Investigation of the Free-space Propagation Operator Eigenfunctions in the Near-field Diffraction

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Abstract— Finite transform, matched with free-space optical fields propagation operator and based on beam expansion of plane waves (1D case), has been considered. The spectrum is also band-limited. We demonstrate that this operator is normal, therefore, its eigenfuntions are orthogonal set. We may consider that resulting functions are eigenmodes. We produced the calculations of eigenfunctions where waves propagate at a distance of one wavelength. Elements of some functions are comparable with a half of wavelength. It is allowing to come closer for solving the problem of superresolution in free space.

1. INTRODUCTION

In recent time there are many attempts to overcome diffraction limit have been produced. It can provide the visualization of elements smaller than half of wavelength. The problems of superresolution are solved using meta-materials [1, 2], nanohole array [3, 4], superoscillation lens optical microscope [5], which includes creation the masks for superoscillation field generation [6], and flat dielectric grating reflectors with focusing abilities [7]. The possibilities of superresolution in far-field have been investigated using hyperlenses, metalenses [8], and optical eigenmodes [9, 10]. In work [11], significant progress has been achieved, where size of imaging object is equals to 25 nm. The facilities of subwave focusing have been demonstrated through development of optical needle [12].

In this work we demonstrate possibilities of creating monochromatic light distribution with details comparable with the wavelength using communication modes [13, 14]. To obtain the desired results, we investigate eigenfunctions of limited optical system. Previously, we study eigenfunctions for the limited system of two lenses where Fourier transform (or Hankel transform in case of radial symmetry) plays the role of propagation operator [15–17]. The eigenfunctions are spheroidal functions if the optical system is Cartesian [15]. The distinction of this work consists in free-space propagation in the near-field diffraction rather than through the lenses, therefore, modeling can be done at a small distance from the input plane.

2. GENERAL THEORY

We consider the propagation of one-dimensional light waves in free space on the basis of scalar diffraction theory. According to this theory, propagation equation based on plane wave expansion, is written as follows [18]:

$$F(u) = \int_{-\infty}^{+\infty} f(x) H_z(u-x)dx, \tag{1}$$

where $f(x)$ — input field, $F(u)$ — output field, $H_z(u-x)$ — the kernel of integral transformation, which depends on propagation distance z:

$$H_z(x) = \frac{1}{\lambda} \int_{-\infty}^{+\infty} e^{ikx\alpha} e^{ikz\sqrt{1-\alpha^2}} d\alpha, \tag{2}$$

where λ — light wavelength, k — wavenumber. This expression is inverse Fourier transform, and the integration is performed over the real axis. In case of infinity limits of integration, there is an analytic form of writing this integral using Hankel function of first kind [19]. We observe that if $|\alpha| > 1$ the plane wave is evanescent.

We also note that formula (1) can be rewritten as a convolution of two functions:

$$F = f * H_z. \tag{3}$$

We consider simple optical system consists of input limited aperture and observable domain at some distance z from it (Figure 1).

We consider only the propagation waves (without evanescent), because evanescent waves cannot pass in free-space at a distance more than third part of wavelength [20]. In other words, we take kernel $h_z(x)$ with finite integral limits instead of $H_z(x)$:

$$h_z(x) = \frac{1}{\lambda}\int\limits_{-1}^{1} e^{ikx\alpha} e^{ikz\sqrt{1-\alpha^2}} d\alpha. \tag{4}$$

Since optical system's input domain limited by width of $2a$, formula (1) can be rewritten as follows:

$$F_a(u) = \int\limits_{-a}^{a} f(x) h_z(u-x) dx = \int\limits_{-a}^{a} f_a(x) h_z(u-x) dx, \tag{5}$$

where $f_a(x)$ — function, that turns into zero outside the integration interval, $F_a(u)$ — field corresponding with it. We bound the width of observable domain by same limits: from $-a$ to a. The sets of functions defined on these domains are equal and form Hilbert space with scalar production:

$$(f, g) = \int\limits_{-a}^{a} f(u) g^*(u) du, \tag{6}$$

where asterisk denotes the complex conjugation.

We try to find the functions that remain their form after propagation at a given distance z. This problem reduces itself to propagation operator eigenfunctions calculation problem. To resolve it, we write operator representations of some integrals. The operator Γ performs integral transformation in the formula (1):

$$F = \Gamma\ [f]\,. \tag{7}$$

Γ_a denotes the operator corresponding to the finite transform in the formula (5):

$$F_a = \Gamma_a\,[f] = \Gamma_a\ [f_a]\,. \tag{8}$$

The function defined on input of limited system $f_a(x)$ can be represented as production of unlimited field $f(x)$ and rectangular function rect $(x/2a)$:

$$\mathrm{rect}\left(\frac{x}{2a}\right) = \begin{cases} 1, & |x| \le 2a, \\ 0, & |x| > 2a. \end{cases} \tag{9}$$

Γ is a normal operator. We show that this property is valid for operator Γ_a too. An operator is normal, if it commutes with its adjoint operator. The adjoint operator of Γ_a is operator Γ_a^*:

$$\Gamma_a^*\ [f] = \int\limits_{-a}^{a} f(u) h_z^*(x-u) du. \tag{10}$$

Using the above relations, we can obtain representation of operator $\Gamma_a^* \Gamma_a$:

$$\Gamma_a^*\left(\Gamma_a f\right) = \left[f(x)\mathrm{rect}\frac{x}{2a}\mathrm{rect}\frac{u}{2a} *_x h_z(x)\right](u) *_u h_z^*(u), \tag{11}$$

where the convolution index means performing the integration by specified variable. Due to convolution properties of commutativity and associativity formula (11) can be rewritten as:

$$\left[f(x)\mathrm{rect}\frac{x}{2a}\mathrm{rect}\frac{u}{2a} *_x h_z(x)\right](u) *_u h_z^*(u) = \left[f(x)\mathrm{rect}\frac{x}{2a}\mathrm{rect}\frac{u}{2a} *_x h_z^*(x)\right](u) *_u h_z(u). \tag{12}$$

The right-hand part of (12) is action of operator $\Gamma_a \Gamma_a^*$. Thus, we have found that operator Γ_a commutes with its adjoint operator and, therefore, is normal. Its eigenfunctions corresponding to different eigenvalues is orthogonal. Hence, we can assume that the resulting basis of eigenfunctions is the set of orthogonal eigenmodes.

The problem of eigenvalues and eigenfunctions is written as follows:

$$\Gamma_a \phi_n = \mu_n \phi_n, \tag{13}$$

where μ_n — eigenvalue, ϕ_n — corresponding eigenfunction, $n \ge 1$ — the number of eigenfunction.

3. COMPUTATION OF EIGENVALUES AND EIGENFUNCTIONS

We solve the problem of eigenvalues and eigenfunctions using sampling of integral transformations by rectangle method. The calculation parameters are $\lambda = 1000\,\text{nm}$, $z = 1\lambda$, $a = 5\lambda$.

The image of eigenfunctions has "step" form, as shown at Figure 2. First of 17 eigenvalues is approximately equal in modulus to 1, other values sharply fall to zero (thus, the figure shows only the first 50 eigenvalues). If we approximate arbitrary field by eigenfunctions corresponding to maximum in modulus eigenvalues, then an approximate error will be minimal.

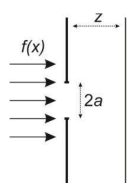

Figure 1: Image of optical system.

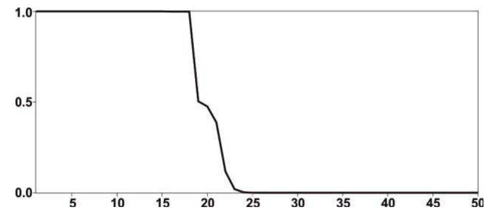

Figure 2: The modules of 50 eigenvalues.

The plots of eigenmodes have shown in Figure 3. It can be seen that fifteenth eigenmode has peaks with FWHM comparable to the half of wavelength. It is a reflection of the fact that the scalar diffraction theory allows approaching to the solution of superresolution problem.

By varying the parameter z, we can establish what kind of fields can pass the given distance without distortion, or, in other words, we can judge the degree of "survival" of optical field.

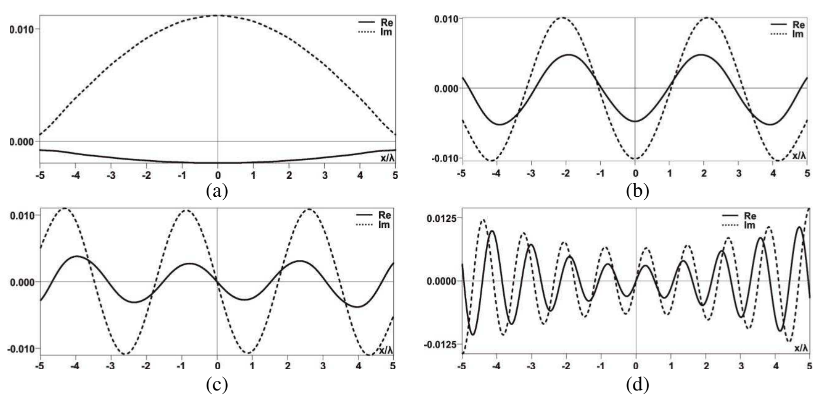

Figure 3: Eigenfunctions: (a) first; (b) second; (c) third; (d) fifteen.

4. CONCLUSION

In this work, we have studied the properties of limited operator of optical signal free-space propagation in near field (at a distance of about one wavelength). We have shown that considered operator is normal and its eigenfunctions are therefore orthogonal.

We perform the calculations of eigenvalues and eigenfunctions of monochromatic optical distribution near-field propagation operator. The beam propagation distance is a parameter of the operator and it significantly changes the set of eigenvalues and eigenfunctions. The eigenvalues modules define corresponding "survival" eigenfunction at a given distance. Thus, resulting calculations allow finding out the number of degrees of freedom in dependence of the distance.

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