{x}}$) as *complex vector* in order to eliminate repeating of the word vector. In the case of matrix composed of space-vector coefficients (e.g., $\underline{\mathbf{A}}$) let us call it *complex matrix* if necessary. The mathematical symbols can be distinguished easily thank to their underlying. From the transformation formula (8) to the system with coordinates based on other state-variable selection

$$\tilde{\underline{\mathbf{x}}} = \mathbf{T}\cdot\underline{\mathbf{x}} \tag{8}$$

the following transformation equations for the system and input matrices can be derived

$$\tilde{\underline{\mathbf{A}}} = \mathbf{T}\cdot\underline{\mathbf{A}}\cdot\mathbf{T}^{-1} \tag{9}$$

$$\tilde{\underline{\mathbf{B}}} = \mathbf{T}\cdot\underline{\mathbf{B}} \tag{10}$$

5. FLUX-FLUX SYSTEM IN THE COMPLEX MATRIX FORM

If we introduce time constants of stator τ_{s} (13) and rotor τ_{r} (14), then Equations (1) and (2) describing the induction machine as a flux-flux system can be reformulated into the complex matrix form (6) and (7) as follows

$$\underline{\mathbf{A}}_{\Gamma\Psi} = \begin{pmatrix} -\frac{1}{\tau_{\mathrm{s}}} - \frac{1}{\frac{\mathrm{L}_\sigma}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{s}}} - \mathbf{j}\cdot\omega_{\mathrm{B}} & \frac{1}{\frac{\mathrm{L}_\sigma}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{s}}} \\ \frac{1}{\frac{\mathrm{L}_\sigma}{\mathrm{L}_\mu+\mathrm{L}_\sigma}\cdot\tau_{\mathrm{r}}} & -\frac{1}{\frac{\mathrm{L}_\sigma}{\mathrm{L}_\mu+\mathrm{L}_\sigma}\cdot\tau_{\mathrm{r}}} - \mathbf{j}\cdot(\omega_{\mathrm{B}} - \omega_{\mathrm{E}}) \end{pmatrix} \tag{11}$$

$$\underline{\mathbf{B}}_{\Gamma\Psi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{12}$$

where

$$\tau_{\mathrm{s}} = \frac{\mathrm{L}_\mu}{\mathrm{R}_{\mathrm{s}}} \tag{13}$$

$$\tau_{\mathrm{r}} = \frac{\mathrm{L}_\mu + \mathrm{L}_\sigma}{\mathrm{R}_{\mathrm{r}}} \tag{14}$$

By applying the transformation Formulas (9) and (10) we can obtain corresponding matrices for any combination of state variables. We could even create a new state variable as linear combinations of the four already existing ones. The approach of transformation would proceed in an unchanged way in any case.

Table 3: Comparison of system description equation sets for the Γ-shaped equivalent circuit.

Variables	$\underline{\mathbf{A}}$	$\underline{\mathbf{B}}$
$\underline{\psi}_\mu$, $\underline{\psi}_{\mathrm{r}}$	$\begin{pmatrix} -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{s}}} & \frac{1}{\frac{1}{20}\cdot\tau_{\mathrm{s}}} \\ \frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} & -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} + \mathbf{j}\cdot\omega_{\mathrm{E}} \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\underline{i}_{\mathrm{s}}$ $\underline{i}_{\mathrm{r}}$	$\begin{pmatrix} -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{s}}} - \mathbf{j}\cdot 20\cdot\omega_{\mathrm{E}} & -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} + \mathbf{j}\cdot 21\cdot\omega_{\mathrm{E}} \\ -\frac{1}{\frac{1}{20}\cdot\tau_{\mathrm{s}}} - \mathbf{j}\cdot 20\cdot\omega_{\mathrm{E}} & -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} + \mathbf{j}\cdot 21\cdot\omega_{\mathrm{E}} \end{pmatrix}$	$\begin{pmatrix} \frac{21}{\mathrm{L}_\mu} \\ \frac{20}{\mathrm{L}_\mu} \end{pmatrix}$
$\underline{i}_{\mathrm{s}}$ $\underline{\psi}_{\mathrm{r}}$	$\begin{pmatrix} -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{s}}} - \frac{1}{\frac{1}{20}\cdot\tau_{\mathrm{r}}} & \frac{1}{\frac{1}{20}\cdot\mathrm{L}_\mu\cdot\tau_{\mathrm{r}}} - \mathbf{j}\cdot\frac{20}{\mathrm{L}_\mu}\cdot\omega_{\mathrm{E}} \\ \frac{1}{\frac{1}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{r}}} & -\frac{1}{\tau_{\mathrm{r}}} + \mathbf{j}\cdot\omega_{\mathrm{E}} \end{pmatrix}$	$\begin{pmatrix} \frac{21}{\mathrm{L}_\mu} \\ 0 \end{pmatrix}$
$\underline{\psi}_\mu$ $\underline{i}_{\mathrm{r}}$	$\begin{pmatrix} -\frac{1}{\tau_{\mathrm{s}}} & -\frac{1}{\frac{1}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{s}}} \\ -\frac{1}{\frac{1}{20}\cdot\mathrm{L}_\mu\cdot\tau_{\mathrm{s}}} - \mathbf{j}\cdot\frac{20}{\mathrm{L}_\mu}\cdot\omega_{\mathrm{E}} & -\frac{1}{\frac{1}{20}\cdot\tau_{\mathrm{s}}} - \frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} + \mathbf{j}\cdot\omega_{\mathrm{E}} \end{pmatrix}$	$\begin{pmatrix} 1 \\ \frac{20}{\mathrm{L}_\mu} \end{pmatrix}$
$\underline{i}_{\mathrm{s}}$ $\underline{\psi}_\mu$	$\begin{pmatrix} -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{s}}} - \frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} + \mathbf{j}\cdot\omega_{\mathrm{E}} & \frac{1}{\frac{1}{21}\cdot\mathrm{L}_\mu\cdot\tau_{\mathrm{r}}} - \mathbf{j}\cdot\frac{21}{\mathrm{L}_\mu}\cdot\omega_{\mathrm{E}} \\ -\frac{1}{\frac{1}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{s}}} & 0 \end{pmatrix}$	$\begin{pmatrix} \frac{21}{\mathrm{L}_\mu} \\ 1 \end{pmatrix}$
$\underline{i}_{\mathrm{r}}$ $\underline{\psi}_{\mathrm{r}}$	$\begin{pmatrix} -\frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{s}}} - \frac{1}{\frac{1}{21}\cdot\tau_{\mathrm{r}}} & -\frac{1}{\frac{1}{20}\cdot\mathrm{L}_\mu\cdot\tau_{\mathrm{s}}} - \mathbf{j}\cdot\frac{20}{\mathrm{L}_\mu}\cdot\omega_{\mathrm{E}} \\ \frac{1}{\frac{20}{21}\frac{1}{\mathrm{L}_\mu}\cdot\tau_{\mathrm{r}}} & \mathbf{j}\cdot\omega_{\mathrm{E}} \end{pmatrix}$	$\begin{pmatrix} \frac{20}{\mathrm{L}_\mu} \\ 0 \end{pmatrix}$

6. INTERDEPENCES AMONG STATE-VARIABLES AND INPUT VECTORS

In order make the differences between the various combinations of state-variables more visible, let us introduce an assumption about the relation between leakage and magnetizing inductance. That should not be much distant from a typical physical reality. Thus, let us suppose

$$\frac{\mathrm{L}_\sigma}{\mathrm{L}_\mu} \stackrel{!}{=} \frac{1}{20} \quad \text{and further} \quad \omega_{\mathrm{B}} = 0 \tag{15}$$

Since the influence of rotating coordinates is symmetrical and independent on selected state-variables let us ignore the reference speed as well. The complex matrices resulting from the assumptions (15) are to be found in the Table 3. We can split the results into two groups according to the number of direct links between the input voltage towards to the state variables. This is depicted in the Figures 2 and 3.

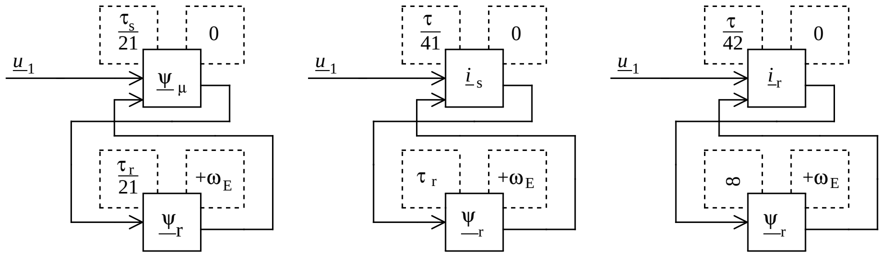

Figure 2: State-variable combinations with one direct link from the input voltage.

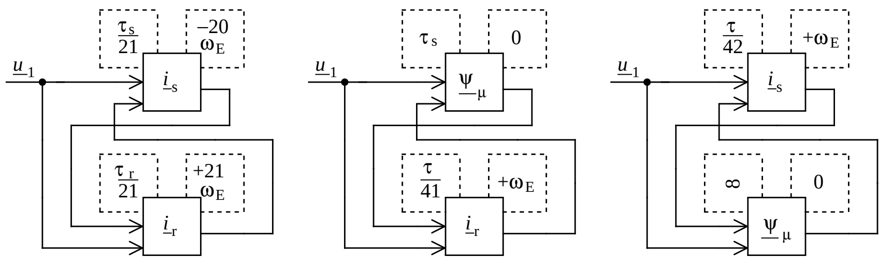

Figure 3: State-variable combinations with two direct links from the input voltage.

7. CONCLUSION

From the Figures 2 and 3 it is obvious, all combination containing the rotor flux $\underline{\psi}_{\mathrm{r}}$ feature one direct link between the input and state variable. In this case, the second selected variable then in the particular combination is controlled indirectly only via the state variable that has a direct link to the input voltage. Thus, for a hierarchical controller design [2] merely these combinations in the Figure 2 are convenient. On the other hand, the lather three combinations in the Figure 3 are rather suitable for a direct torque control method [3]. It is to be noted, the time constant for particular variables varies strongly depending on its counterpart (see the top left dashed boxes).

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