Characteristics of Femtosecond Pulse in Silicon Nanowire Embedded Photonic Crystal Fiber: Variational Approach

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Abstract— In this paper, we investigate the propagation characteristics of femtosecond pulses using the proposed silicon nanowire embedded photonic crystal fiber (SN-PCF). The pulse propagation in SN-PCF is governed by the well known higher order nonlinear Schrödinger (HNLS) equation. Using variational analysis, we study the propagation characteristics of an hyperbolic secant pulse, namely, pulse amplitude, width, chirp and phase.

1. INTRODUCTION

Thanks to the advancement in nanotechnology, the realization of nanowire in fiber optics called photonic nanowire (PN) has become a reality [1]. PN with core diameter less than 1 μm, which is lesser than the optical wavelength, has attracted a significant attention owing to the myriad of interesting optical properties such as tight mode optical confinement, large waveguide dispersion, high effective nonlinearity, etc. [2]. In addition, the dispersion due to this waveguide can be changed by varying the core diameter, ultimately, facilitating positive, negative and zero dispersion operations [3]. This waveguide with tailored dispersion finds applications in soliton-self compression [4], dispersion compensation, supercontinuum generation [5–7] and biosensing [8] due to its smaller core diameter.

In recent times, PNs have been fabricated from a variety of high-low index glasses such as silica glass [6, 9], chalcogenide glass [10] and silicon [11, 12]. However, of these materials, silicon is highly preferred to the rest for the reasons mentioned in what follows. First, silicon exhibits excellent transmission properties in the near infra-red wavelength range and, in particular, in the communication window. When compared to silica, silicon possesses a large nonlinearity by four order [13]. Recently, single mode operation, group velocity and waveguide dispersion have been studied experimentally using silicon nanowires [14]. Analysis of modal ellipticity and modal hybridness in a silicon nanowire has been presented [15]. Silicon waveguides find many applications in Raman lasers [16], supercontinuum generation [17, 18], all-optical regeneration [18] and pulse compression [4]. It is observed that the PN provides enhanced optical properties when embedded with a photonic crystal fiber (PCF) and the resulting waveguide is known as nanowire embedded PCF or PCF-photonic nanowire (PCF-NW) [19]. In this work, we use a silicon nanowire embedded PCF (SN-PCF) as a medium of ultrashort pulse propagation and study the dynamics of pulse parameters.

The propagation of intense light pulses in a SN-PCF can induce a host of nonlinear phenomena, such as third-order electronic (Kerr) and phonon (Raman) nonlinearities, self-steepening and the two-photon absorption (TPA) process. TPA, in turn, creates free carriers whose presence leads to additional losses through free-carrier absorption (FCA) and refractive-index changes through free-carrier dispersion (FCD). The combined effects of nonlinear phenomena and the fiber chromatic dispersion lead to many interesting dynamical processes which are difficult to understand in terms of the original pulse field, but can be easily understood by applying a collective variable (CV) approach. For example, the dynamics of the propagation of light pulses in an optical fiber can be completely described by a field, say U, which is a solution of corresponding nonlinear partial differential equation. However, for some nonlinear systems, exact solutions are not available and these systems are,in general, termed as nonintegrable systems. Hence, to study the pulse dynamics in these systems, we introduce a set of variables associated with these systems called collective variables.

In this paper, we adopt the CV approach to study the dynamics of the pulse in the proposed SN-PCF. In general, collective variables (CVs) may represent the amplitude of a pulse, its temporal position, the pulse width, and so on. The number of CVs that can be introduced into the system is usually determined by the physics under consideration. Then one must derive a transformation which allows us to express the original field equation in terms of CVs. In other words, one must derive the equations of motion for the CVs, whose solutions will explicitly yield the complete dynamics of the nonlinear localized modes under consideration. In this paper, we consider higher-order dispersions, linear and nonlinear losses like TPA, FCA, and FCD and the nonlinear effects, namely, Kerr effect, self-steepening and Raman effect because of its resonant nature in silicon.

The paper is organized as follows. In Section 2, we model the pulse propagation in SN-PCF by modified higher-order nonlinear Schrödinger equation. We derive the propagation characteristics of an hyperbolic secant pulse, namely, pulse amplitude, temporal position, width, chirp, frequency and phase in Section 3. Finally, we summarize the findings in Section 4.

2. THEORETICAL MODEL

The propagation of an intense optical pulse (fs) through a SN-PCF is governed by the extended nonlinear Schrödinger equation with self-steepening (SS) and stimulated Raman scattering (SRS) [20]

$$\frac{\partial U}{\partial z}+i\frac{\beta_2}{2}\frac{\partial^2 U}{\partial t^2}-\frac{\beta_3}{6}\frac{\partial^3 U}{\partial t^3}=i\gamma|U|^2U-\Gamma|U|^2U-\frac{\alpha}{2}U-\frac{\sigma}{2}N_cU-\gamma_s\frac{\partial}{\partial t}(|U|^2U)-i\gamma_R U\frac{\partial}{\partial t}|U|^2, \quad (1)$$

where U, β_2, β_3, γ, Γ, α, σ and N_c represent the slowly varying field amplitude, second-order dispersion, third-order dispersion, nonlinear Kerr coefficient, TPA coefficient, linear loss parameter, FCA coefficient, and free carrier density, respectively. Since the TPA-induced free-carrier density N_c has a profound effect on the pulse amplitude, the dynamic nature of N_c is included by solving the rate equation [21],

$$\frac{dN_c}{dt}=\frac{\beta_{TPA}}{2h\nu_0A_{\mathit{eff}}^2}|U(z,t)|^4-\frac{N_c}{\tau_c}, \quad (2)$$

where τ_c is the carrier life time, $h\nu_0$ is the photon energy at the incident wavelength, A_{eff} is the effective mode area, and $\beta_{TPA}=2\Gamma A_{\mathit{eff}}$ is the usual TPA parameter.

For a short optical pulse ($t_0\ll\tau_c$), one can ignore τ_c, as carriers do not have enough time to recombine over the pulse duration. In this situation, it is possible to solve Eq. (2) analytically for a given optical field. We assume that the hyperbolic secant shape of the input pulse remains unchanged during propagation but allows its parameter to evolve with the propagation distance z. In this case, a suitable form of the optical field is

$$U(z,t)=x_1(z)sech\left[\frac{t-x_2(z)}{x_3(z)}\right]\exp\left[i\frac{x_4(z)(t-x_2(z))^2}{2}-ix_5(z)(t-x_2(z))+ix_6(z)\right], \quad (3)$$

where x_1, x_2, x_3, x_4, x_5 and x_6 represent the pulse amplitude, temporal position, width, chirp, frequency and phase, respectively and all of them vary with z. For this pulse shape, we can solve Eq. (2) analytically, and the carrier density is found to be

$$N_c=\left[\frac{\beta_{TPA}x_1^4x_3}{2h\nu_0a_{\mathit{eff}}^2}\right]\left[\frac{2}{3}+\tanh\left(\frac{t-x_2}{x_3}\right)-\frac{1}{3}\tanh^3\left(\frac{t-x_2}{x_3}\right)\right] \quad (4)$$

3. DYNAMICS OF PULSE PARAMETERS

To solve Eq. (1) with the variational technique, we first find the Lagrangian and the Rayleigh dissipation function (RDF) associated with it. They are given by

$$\begin{aligned} L &= \frac{1}{2}\left[UU_z^*-U^*U_z\right]+i\frac{\beta_2}{2}|U_t|^2-\frac{\beta_3}{6}\left[U_t^*U_{tt}-U_tU_{tt}^*\right]+i\frac{\gamma}{2}|U_t|^4 && (5)\\ R &= \Gamma\left[UU_z^*-U^*U_z\right]|U|^2+\frac{\alpha}{2}\left[UU_z^*-U^*U_z\right]+\frac{1}{2}\sigma N_c\left[UU_z^*-U^*U_z\right]\\ &\quad+\gamma_s\left[U_z^*\frac{\partial|U|^2U}{\partial t}-U_z\frac{\partial|U|^2U^*}{\partial t}\right]+i\gamma_R\frac{\partial|U|^2}{\partial t}\left[UU_z^*+U^*U_z\right] && (6)\end{aligned}$$

The reduced form of Lagrangian and RDF is obtained by integrating them over time [22]:

$$L_g=\int_{-\infty}^{\infty}Ldt \quad (7)$$

and

$$R_g = \int_{-\infty}^{\infty} R dt \tag{8}$$

With the help of Eqs. (3)–(7), we obtain the following explicit expressions of L_g and R_g:

$$\begin{aligned} L_g &= \frac{i\beta_2 x_1^2}{3x_3}\left[1 + x_3^2\left(\frac{x_3^2 x_4^2 \pi^2}{4} + 3x_5^2\right)\right] + \frac{i\beta_3 x_1^2}{x_3 x_5}\left[\frac{1}{x_3^2} + x_5^2 + \frac{1}{4}x_3^2\pi^2\right] + \\ &\quad -ix_1^2\left[2x_3x_5\dot{x_2} - 2x_3\dot{x_6} - \frac{x_3^3\pi^2\dot{x_4}}{12}\right] + i\frac{2}{3}\gamma x_1^4 x_3 \end{aligned} \tag{9}$$

Using the given ansatz (19), the reduced RDF can be obtained as

$$\begin{aligned} R_g &= \left[-2ix_3x_5\dot{x_2} - \frac{i}{12}\pi^2 x_3^3\dot{x_4} - 2ix_3\dot{x_6}\right]\left[\frac{4}{3}\Gamma x_1^4 + \alpha x_1^2 + \frac{2}{3}\sigma\left(\frac{\beta x_1^6 x_3}{2h\nu_0 a_{eff}^2}\right)\right] + \frac{2}{3}ix_3^3\Gamma x_1^4\dot{x_4} \\ &\quad -2i\gamma_s x_1^3 x_3\left[\frac{2x_1x_4\dot{x_2}}{3} + x_1\dot{x_5} + \frac{4}{3}x_5\dot{x_1} + \frac{x_1x_5\dot{x_3}}{3x_3}\right] - \frac{16i\gamma_R x_1^4\dot{x_2}}{15x_3} \end{aligned} \tag{10}$$

The Euler-Lagrange equation in the form

$$\frac{d}{dt}\left(\frac{\partial L_g}{\partial q_z}\right) - \frac{\partial L_g}{\partial q} + \frac{\partial R_g}{\partial q_z} = 0, \tag{11}$$

where $q = x_1$, x_2, x_3, x_4, x_5 and x_6 and the suffix z indicates the corresponding derivatives. Applying the generalized EulerLagrange equation, we get a set of six coupled ordinary differential equations for each q represent the dynamics of pulse parameters can be obtained as follows:

$$\dot{x}_1 = \frac{\beta_2}{2}x_1x_4 + \frac{\beta_3}{2}x_1x_4x_5 - 2\Gamma x_1^3\left[\frac{1}{\pi^2} + \frac{1}{3}\right] - \frac{\alpha}{2}x_1 - \frac{\beta x_1^5 x_3\sigma}{6h\nu_0 a^2} \tag{12}$$

$$\dot{x}_2 = \beta_2 x_5 + \frac{\beta_3}{24}x_3^2x_4^2\pi^2 + \frac{\beta_3}{2}\left[x_5^2 + \frac{1}{3x_3^2}\right] + \gamma_s x_1^2 \tag{13}$$

$$\dot{x}_3 = -\beta_2 x_3x_4 - \beta_3 x_3x_4x_5 + \frac{4\Gamma x_1^2 x_3}{\pi^2} \tag{14}$$

$$\dot{x}_4 = \beta_2\left[x_4^2 - \frac{4}{x_3^4\pi^2}\right] + \beta_3 x_5\left[x_4^2 - \frac{4}{x_3^4\pi^2}\right] - \frac{4\gamma x_1^2}{\pi^2 x_3^2} - \frac{4\gamma_s x_1^2 x_5}{x_3^2\pi^2} \tag{15}$$

$$\dot{x}_5 = \frac{-2\gamma_s x_1^3 x_4}{3} - \frac{8\gamma_R x_1^3}{15x_3^2} \tag{16}$$

$$\dot{x}_6 = \beta_2\left[\frac{2}{3x_3^2} - \frac{x_5^2}{2}\right] - \frac{\beta_3 x_5}{3}\left[x_5^2 - \frac{x_3^2x_4^2\pi^2}{24}\right] + \frac{5\gamma x_1^2}{6} - \frac{\gamma_s x_1^2 x_5}{6} \tag{17}$$

The variational Eqs. (12)–(17) show clearly how the pulse parameters change during the propagation inside a SN-PCF and how they are coupled with each other. From the above equations, it is very clear that the linear and nonlinear effects affect a particular pulse parameter. For instance, pulse width in Eq. (14) is affected directly not only by the dispersion parameter and TPA but also by the TOD. In general, dispersion may lead to pulse broadening or compression depending on the sign of β_2. On the other hand, TPA always results in pulse broadening. But, TOD creates asymmetrical pulse broadening in the pulse. This TOD affects all the parameters except frequency of the pulse which is evident from the above equations of motions. The self-steepening affects the position, chirp, frequency and phase of the pulse. The frequency of the pulse changes due to the the Raman term as is evident in Eq. (16).

4. CONCLUSION

In this work, we have successfully derived a set of equations of motions involving various pulse parameters, namely, amplitude, temporal position, width, chirp, frequency and phase, for the modified NLSE by adopting collective variable approach. From these equations, one could easily understand the influence of all the important physical effects over the pulse dynamics.

ACKNOWLEDGMENT

KSN wishes to thank CSIR [No. 03(1264)/12/EMR-II] and DST [No. SR/FTP/PS-66/2009], Government of India, for the financial support through the sponsored projects. SS wishes to thank DST [No. SR/S2/LOP-0014/2012], Government of India, for the financial support through the sponsored project.

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