# Superresolution Based on the Methods of Extrapolation

**B. A. Lagovsky**[1], **A. B. Samokhin**[1], **and Y. V. Shestopalov**[2]

[1]Department of Applied Mathematics
Moscow State Technical University of Radio Engineering, Electronics and Automatics, Russia
[2]University of Gävle, Sweden

**Abstract**— A new method of signal processing by smart antennas is proposed and justified. It allows to improve the accuracy of angle measurement and to restore the image of the object with superresolution. The method is based on the extrapolation of the signals received by each element of the antenna array, outside the aperture. This allows introducing new virtual elements and thus synthesizing significantly larger antenna array. The method is tested in numerical experiments using a mathematical model and the maximum effective angular resolution is found for different cases and objects. Algorithms based on the method of digital aperture synthesis provide angular superresolution 3–7 times greater than that according to the Rayleigh criterion for a signal/noise ratio of 12–13 dB

## 1. INTRODUCTION

It is well known that to improve the quality of the target image identification we need to improve the angular resolution. Similar problems are especially important in remote sensing. The most important modern trend to improve surveillance and goniometric systems is obtaining images of objects with high angular resolution. For targets located in far zone, the angular resolution based on Rayleigh criterion is usually represented as

$$\delta\theta \cong \lambda/d, \tag{1}$$

where $d$ is a size of antenna and $\lambda$ is the wavelength. An important way to improve radar systems based on smart antennas is enhanced accuracy of angle measurements and increase in the angular resolution. In this regard, the current problem is to restore the image of the object with superresolution. This inverse problem is reduced to a Fredholm integral equation

$$U(\alpha) = \int_{\Omega} F(\alpha - \varphi) I(\varphi) d\varphi, \tag{2}$$

where $U(\alpha)$ is a signal received as a result of scanning at angle $\alpha$, $\Omega = [\alpha_1, \alpha_2]$ is the angular region of location of the signal source, $I(\alpha)$ is the desired angular distribution of the reflected signal amplitude, and $F(\alpha, \varphi)$ is the radiation pattern (RP). Algebraic methods for solving problem (2) presented in [1–7] prove to be promising. They consist in representing approximate solutions in the form of finite expansions given by a sequence of functions with unknown coefficients; however it is known that this solution technique may be unstable. A new method of signal processing by smart antennas proposed in this study is based on the extrapolation of the signals received by each element of the antenna array outside the aperture.

## 2. PROBLEM STATEMENT

For smart antennas (SAs), a different approach to the problem is used, without direct reduction to Equation (2), which increases the stability of the algorithms. To simplify the expressions obtained we consider a large linear SA with $2N+1$ elements. The amplitudes of the currents at the emitters assumed to be equal and the area source location $\Omega$ one-dimensional with the dimensions smaller than the width of the SA beam, $\theta_{0,5}$. Then RP is

$$F(\phi - \alpha) = \sum_{n=-N}^{N} \exp(ikdn(\phi - \alpha) \tag{3}$$

where $k = 2\pi/\lambda$, $d$ is the distance between adjacent emitters, and $\alpha$ is the direction of the SA maximum. Signal (2) for the formulated conditions is converted to the form

$$U(\alpha) = \sum_{n=-N}^{N} \exp(-ikdn\alpha) \int_{\Omega} \exp(ikdn\phi) I(\phi) d\phi = \sum_{n=-N}^{N} C_n \exp(-ikdn\alpha) \tag{4}$$

$$C_n = \int_{\Omega} \exp(ikdn\phi) I(\phi) d\phi, \quad n = 0, \pm 1 \ldots \pm N \tag{5}$$

where coefficient $C_n$ is a signal received by the SA element with number $n$. Generalization of (4) and (5), particularly to two-dimensional problems, is the case where the main difficulty is encountered. Kernel (2) in the form (3) is degenerate. It is easy to show that in this case the solution $I(\alpha)$ is expressed in terms of an expansion involving coefficients $C_n$ as

$$I(\alpha) = A \sum_{n=-N}^{N} \exp(-ikdn\alpha) C_n + \Psi(\alpha), \tag{6}$$

where $\psi(\alpha)$ is an arbitrary function orthogonal to all the kernel eigenfunctions, i.e., functions from (6) on the interval $[-\lambda/d;\ \lambda/d]$, where $A$ is the normalization factor. The specific form of function $\psi(\alpha)$ cannot be determined directly from the conditions of the problem and *a priori* information is required. Approximate solution, the first term in (6), repeats waveform $U(\alpha)$ and therefore permits establishing a correspondence to the Rayleigh criterion.

## 3. MAIN RESULTS

Consider one of the opportunities to increase angular resolution of SA. Unlike conventional phased arrays, the signals received by each transducer SA specified by $C_n$ can be recorded in digital form and applied then in digital processing.

From (5) it follows that the difference between the values of $C_n$, $n = 1, 2, \ldots$ are determined only by the spatial position of the individual SA emitters. Therefore, by analyzing the location of a large number of emitters and the values of the corresponding coefficients $C_n$, we can find a dependence, with a certain accuracy, of the received signal $C_j$ on the coordinates of the radiator. The solution of this problem allows one to estimate the value of the received signal outside the SA aperture. Next, using the predicted values $C_j$, $j = \pm(N + 1) \ldots \pm M$ together with previously measured values $C_j$, $j = -N, -N + 1, \ldots, +N$, one can extend the summation in (3) to $2M + 1$ terms. In the end we obtain a virtual RP with $2M + 1$ emitters having the aperture increased in $M/N$ times and the beam width $\theta_{0,5}$ decreased by the same order of magnitude, so that the angle accuracy and angular resolution increases correspondingly. As a result, the restored with superresolution approximate image of the object is represented as

$$I(\alpha) \cong \sum_{n=-N}^{N} \exp(-ikdn\alpha) C_n, \tag{7}$$

Thus, approximate solution (2)–(4) is reduced to the determination of values of the maximum possible number of complex coefficients $C$ outside the SA aperture.

Estimation of the coefficients $C$ for virtual SA items (i.e., outside the actual aperture) can be performed using extrapolation (linear, polynomial, spline, and so on) with different indicators of computational complexity and performance. Specialized numerical methods make it possible to carry out more effectively prediction of the behavior of diverse forms of dependence than using, e.g., polynomials. Preliminary studies show that the dependence of the values of coefficients $C$ on the distance to the center of SA is characterized as a whole, regardless of source, as a non-periodic oscillating smooth curve. Such a firmly established regularity responds well to extrapolation using linear Berg prediction and with relatively small error if the dependence is almost periodic or oscillating. With a sufficiently large number of initial points the technique can be used successfully for a much greater distance from the target area compared with polynomial or spline extrapolation. In the algorithm based on the Berg method and developed in this work, a polynomial regression builds on the initial data which reduces the effect of random errors in the specification of initial values of $C$ on the results of extrapolation.

**Example 1. Point source**. In many respects the quality of extrapolation, in particular, the size range outside the predetermined interval in which the extrapolated signal does not exceed the admissible error is determined by the signal type, more precisely, its angular size. For a point source all the coefficients $C$, as follows from (2), are

$$C_n = A\exp(ikdn\alpha_0), \quad n = \pm(N+1)\ldots\pm\infty, \tag{8}$$

where $\alpha_0$ is the angular position of the source and $A$ is the amplitude coefficient. The values of $\alpha_0$ can be found for each $n$ from the expression

$$C_n + C_{-n} = 2A\cos(kdn\alpha_0), \quad n = 1, 2\ldots N. \tag{9}$$

*Remark.* It should be noted that an $\alpha_0$ determines the source location and one cannot perform further synthesis of the aperture, at least for large signal/noise ratio (SNR) at each SA element. If the presence of random component becomes visible, a preference should be given to the digital processing algorithm based on aperture synthesis. A parameter extrapolation method searches for expression describing an experimental dependence $C_j$, $j \in [-N, N]$, taking into account the fact that the results are obtained with certain random error.

**Example 2**. Of practical interest is the problem of obtaining superresolution radar observations at two closely spaced point objects. In this case, assuming a predetermined target information in the form of two point objects it is useful to number aggregate signals received by SA symmetric elements using indices $n$ and $-n$:

$$\begin{matrix} C_n + C_{-n} = 2A_1\cos(kdn\alpha_1) + 2A_2\cos(kdn\alpha_2), \quad n = 1, 2\ldots N \\ c_0 = A_1 + A_2 \end{matrix}, \tag{10}$$

where $\alpha_{1,2}$ and $A_{1.2}$ are unknown angular coordinates of the objects and the amplitudes of the received signals. The resulting system of $N+1$ nonlinear equation with unknown $\alpha_{1,2}$ and $A_{1.2}$ is overdetermined. Its solutions must be sought in the form of a minimum amount of the standard deviation of the left and right sides, reducing thus the effect of noise and random noise. Note that the remark made above applies here even to a greater extent because numerical solution of system (1) yields additional errors.

## 4. SIMULATION RESULTS

Characteristics of increasing angular resolution and its limits were investigated using a mathematical model. Considered RP were formed by flat equidistant SA with $31 \times 31$ elements. At the first stage the quality of extrapolation coefficients $C$ were analyzed.

The results of image reconstruction of two identical point sources are shown in Figure 1. The algorithm is allowed to calculate with minor error the values of coefficients $C$ for a synthetic SA, the size of which is ten times greater than the size of the original SA. Figure 2 shows the solutions found with a low level of noise.

It is known that inverse problems are very sensitive to the presence of errors in the original data. Therefore, in addition to estimating the degree of excess of the Rayleigh criterion, an important

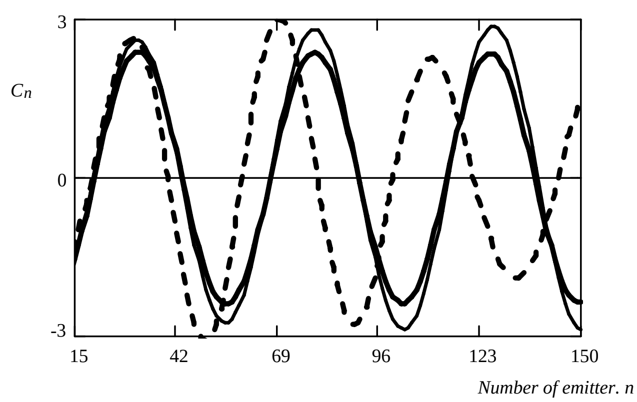


Figure 1: The values of the real part of $C_n$ outside the aperture: the bold solid line — the true values, dashed — extrapolation using splines, and thin solid — Berg extrapolation.

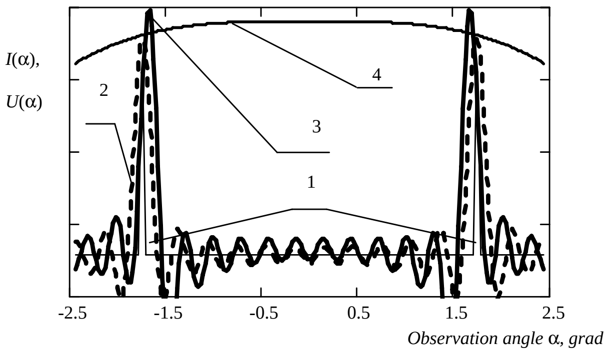


Figure 2: Results of image reconstruction of signal sources. 1 — intensity distribution of the true source; dashed 2 — signal received from the real SA; bold 3 — signal obtained on the basis of synthesis SA; 4 — real signal $U(\alpha)$.

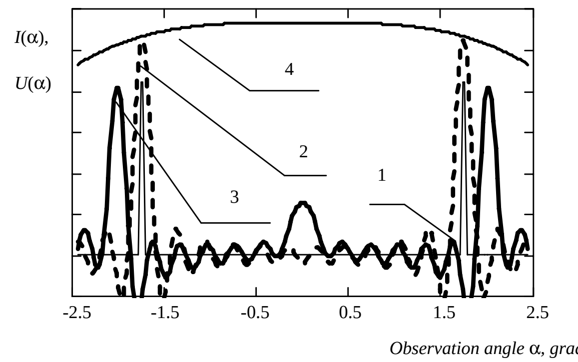


Figure 3: Results of image reconstruction at $q = 14\,\text{dB}$. 1 — intensity distribution of the true source; 2 — signal received from the real SA with 300 elements; 3 — the signal obtained by synthesis SA; 4 — signal $U(\alpha)$.

characteristic of solutions is the minimum SNR at which the decision with superresolution is still possible.

Stability of the solutions and the quality of image reconstruction have been also investigated in model simulations. Figure 3 shows the solutions found with a significant level of noise when SNR $q = 14\,\text{dB}$. For the elements starting with the number $N = 80$ values of the coefficients $C$ (Figure 2) undergo significant changes as compared to the true values. Nevertheless, the quality of the reconstructed image is still good.

Due to the effects of noise the reconstructed image was somewhat offset in relation to the true one having a slightly increased amplitude of the oscillating component. With further decrease of $q$ an increase in the source position error increases the oscillating component. Image quality begins to depend on each particular implementation of the noise component. When the signal/noise ratio is less than 10 dB the image recovery becomes impossible.

## 5. CONCLUSION

The proposed method of signal processing based on the digital aperture synthesis improves the accuracy of angle measurements and provides the recovery of images of objects with superresolution. The results of numerical studies have shown that the angular resolution sharply increases in SNR ranging from 12–13 dB, i.e., at substantially lower values than those provided by the known methods and algorithms. Implementation of this method as a software package in computer codes applied for the design of new systems will simplify technical solutions and reduce the costs. The developed fast algorithms of the aperture synthesis can be used in the real-time mode.

## REFERENCES


1. Quinquis, A., E. Radoi, and F. Totir, "Some radar imagery results using superresolution techniques," *IEEE Trans. Antennas and Propagation*, Vol. 52, No. 5, 1230–1244, 2004.
2. Lagovsky, B. A., "Image restoration of the objects with superresolution on the basis of spline – Interpolation," *PIERS Proceedings*, 989–992, Moscow, Russia, Aug. 2012.
3. Lagovsky, B. A., "High performance angular resolution algorithm for radar systems," *PIERS Proceedings*, 1637–1641, Moscow, Russia, Aug. 18–21, 2009.
4. Lagovsky, B. A., "Algorithm for the determination of targets coordinates in structure of the multiple targets with the increased effective resolution," *PIERS Proceedings*, 1642–1645, Moscow, Russia, Aug. 18–21, 2009.
5. Herman, M. A. and T. Strohmer, "High-resolution radar via compressed sensing," *IEEE Trans. Signal Processing*, Vol. 57, No. 6, 2275–2284, 2009.
6. Lagovsky, B. A., "Superresolution: Simultaneous orthogonalization of function systems describing the received signal and its source," *PIERS Proceedings*, 993–996, Moscow, Russia, Aug. 18–21, 2009.
7. Lagovsky, B. A. and A. B. Samokhin, "Image restoration of two-dimensional signal sources with superresolution," *PIERS Proceedings*, 315–319, Moscow, Russia, Aug. 18–21, 2009.