An Formation Algorithm of the Synthetic Aperture in an Automotive Radar with Use of the MUSIC Algorithm

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Abstract— This paper proposes an implementation of MUSIC algorithm in order to increase a range resolution after processing of synthetic aperture radar (SAR) image. SAR system operates in small ranges and uses a frequency modulated continuous wave (FMCW) in radars, offers a low-cost, low power, small earth observation system that can be placed on different types of mobile platforms. SAR operates in the side looking mode. Modified wavenumber domain algorithm (WDA) is used. WDA gives a clear sinc function on both coordinates of image. Post-processing is performed in cut-out piece of focused image after WDA. Simulation conditions are reached at the expense of available equipment intended for experimental verification.

1. INTRODUCTION

Since the middle XX century a synthetic aperture radar (SAR) is an important tool to obtain a radar image of terrain with objects located on it. The radar image is close in quality to the aerial photographs over long distances regardless of the time of day and weather conditions [1, 2]. In recent years the interest in the development of SAR systems operating on small distances has been increased [2–6]. These systems allow solving many problems in the field of earth observation, such as: monitoring flows and flood water or areas in which gas pipes and electrical power lines are buried. Operation in small ranges extends the list of possible platforms which can be used to arrange on them this system including unmanned drones, vehicles [6–10]. Moreover, this system can be applied for automatic control of the vehicle where it is important to monitor the side looking area during the movement.

In classic mode of operation the SAR use pulse signals. The use of the pulse radar by small distances imposes a number of considerable limitations. One of them is locking of receiver during transmission time of radiated signal and as a result the formation of "dead zone" in the range direction. Further the signal processing requires enough fast ADC which leads to the increase of system cost. Systems with FMCW (Frequency Modulated Continuous Wave) signal require less high-speed ADC, consumes less power and removes "dead zones". Thus the combination of FMCW radars and SAR systems allows implementing compact, low-power, low-cost earth observation systems which can be placed on different types of mobile platforms.

The problem of increase of the azimuth resolution can be overcome by applying the synthetic aperture radar processing techniques. Range resolution depends on the bandwidth of the transmitted signal and therefore is limited by radar hardware components. The most intuitive way in order to enhance the range resolution the probe signal bandwidth can be increased or the interferometric wavenumber shift principle can be used. At the same time it is necessary to apply two different acquisitions with different viewing angles [11]. In addition there is alternative way based on the modern spectral estimation techniques like MUSIC (Multiple Signal Classification) which was first proposed to determine the angle of the arrival of signal [12]. Direct application of this algorithm to the problem has a number of drawbacks. Firstly it is desirable to know a priori the number of echo signals. Secondly the method requires high computational costs. Thirdly the MUSIC response does not give a true estimation of the power spectral density, i.e., the algorithm will give the exact position of the target but does not retain an information about power of signal reflected from the target. This paper proposes an algorithm for SAR data processing using MUSIC method implemented on the basis of the finished image obtained by Wavenumber Domain Algorithm (WDA) at the expense of the post-processing of specified cut-out of window within the image.

2. DATA MODEL

The radar signal at the out-put of FMCW transmitter can be presented as:

$$S(t) = \exp\left[j2\pi f_c t + j\pi\beta t^2\right], \tag{1}$$

where f_c is a carrier frequency; $\beta = \Delta F/\text{PRI}$ is the frequency modulation rate; ΔF is a signal bandwidth; PRI is a pulse repetition interval; t is a signal time, $0 \le t \le \text{PRI}$.

A side look mode of the radar mounted on a car platform is considered. The geometry of such SAR system is represented in Fig. 1. The reflected signal from the point target can be presented as:

$$S_r(t, t_{az}) = \exp\left[j2\pi f_c\left(t - 2R(t, t_{az})/c\right) + j\pi\beta\left(t - 2R(t, t_{az})/c\right)^2\right] \tag{2}$$

where $-L_{\text{SAR}}/2V \le t_{az} \le L_{\text{SAR}}/2V$; L_{SAR} is an aperture length; c is the light speed; V is a velocity of the car; $R(t, t_{az})$ is range of target which depends on both azimuth time and range time: $R(t, t_{az}) = \sqrt{R_0^2 + V^2(t + t_{az})^2}$; where R_0 is minimal distance between radar and target.

After multiplication of the received signal with the reference signal (complex conjugate version of transmitted signal) a beat signal is arisen. The beat signal can be written as:

$$S(t, t_{az}) = \exp\left[-j\beta\frac{4\pi R(t, t_{az})}{c}t\right] \times \exp\left[-\frac{j4\pi f_c R(t, t_{az})}{c}\right] \times \exp\left[\frac{j4\pi\gamma R^2(t, t_{az})}{c^2}\right]. \tag{3}$$

The first exponent in (3) determines the range to the target, the second exponent is a Doppler shift in the spectrum along the azimuth, and the third exponent is called the residual video phase (RVP) [5]. For processing of the beat signal the method WDA can be used [5]. A block diagram of this algorithm is shown in Fig. 2. All formulas describing the algorithm are presented in [5].

The formula which describes the final image for a single target is shown below (4) where γ is a reflection coefficient; f_0 is a beat frequency; t_{az} is a azimuth time by which the distance between radar and target is minimal:

$$S(f_r, t_{az}) = \gamma \operatorname{sinc}(f_r - f_0) \operatorname{sinc}(t_{az} - t_0). \tag{4}$$

The simulation result of the considered algorithm for the two point targets with coordinates in range and azimuth [23 m, 5.5 m] for the first target and [26 m, 5.5 m] for the second target is shown in Fig. 4(a). Simulation parameters are presented in Table 1. These parameters are taken according to available equipment for the experimental proof. Simulation results show that targets can't be resolved in range direction due to the fact that the range resolution which is defined as $\rho_x = c/2\Delta F = 3.75\,\text{m}$ and it is greater than the distance between targets. To solve this problem we have performed the post-processing of the final image using the method MUSIC described below.

3. MUSIC POST-PROCESSING

As previously mentioned this method has been proposed for estimation of the angle of arrival of the signal which is generally described in the following way. Let's assume that there is a parameter

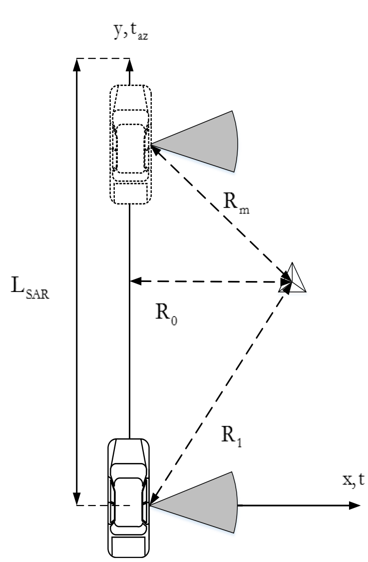

Figure 1: The geometry of the SAR system.

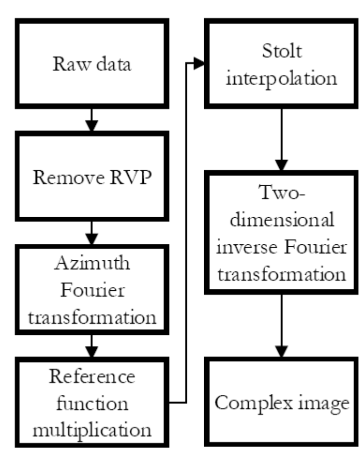

Figure 2: Block diagram of the WDA algorithm.

Table 1: System parameters.

Sampling frequency	200 kHz
Carrier frequency	10 GHz
Pulse repetition interval	320 µs
Signal bandwidth	40 MHz
Car velocity	11.1 m/s
Aperture length	11.1 m

θ which has to be evaluated in the input array $\mathbf{X}$. Data model with L different values of θ at the input of such system can be presented as:

$$\mathbf{X} = \mathbf{A} \cdot \mathbf{S} + \mathbf{W}, \tag{5}$$

where $\mathbf{A} = (\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \ldots\ \mathbf{a}(\theta_L))$ are directing vectors which are corresponded to L values and described the signal shape depending on θ_L (for the frequency estimation $\mathbf{a}(\theta_k) = \exp(j \cdot 2\pi \cdot \theta_L \cdot t)$), $\mathbf{S} = (s_1\ s_2\ \ldots\ s_L)$ is an array of complex envelopes of the signal, $\mathbf{W}$ is the white Gaussian noise.

Before calculation of the correlation matrix for the signal model two main assumptions are done:

- signal and noise must be uncorrelated;
- full rank of autocorrelation matrix of the signal must be greater than the number of L.

After calculation of the correlation matrix $\mathbf{R} = E\{\mathbf{X}\mathbf{X}^H\}$ a singular decomposition of the matrix gives a matrix of the eigenvectors $\mathbf{V}$ where the first L columns correspond to the signal subspace, the remaining T-L columns where T is the size of the array $\mathbf{X}$ correspond to the noise. On the basis of the orthogonality property of the noise and signal the noise eigenvectors are used to obtain a final function (6) for estimation values θ:

$$\mathbf{P}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \cdot \mathbf{V}_{T\text{-}L} \cdot \mathbf{V}_{T\text{-}L}^H \cdot \mathbf{a}(\theta)} \tag{6}$$

Accordingly, the output should give L peaks with the corresponding values θ_L.

Now we should consider this method as applied to the SAR system. After the imaging it is necessary to obtain a more detailed picture within a given piece of an image of size $N \times M$. At first it is essential to estimate the correlation matrix of the signal in a given window. Before evaluation of the correlation matrix we must restore its rank because just one measurement was done [11]. A method of Spatial Smoothing is used to restore rank of the matrix. This method is represented in Fig. 3.

Matrix $\mathbf{X}(N, M)$ in Fig. 3 represents the result of 2D inverse Fourier transform of window of the function $\mathbf{S}(N, M)$ by the use of (4). Further a window is formed with size $m_1 < N$ and $m_2 < M$ and then every possible position K is scanned by this window. For each position an one-dimensional vector is formed as shown in Fig. 3 and then the correlation matrix is calculated according to (7):

$$K = (N - m_1 + 1) \cdot (M - m_2 + 1)\,, \quad \mathbf{R} = \sum_i^K \mathbf{X}_i \cdot \mathbf{X}_i^H \tag{7}$$

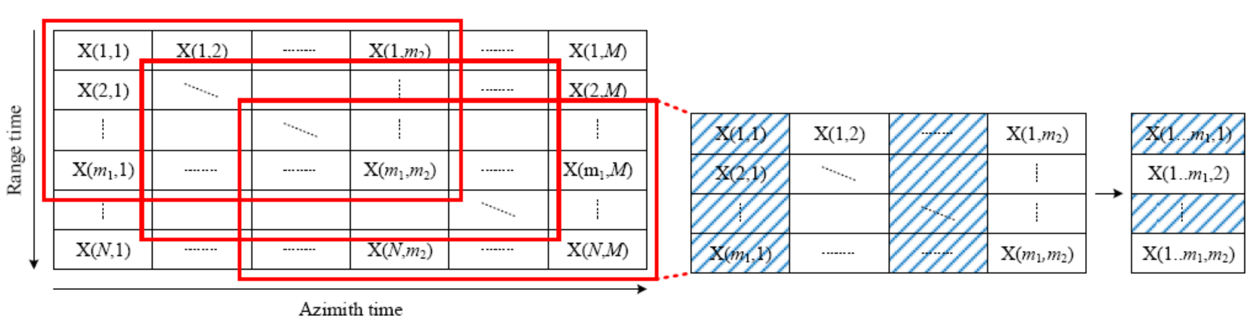

Figure 3: The method of Spatial Smoothing over 2D SAR data.

As mentioned above an array of eigenvectors $\mathbf{V}_n = \mathbf{V}(m_1 \cdot m_2,\, m_1 \cdot m_2 - 2L)$ corresponding to the noise subspace is formed after the singular decomposition of the matrix. Number of targets L can be estimated by the methods AIC, MDL [13]. In addition according to [14] the vectors $\mathbf{V}(m_1 \cdot m_2,\, m_1 \cdot m_2 - p_1 p_2)$ accurately refer to the noise subspace where p_1 and p_2 are all possible positions of the windows along the range and the azimuth, i.e., the estimation of the number of targets is not required.

Before calculation of the final function of MUSIC an array of the directing vectors are formed as:

$$\mathbf{a}\left(f_0, t_0\right) = \left[1 \; e^{jf_0} \;\; \ldots \;\; e^{jf_0(m_1-1)} \; e^{jt_0} \;\; \ldots \;\; e^{jf_0(m_1-1)} \; e^{jt_0(m_2-1)}\right] \tag{8}$$

Accordingly if it is necessary to keep the scale of the cut window, there is a need to form the $N \times M$ directing vectors where each vector will have its own frequency f_0 over range and azimuth delay t_0. The MUSIC finite function is calculated as:

$$P\left(f_0, t_0\right) = \frac{1}{\mathbf{a}^H\left(f_0, t_0\right) \cdot \mathbf{V}_n \cdot \mathbf{V}_n^H \cdot \mathbf{a}\left(f_0, t_0\right)} \tag{9}$$

Then to obtain the final image like Fig. 3 but in reversed order it is essential to form a matrix of size $N \times M$ from one-dimensional vector $P(f_0, t_0)$. The results of simulation with the same system parameters before and after the post-processing by the MUSIC method are shown in Fig. 4.

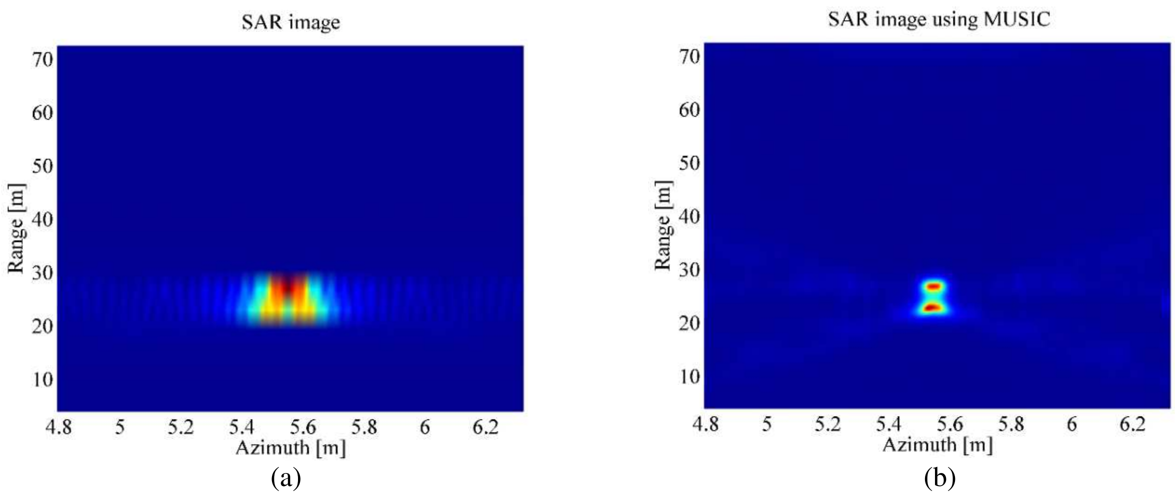

Figure 4: SAR image (a) before and (b) after the image post-processing using the MUSIC method.

4. CONCLUSION AND FUTURE WORK

The post-processing of the final image obtained by the synthetic aperture using the MUSIC method provides a more accurate picture and increases the range resolution more than two times in comparison with the original image.

One of the drawbacks of the MUSIC method is a disproportion of the power of signal reflected from the target to the output function of the method. Further researches will take into account the different reflection coefficients to obtain an accurate picture for 3-dimensional targets with a complex shape. In addition the simulation results show that it is not necessary to estimate the number of targets separately by the use of the Spatial Smoothing method and as a result a requirement to have a priori information about the number of targets is eliminated. According to [14] this statement is true only for the cases when the window size is approximately equaled $m_1 \approx 2N/3$ and $m_2 \approx 2M/3$. Therefore in the future it is necessary to assess more accurately a formula for calculation of the optimal window size.

To compare results an experiment by the conditions presented in Table 1 is planned to perform in a following research. Taking into consideration the high computational costs simplified algorithms will also be developed to get an accurate picture in real time. In addition it is planned to consider features of the image processing in the slanting SAR mode for imaging in other sector of view.

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