Secondary Instabilities of Steady Stationary Solution in Wide-aperture Lasers with Negative Detuning

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Abstract— Pattern formation and the evolution of complexity in spatially extended dynamical systems has been the subject of many investigations in widely diverse branches of science. In nonlinear optics, such studies have been motivated by the goal of developing a general understanding to describe the emergence of periodic and quasiperiodic patterns, vortices or spatial solitons using various nonlinear systems as well as gas, solid-state and semiconductor lasers. These systems are very complex devices having a rich spatiotemporal dynamics. The control parameter that determines the mechanism, which give rise to the transverse structure formation, is the Fresnel number.

In this work, we report on numerical and analytical investigations of the wave instability occurring for negative detuning (cavities tuned above resonance) in lasers class B. In that case, relations amongthe decay rate of polarization ,the decay rate of population difference and the cavity rate were $\gamma_\perp \gg \gamma_\| \gg \kappa$. We use the Maxwell-Bloch equations to describe the dynamics of transverse patterns in single longitudinal mode lasers, where the active medium is composed of two level atoms inside a cavity with flat mirrors. An important parameter for determining the behavior of the system is the cavity detuning of field frequency from the atomic resonance transition frequency.

With increase of the pump level the homogeneous solution becomes unstable against spatiotemporal perturbations with a finite wavelength. We have numerically simulated the dynamics of laser field above the second laser threshold in one- and two-dimensional cases. In the result of stability loss of steady solution, the regime that evolves in the system consists of a sequence of switches between traveling wave solutions with different wave numbers. The laser intensity presents the areas of nearly constant values which interrupted by bursts of large amplitude oscillations. The sequence of wave numbers can be predicted as perturbation with highest growth rate.

We use analytical and numerical approaches to perform the linear stability analysis. We found that the regime of dynamical switches is the result of secondary instabilities. By both methods, the evolution may be predicted with a relatively good precision.

1. INTRODUCTION

Pattern formation and the evolution of complexity in spatially extended dynamical systems has been the subject of many investigations in widely diverse branches of science. In nonlinear optics, such studies have been motivated by the goal of developing a general understanding to describe the emergence of periodic [1] and quasiperiodic patterns [2], vortices [3] or spatial solitons [4] using various nonlinear systems as well as gas [5], solid-state[6] and semiconductor lasers [7]. These systems are very complex devices having a rich spatiotemporal dynamics. The control parameter that determines the mechanism, which gives rise to the transverse structure formation, is the Fresnel number.

In this work, we report on numerical and analytical investigations of the wave instability occurring for negative detuning (cavities tuned above resonance) in lasers class B. In that case, relations among the decay rate of polarization, the decay rate of population difference and the cavity ratewere $\gamma_\perp \gg \gamma_\| \gg \kappa$. The large variety of spatio-temporal dynamics have been observed, including vortex lattices, high ordered structures, turbulence. In addition,class B lasers play a significant role in industrial applications, first, in communications. Therefore, the study of transverse laser field behavior becomes a relevant problem since it allow to determine the spatial coherence and the brilliance of the laser beam.

2. LINEAR ANALYSIS AND SIMULATION

We use the Maxwell-Bloch equations to describe the dynamics of transverse patterns in single longitudinal mode lasers, where the active medium is composed of two level atoms inside a cavity with flat mirrors. This equations are normalized and take into account diffraction in one transverse direction x:

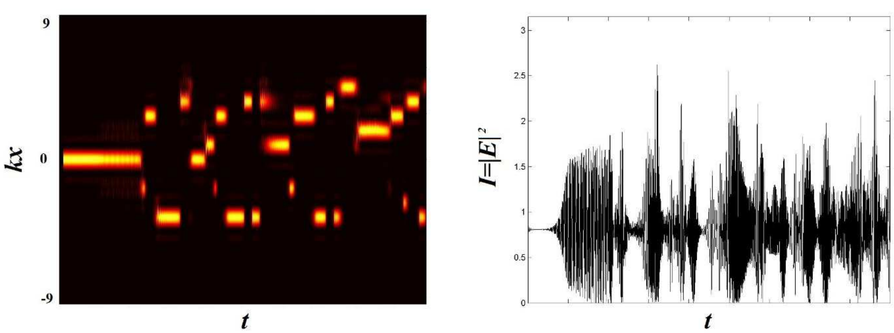
$$\begin{aligned}\frac{\partial E}{\partial t} &= ia\frac{\partial^2 E}{\partial x^2} + \sigma\left(P - E\right),\\ \frac{\partial P}{\partial t} &= -\left(1 + i\delta\right)P + DE,\\ \frac{\partial D}{\partial t} &= -\gamma\left[D - r + \frac{1}{2}\left(E^* P + EP^*\right)\right].\end{aligned} \tag{1}$$

E, P, D are obtained through normalization of the electric field, macroscopic polarization and the population inversion, respectively; $\gamma = \gamma_{||}/\gamma_{\perp}$, $\sigma = k/\gamma_{\perp}$, where k, $\gamma_{\perp}$ and $\gamma_{||}$ are the relaxation rates of field, polarization and inversion, respectively; $\delta = (\omega_{21} - \omega)/\gamma_{\perp}$ is the normalized detuning of field frequency from atomic resonant frequency; $a = c^2/\left(2\omega\gamma_{\perp}d^2\right)$ is diffraction parameter, where$d$is typical spatial size; r is pumping parameter.

An important parameter for determining the behavior of the system is the cavity detuning of field frequency from the atomic resonance transition frequency.

For positive detuning, the model theoretically predicts the off-axis radiation. For negative detuning, it predicts the paraxial radiation. In this work, we consider the dynamics for negative detuning. In this case, the spatially homogeneous solution is observed in the system above the lasing threshold. But, with exceeding of pumping, this mode becomes unstable against spatiotemporal perturbations with a finite wavelength (second lasing threshold).

In the result of stability loss of steady solution, the regime evolving in the system consists of a sequence of switches (Fig. 1) between traveling wave solutions with different wave numbersas described in [8].



(a) (b)

Figure 1: The time evolution of spatial Fourier spectrum (a) and intensity (b) at the parameters of the system: $\sigma = 0.03$, $\gamma = 5 \cdot 10^{-4}$, $\delta = -0.5$, $r = 2$, $a = 0.001$

An example of time dependence of spatial spectrum is presented in Fig. 1(a). The sequence of wave vectors can be predicted using linear stability analysis as perturbation with highest growth rate.

For this purpose, we apply a new analytical approach described in [9]. The wave number and frequency of mode with highest growth rate can be written as

$$q_{uns} = \sqrt[4]{\frac{2\gamma\sigma I_{st}}{\left(1 + \delta^2\right)a^2}}, \tag{2}$$

$$\Omega_{uns} = \sqrt{\frac{4\gamma\sigma I_{st}}{\left(1 + \delta^2\right)}}; \tag{3}$$

Also, we use a numerical approach to perform the linear stability analysis. At first, the system has been linearized around the homogeneous solution. The real part of eigenvalues versus wave number is plotted in Fig. 2. That curve is symmetric for negative wave numbers. The highest growth rate obtained numerically corresponds to the perturbation with the wave number $q_{uns} =$

2.28. According to (2), the analytical value of unstable wave number with the highest growth rate is $q^T_{uns} = 2.44$. As we can see, the difference is small and the results are very close to nonlinear simulation (Fig. 1(a)), where $q = 2.16$. In one-dimension case, we have two same directions for traveling waves, apparently the system selects one of them. Then we linearize the system around the next traveling wave solution with a certain wave number and so on.

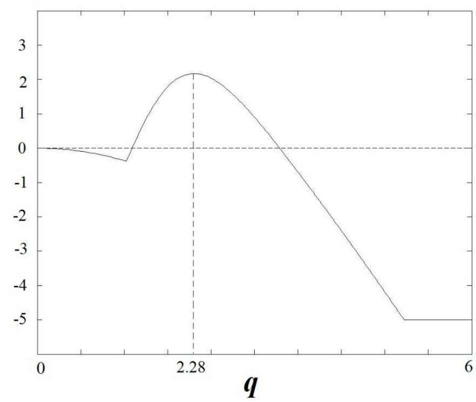


Figure 2: The growth rate of perturbation versus wave number.

Unlike dynamics in class A lasers, this case is more complex. The some noise exists in spatial spectrum and the laser intensity presents irregular large amplitude oscillations for all time evolution. But the sequence of wave numbers is repeated from cycle to cycle.

In two-dimensional simulation, we found that dynamics have not switches, but it is still irregular. In that case, a number of possible wave vector directions with the highest growth rate is infinite. And the ring-shaped image is obtained in the spatial spectrum at a linear stage of perturbation growth (Fig. 3(a)). After that, the optical turbulence takes place as shown in Fig. 3(b).

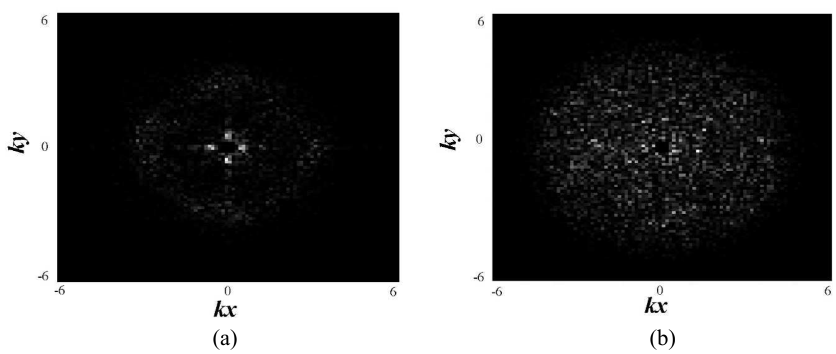


Figure 3: Spatial Fourier spectrum at a linear stage of perturbation growth (a) and optical turbulence (b), both with clipped central component.

3. CONCLUSION

Thus, we found that the regime of dynamical switches is the result of secondary instabilities. By both methods, the evolution may be predicted with a relatively good precision.

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