# Railway Traction Vehicle Longitudinal Velocity Estimation by Kalman Filter

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**Abstract**— Important task of railway traction vehicles is to achieve maximal tractive effort at any conditions if required. For this purpose the vehicles have slip controllers. Some types of these controllers need the vehicle longitudinal velocity value as an input. The vehicle longitudinal velocity is difficult to measure directly because the vehicle longitudinal velocity could not be directly determined from wheel velocity. When the wheel is driven its velocity is higher than the vehicle longitudinal velocity because of the slip phenomenon. The paper describes method that provides possibility to estimate the vehicle velocity on base of measured wheel velocity. Estimation of the vehicle longitudinal velocity is enabled by Kalman filtration. The method is verified by MatLab simulation using real measured data.

## 1. INTRODUCTION

The maximal value of railway traction vehicle tractive effort that can be transmitted between wheels and rails depends on the adhesion coefficient value. For this purpose locomotives have a slip controller. The slip controller maintains the value of the adhesion coefficient on appropriate value and enables maximizing the transmitted force. The adhesion coefficient is changed by the value of the slip velocity. Some of the slip controllers need to know the train velocity to work properly [1]. The train velocity is typically determined from the locomotive wheel velocity.

Wheel velocity of railway traction vehicles is measured by incremental encoders. The encoders are typically mounted on wheelsets in case of vehicles with DC motors on motor shaft in case of vehicles with induction motors. The encoders are exposed to high mechanical stress and electromagnetic interference that have influence to velocity measurement accuracy. Therefore the incremental encoders are typically of magnetic types with low number of pulses per revolution to be robust and reliable [2, 3]. By measuring wheel velocity a vehicle longitudinal velocity value cannot be calculated directly when all wheels are driven. The reason is the slip velocity that occurs on every driven wheel. Therefore measured wheel velocity is higher than actual vehicle velocity. For purpose of vehicle velocity determination some methods exist. These methods are typically based on evaluation of the slowest wheelset velocity. But this type of methods could fail when high value of slip velocity occurs synchronously on all wheelsets. For locomotive longitudinal velocity estimating the Kalman filter could be used [4].

In the paper the railway traction vehicle longitudinal velocity estimation based on the Kalman filter and vehicle state-space model are described. The Kalman filter and a vehicle model are discussed in detail. The estimated vehicle longitudinal velocity is based on measured wheel velocity without need of any other additional information. Simulations using real measured data are performed.

## 2. KALMAN FILTER

Kalman filter can estimate the linear dynamic system states disturbed by a noise. As the Kalman filter inputs measurable values related to these states are used. The Kalman filter needs the system model and input that is related to some states of the system to work properly. The paper deals with the Kalman filter utilization to estimate the train velocity. The model inputs should be the whole locomotive tractive effort or motor torques of individual motors and measured wheel velocity values. Model output should be the train velocity.

### 2.1. Kalman Filter Equations

The general equations for a discrete time dynamic system are:

$$x[n+1] = A \cdot x[n] + B \cdot u[n] + w[n] \tag{1}$$

$$y[n+1] = C \cdot x[n] + v[n] \tag{2}$$

where $x$ is the state vector, $u$ are measured system inputs, $w$ are random dynamic disturbances, $y$ are sensor outputs, $v$ is random sensor noise, $A$ is system matrix, $B$ is input matrix, and $C$ is output matrix.

The Kalman filter is based on matrixes $A$ and $C$. The system does not include matrix $B$ because the motor torques are considered as part of dynamic disturbances. The Kalman filter algorithm isdescribed by equations [5]:

$$K = \frac{P \cdot C^T}{C \cdot P \cdot C^T + R} \tag{3}$$
$$x_h = x_h + K \cdot (z_n - C \cdot x_h) \tag{4}$$
$$P = P - K \cdot C \cdot P \tag{5}$$
$$x_h = A \cdot x_h \tag{6}$$
$$P = A \cdot P \cdot A^T + Q \tag{7}$$
$$P = \frac{1}{2}\left(P + P^T\right) \tag{8}$$

where $K$ is the Kalman gain matrix, $P$ is state estimation uncertainty covariance matrix, $R$ is measurement uncertainty covariance matrix, $x_h$ is system state vector, $z_n$ are measured values,and $Q$ is process noise covariance matrix.

### 2.2. System Model

The system model is a three mass model. It contains an induction motor, a gear, and a wheel that represents the wheelset.The model is illustrated in Figure 1. Figure 1 shows three main parts of the model that are induction motor, gear, wheel, and further connections between the parts.

Block diagram of the Kalman filter connection into the whole system is depicted in Figure 2. The system consists of a train which input is the required tractive effort and its output is the wheel velocity. Wheel velocities are used to calculate the minimal value of them and this value is connected to the Kalman filter input. Kalman filter output represents the estimated train velocity that could be used, e.g., as a slip controller input.

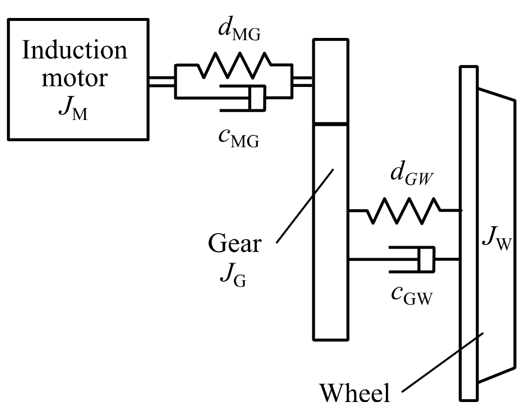


Figure 1: System model block diagram.

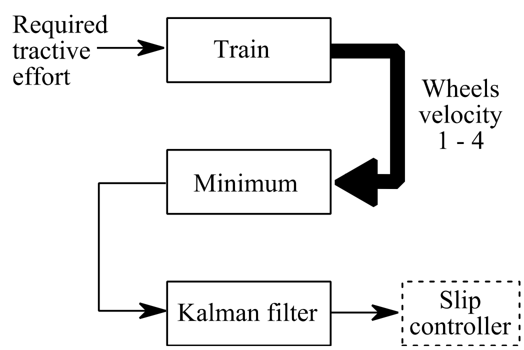


Figure 2: Kalman filter and system connection.

The model is based on equations that describe the system model shown in Figure 1:

$$T_M = J_M \ddot{\varphi}_M + d_{MG}(\dot{\varphi}_M - \dot{\varphi}_G) + c_{MG}(\varphi_M - \varphi_G) \tag{9}$$
$$T_G = -J_G \ddot{\varphi}_G - d_{MG}(\dot{\varphi}_G - \dot{\varphi}_M) - d_{GW}(\dot{\varphi}_G - \dot{\varphi}_W) - c_{MG}(\varphi_G - \varphi_M) \tag{10}$$
$$T_W = J_W \ddot{\varphi}_W + d_{GW}(\dot{\varphi}_W - \dot{\varphi}_G) \tag{11}$$

where $T_M$ is the motor torque, $T_G$ is torque transmitted by the gear, $T_w$ is torque on the wheel, $J_M$ is the motor moment of inertia, $J_G$ is the gear moment of inertia, $J_W$ is the wheel moment of inertia, $d_{MG}$ is an element of damping between the motor and the gear, $d_{GW}$ is an element of damping between the gear and the wheel, $c_{MG}$ is an element of stiffness between the motor and the gear, $\ddot{\varphi}_M$, $\ddot{\varphi}_G$ and $\ddot{\varphi}_W$ are the motor, gear, and wheel accelerations, $\dot{\varphi}_M$, $\dot{\varphi}_G$, and $\dot{\varphi}_W$ are velocities of the parts, and $\varphi_M$, $\varphi_G$, and $\varphi_W$ are displacements of the parts.

## 3. SIMULATION RESULTS

The simulations are using real measured data. An example of measured data is shown in Figure 3. Here Figure 3 measured velocities for four locomotive wheelsets are shown. All wheelsets are driven and all wheelsets have a high value of the slip velocity in different time. The high value of the slip velocity is shown as high peaks in measured data. The values of the peaks are limited by a readhesion controller that decreases motor torque when the high value of the slip is detected. If the readhesion controller does not limit the motor torque, the slip velocity could increase uncontrollably. When the wheels have a high value of slip velocity, the adhesion coefficient value decreases, therefore decreases the maximal value of force that can be transmitted between wheels and rails. Another disadvantage of the slip velocity high value lies in higher mechanical wear of wheels and rails and in an extreme case some locomotive parts could be damaged.

The Kalman filter performance depends on the measurement uncertainty covariance matrix $R$ and process noise covariance matrix $Q$. The covariance matrix settings used for simulation are summarized in Table 1. The matrix $Q$ size is of $6 \times 6$ and matrix $R$ size is of $1 \times 1$.

The data processed by the Kalman filter are shown in Figure 4 in terms of the Kalman filter input and output data. The input data is the minimal value of velocities obtained from the four

| Covariance matrix | Values |
|---|---|
| $Q$ | $\begin{bmatrix} 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$ |
| $R$ | 500 |

Table 1: Covariance matrices.

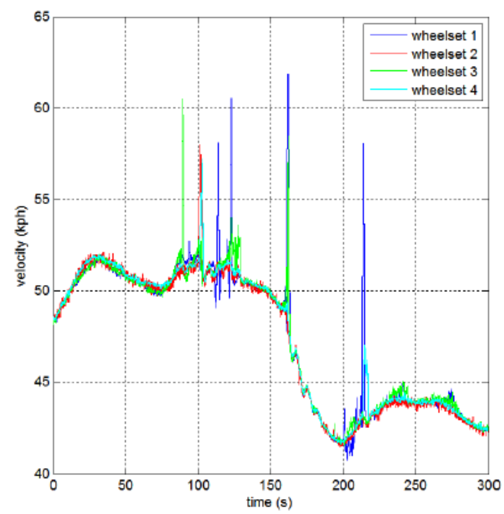


Figure 3: Measured data.

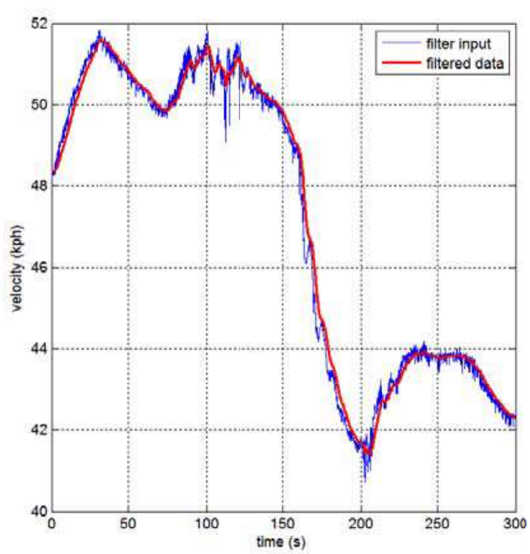


Figure 4: Kalman filter input and output.

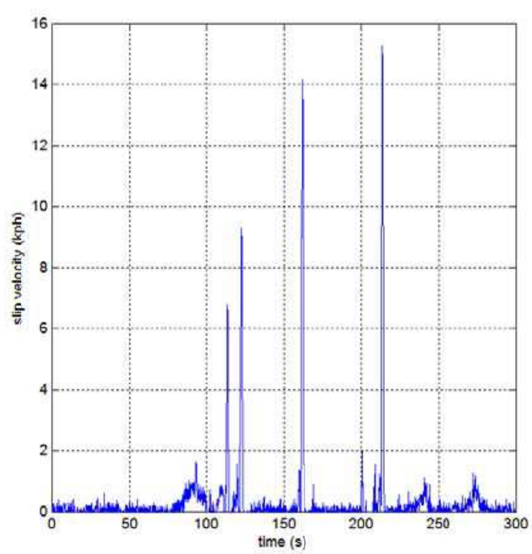


Figure 5: Slip velocity for the first wheelset.

wheelsets. The data in Figure 4 correspond to data in Figure 3.

Figure 5 displays the calculated slip velocity of the first wheelset with high value of the slip velocity. Data in Figure 5 correspond to data in Figure 3.

## 4. CONCLUSION

The paper describes railway traction vehicle longitudinal velocity estimation using Kalman filter. The Kalman filter input data is minimal velocity from the four wheelsets. All wheelsets are driven, therefore the measured velocity is higher than train velocity and the input data contains high value of slip velocity. The velocities were measured on locomotive. The vehicle longitudinal velocity is intended for vehicle slip controller that enables transfer of maximal force between wheels and rails if it is required. Kalman filter equations are described. Performance of the Kalman filter depends on measurement uncertainty covariance matrix $R$ and process noise covariance matrix $Q$. If the matrices are not set properly the filter could not estimate the velocity correctly. The matrices setting are presented in the paper. The Kalman filter works with vehicle mathematical model. The model is three mass model and contains the motor, the gear and the wheel.

## ACKNOWLEDGMENT

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