Spontaneous Formation of Square Optical Vortex Lattice in a Transverse Section of Broad-area Laser

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Abstract— The square vortex lattice is studied using the full Maxwell-Bloch system. Properties and conditions of formation stationary and nonstationarylattices are investigated. The frequency of vortex oscillations and necessary conditions for these oscillations are obtained.

1. INTRODUCTION

The paper studies the spontaneous formation of square optical vortex lattice (SVL) in broad-area lasers theoretically. The possible motion of vortices is also studied. The nonlinear interaction of numerous transverse modes in broad-area lasers can lead to the spontaneous formation of complex spatiotemporal optical structures including the SVL. The SVL is experimentally observed in the $Nd:YVO_4$ laser [1–3], in a broad-area CO_2 laser operating at a single longitudinal mode [4], in the solid-state laser with optical pumping on a chip $LiNdP_4O_{12}$[5], in a Na_2 laser [6], in the vertical cavity surface emitting laser [7].

2. BASIC EQUATIONS

We studied the formation of SVL both analytically and numerically using the full Maxwell-Bloch model:

$$dE/dt = \sigma\left(P - E\right) + ia\Delta E;\ dP/dt = -\left(1 + i\delta\right)P + DE;\ dD/dt = -\gamma[D - r + 0.5\left(E^*P + EP^*\right)], \tag{1}$$

where Δ is the transverse two-dimensional Laplace operator, E, P are, respectively, the dimensionless electric field and polarization amplitudes, D is the dimensionless population inversion. σ, γ are the dimensionless electric field and inversion decay rates, respectively, r is the pump parameter, δ is the detuning.

3. TRAVELLING WAVE SOLUTION

The simplest solution of Equations (1) is travelling wave solution [8]:

$$E = E_0 \exp\left(i\vec{k}\vec{r} - i\Omega t\right),\ P = P_0 \exp\left(i\vec{k}\vec{r} - i\Omega t\right),\ D = D_0, \tag{2}$$

where $E_0 = \sqrt{r - D_0}$, $P_0 = \frac{(\sigma + i(-\Omega + ak^2))}{\sigma}E_0$, $\Omega = \frac{\delta\sigma + ak^2}{\sigma + 1}$, $D_0 = 1 + \left(\frac{\delta - ak^2}{\sigma + 1}\right)^2$.

Here, $\vec{k}$ is the transverse component of radiation wave vector. In case $\delta \leq 0$, the most attracting solution is the steady-state solution (travelling wave with $k = 0$), which corresponds to the radiation along the optical axis of the resonator. If $\delta > 0$, the travelling wave with $k = \sqrt{\frac{\delta}{a}} \equiv k_0$ is the most attracting solution, which corresponds to the radiation withsome small angle to the optical axis. In this paper, we consider the case $\delta > 0$. Then, solution (2) has the form:

$$E = P = \sqrt{r - 1}\exp\left(i\vec{k}_0\vec{r} - i\delta t\right), \quad D = 1 \tag{3}$$

4. SQUARE VORTEX LATTICE SOLUTION

Transverse plane (x, y) is isotropic. The direction of vector $\vec{k}_0$ is arbitrary. A solution in form of superposition of traveling waves of form (3) is possible. In broad-area lasers,the square vortex latticeis observedas a result of four waves nonlinear interaction. Let us find the solution of Equation (1) in the form of superposition of four waves:

$$E(t,r) = \sum_{j=1}^{4} E_j(t)\exp\left(i\left(\vec{k}_j\vec{r}-\delta t\right)\right),\quad P(t,r)=\sum_{j=1}^{4} P_j(t)\exp\left(i\left(\vec{k}_j\vec{r}-\delta t\right)\right), \tag{4}$$
$$D(t,r) = D_0+\sum_{j=1}^{4} d_{j,j}(t)\exp\left(2i\vec{k}_j\vec{r}\right)+\sum_{j=1}^{4} d_{j,j+1}(t)\exp\left(i\left(\vec{k}_j+\vec{k}_{j+1}\right)\vec{r}\right),$$

where $\vec{k}_1 \uparrow\downarrow \vec{k}_2$, $\vec{k}_3 \uparrow\downarrow \vec{k}_4$, $\vec{k}_1 \perp \vec{k}_3$, $j = 1,2,3,4,1,2,\dots$

After substituting (4) in original system (1) and taking into account only the lowest-order harmonics, original system (1) is reduced to the system of ordinary differential equations describing interaction of four waves:

$$\begin{aligned}
\partial_t E_j &= \sigma\left(P_j - E_j\right)\\
\partial_t P_j &= -P_j + D_0 E_j + E_{j+1}d_{j,j} + E_{j+2}d_{j,j+3} + E_{j+3}d_{j,j+1}\\
\partial_t D_0 &= -\gamma\left[D_0 - r + \frac{1}{2}\sum_{j=1}^{4}\left(E_j^* P_j + E_j P_j^*\right)\right]\\
\partial_t d_{j,j} &= -\gamma\left[d_{j,j} + \frac{1}{2}\left(E_j^* P_{j+2} + E_{j+2}P_j^*\right)\right]\\
\partial_t d_{j,j+1} &= -\gamma\left[d_{j,j+1} + \frac{1}{2}\left(E_j P_{j+3}^* + E_{j+1}P_{j+2}^* + E_{j+2}^* P_{j+1} + E_{j+3}^* P_j\right)\right]
\end{aligned} \tag{5}$$

System (5) has several equilibrium points. Each equilibrium point matches different spatiotemporal structures: single travelling wave, standing wave or SVL. The SVL solution has a form:

$$|E_j|^2 = |P_j|^2 = \frac{r-1}{5},\quad D_0 = 1+\frac{r-1}{5},\quad d_{j,j} = \frac{1-r}{5}\exp\left(i\left(\phi_j-\phi_{j+2}\right)\right),\quad d_{j,j+1}=0 \tag{6}$$
$$(\phi_1+\phi_3)-(\phi_2+\phi_4)=\pi \tag{7}$$

The SVL is the result of interference of four travelling waves with the phase condition (7). The SVL consists of first order optical vortices. Moreover, the SVL solution is stable for the pump parameter r near the threshold value $r \geq r_{th}$. Distance between neighboring vortices could be founded:

$$\lambda_{SVL} = \pi\sqrt{a/\delta} \tag{8}$$

5. NUMERICAL SIMULATION

For numerical simulation of Equation (1), we used the split step Fourier method. Random initial conditions were used. For $\delta > 0$, the nonstationary process of SVL formation was observed beginning with the appearance of numerous random vortices. In sufficiently long time, they occupy the stationary positions and form the ordered structure — SVL (Figs. 1(a), (b)). Relation (8) is in a good agreement with our numerical results. For $r \approx r_{th}$, the SVL is stationary. In the far field, we obtain four bright spots corresponding to four travelling waves (Fig. 1(c)).

The SVL becomes unstable with increase of $r > r_{cr}$. The critical value of pump $r = r_{cr}$ depends on parameters of model (1). In this case, the motion of vortices around equilibrium points is observed (Figs. 2(a), (b)). In the far field, we obtain eight bright spots (Fig. 2(c)).

The radius R of the vortex trajectory depends on the pump value $R \sim sqrt\,(r - r_{cr})$. This type of stability loss is close to the supercritical Hopf bifurcation.

6. ANALYTICAL DESCRIPTION OF OSCILLATIONS IN VORTEX LATTICE

Eight bright spots in the far field (Fig. 2(c)) correspond to two square vortex lattices. As we can see in Fig. 2(c), the secondary SVL have a wavenumber $k = \sqrt{2}k_0$. Using relations (2) and (4), we can find the secondary SVL solution in the analogous form. Superposition of two SVLwith different wave numbers $k = k_0$ and $k = \sqrt{2}k_0$ and appropriate different frequencies gives the oscillating SVL. A frequency of these oscillations was calculated from (2):

$$\omega = \delta/\left(\sigma+1\right) \tag{9}$$

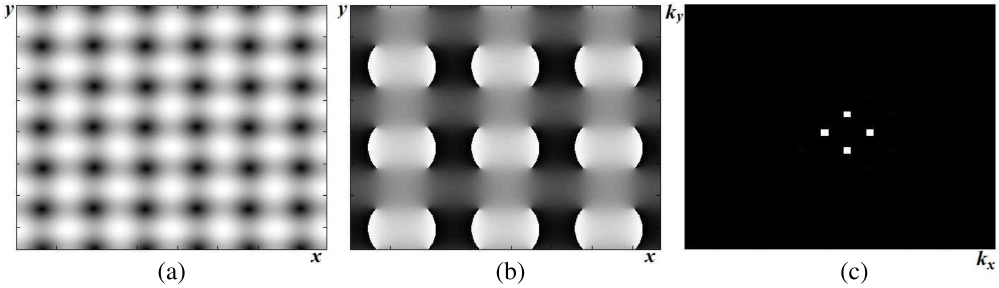

Figure 1: Stationary square vortex lattice: (a) intensity distribution, (b) phase distribution and (c) far field. Here, x, y are transverse coordinates and k_x, k_y are appropriate wave numbers.

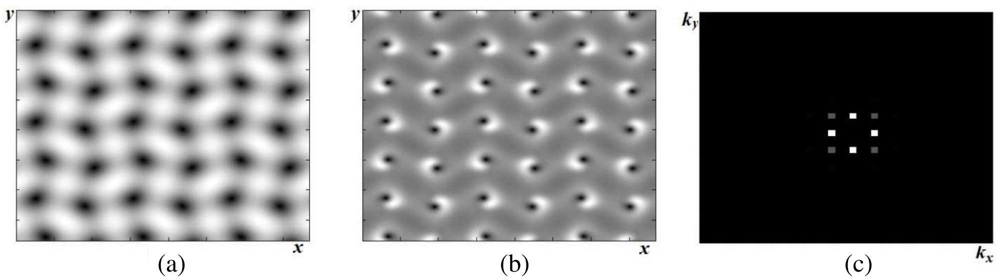

Figure 2: Stationary square vortex lattice: (a) intensity distribution, (b) $|P(x,y)|$ and (c) far field.

This formula is in a very good agreement with our numerical results. Also from (2) we obtained the necessary condition for the existence of these two square vortex lattices: $r \geq 1+\omega^2$. The vortex motion is observed only if this condition is satisfied.

7. CONCLUSION

We studied the formation of SVL both analytically and numerically using the full Maxwell-Bloch model. The single travelling wave solution was investigated. Interaction of four travelling waves was investigated as well. The square vortex lattice solution was obtained analytically. A distance between neighboring vortices were found analytically. The split step Fourier method was used for numerical simulation of Maxwell-Bloch equations. Formation of SVL was observed numerically. For $r \approx r_{th}$, the SVL is stationary. The SVL becomes unstable with increase of $r > r_{cr}$. In this case, the motion of vortices around equilibrium points is observed. In the far field, we obtain eight bright spots. Eight bright spots in the far field correspond to two square vortex lattices. This simple idea allows us to calculate the frequency of vortex oscillations. Also we obtained the necessary condition for the existence of these two square vortex lattices.

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