A Hybrid Scheme for Solving Transient Electromagnetic Problems with Conductors

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Abstract— Transient electromagnetic (EM) scattering by conducting objects is formulated by time-domain electric field integral equation (TDEFIE). Conventionally, the TDSIE is solved by combining the method of moments (MoM) in spatial domain and a march-on in time (MOT) scheme in temporal domain. We propose a hybrid scheme in which the Nyström method is used in spatial domain while the MoM with Laguerre function as basis and testing functions is employed in temporal domain. A numerical example for EM scattering by a conducting cube is presented to demonstrate the approach and its merits can be observed.

1. INTRODUCTION

Solving transient electromagnetic (EM) problems is essential in many applications such as the design of radar cross section of aircrafts and design of some antennas or microwave circuits. The problems are usually of broadband, nonlinear, and time-varying characteristics and can only be described by time-domain governing equations. Although time-domain differential equations can be used to solve the problems, the time-domain integral equations (TDIEs) may be preferred in many situations due to their unique merits [1]. The TDIEs could be in different forms and they are electric field integral equation (EFIE), magnetic field integral equation (MFIE), and combined field integral equation (CFIE), in a time-domain form, if objects are conducting or homogeneously penetrable.

Those TDIEs are usually solved by combining the method of moments (MoM) in spatial domain and a march-on-in-time (MoT) scheme in temporal domain. The MoM requires a well-designed basis function like the divergence-conforming Rao-Wilton-Glisson (RWG) basis function [2] or curl-conforming edge basis function [3], resulting in an inconvenient implementation. On the other hand, the MoT scheme is conventionally inefficient and inherently unstable due to the low- and high-frequency modes of creeping into the solution [4]. The late-time instability of MoT has not been fully solved albeit many improved versions have been developed in recent years [5].

We propose a novel hybrid scheme to efficiently solve the time-domain EFIE for transient EM problems with conductors. The Nyström method instead of the traditional MoM is used in spatial domain while the MoM instead of the conventional MoT is employed in temporal domain. The benefits of Nyström method in spatial domain include the simple mechanism of implementation, removal of basis and testing functions, and allowance of using nonconforming and low-quality meshes [6]. In time domain, we use the MoM with Laguerre basis and testing functions which was proposed by Jung et al. [7] and it is superior to the MoT scheme because the Laguerre function can naturally enforce the causality. Using a Galerkin's testing, we can eliminate the numerical instability through a march-on-in-degree (MoD) procedure. We present a numerical example to demonstrate the hybrid scheme and its robustness has been observed.

2. FORMULATIONS

Consider the scattering of transient EM wave by a conducting object with a surface S in free space. The problem can be formulated by the TDEFIE, or the tangential component of total electric field vanishes on the object surface. The scattered electric field $\mathbf{E}^{sca}(\mathbf{r},t)$ is related to the magnetic vector potential $\mathbf{A}(\mathbf{r},t)$ and electric scalar potential $\Phi(\mathbf{r},t)$ by

$$\mathbf{E}^{sca}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t) - \nabla\Phi(\mathbf{r},t) = -\frac{\mu_0}{4\pi}\int_S \frac{1}{R}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{r}',\tau)\,dS' - \frac{1}{4\pi\epsilon_0}\nabla\int_S \frac{1}{R}q(\mathbf{r}',\tau). \tag{1}$$

In the above, ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively, $\mathbf{J}(\mathbf{r},t)$ and $q(\mathbf{r}',t)$ is the current density and charge density induced on the conductor surface, respectively, $R = |\mathbf{r}-\mathbf{r}'|$ is the distance between an observation point $\mathbf{r}$ and a source point $\mathbf{r}'$, and $\tau = t - R/c$ is the retarded time with c being the speed of light in free space. By using the continuity equation and introducing a new source vector $\mathbf{e}(\mathbf{r}, t)$ which is related to the charge density by $q(\mathbf{r}, t) = -\nabla \cdot \mathbf{e}(\mathbf{r}, t)$, we can write the TDEFIE as

$$\left[\mu_0 \int_S \frac{1}{R} \frac{\partial^2 \mathbf{e}(\mathbf{r}', \tau)}{\partial t^2} dS' - \frac{1}{\epsilon_0} \nabla \int_S \frac{1}{R} \nabla' \cdot \mathbf{e}(\mathbf{r}', \tau)\, dS' \right]_{tan} = 4\pi \mathbf{E}^{inc}(\mathbf{r}, t)\big|_{tan}, \quad \mathbf{r} \in S \tag{2}$$

or

$$\left[\mu_0 \int_S \frac{1}{R} \frac{\partial^2 \mathbf{e}(\mathbf{r}', \tau)}{\partial t^2} dS' + \frac{1}{\epsilon_0} \int_S \nabla\nabla \left(\frac{1}{R}\right) \cdot \mathbf{e}(\mathbf{r}', \tau)\, dS' \right]_{tan} = 4\pi \mathbf{E}^{inc}(\mathbf{r}, t)\big|_{tan}, \quad \mathbf{r} \in S. \tag{3}$$

3. HYBRID SCHEME OF SOLVING THE TDEFIE

We mesh a conductor surface into N small triangular patches and ΔS_n is the area of the nth patch. The unknown source vector $\mathbf{e}(\mathbf{r}', t)$ can then be expanded as

$$\mathbf{e}\left(\mathbf{r}', t\right) = \sum_{n=1}^{N} e_n(t) \mathbf{e}_n(\mathbf{r}'). \tag{4}$$

If we apply the Nyström method in spatial domain, i.e., the integration over a small triangular patch is replaced with a summation under a quadrature rule provided that the integrand is regular, then we have

$$\left[\mu_0 \sum_{n=1}^{N}\sum_{q=1}^{Q} \frac{w_q}{R} \frac{\partial^2 e_n(\tau)}{\partial t^2} \mathbf{e}(\mathbf{r}'_{nq}) + \frac{1}{\epsilon_0} \sum_{n=1}^{N}\sum_{q=1}^{Q} w_q \nabla\nabla\left(\frac{1}{R}\right) \cdot \mathbf{e}(\mathbf{r}'_{nq}) e_n(\tau)\right]_{tan} = 4\pi \mathbf{E}^{inc}(\mathbf{r}, t)\big|_{tan} \tag{5}$$

where $\mathbf{e}(\mathbf{r}'_{nq}) = \mathbf{e}_{nq} = e^u_{nq}\hat{u} + e^v_{nq}\hat{v}$ is the value of unknown source vector at the qth quadrature point in the nth patch and it has two independent components e^u_{nq} and e^v_{nq} in a local coordinate system $\{u, v, w\}$ established over the patch plane. If we choose two orthogonal unit tangential vectors $\mathbf{t}^{(l)}(\mathbf{r}_{mp}) = \mathbf{t}^{(l)}_{mp} = t^{(l)}_{ump}\hat{u} + t^{(l)}_{vmp}\hat{v} + t^{(l)}_{wmp}\hat{w}$ at the pth quadrature point in the mth observation patch $(l = 1, 2)$ to test the above equation, then we have

$$\sum_{n=1}^{N}\sum_{q=1}^{Q} \left[\frac{\mu_0}{4\pi} \frac{\partial^2 e_n(\tau)}{\partial t^2} a^{(l)}_{mpnq} + \frac{e_n(\tau)}{4\pi\epsilon_0} b^{(l)}_{mpnq}\right] = \mathbf{t}^{(l)}_{mp} \cdot \mathbf{E}^{inc}(\mathbf{r}, t) \tag{6}$$

where

$$a^{(l)}_{mpnq} = \frac{w_{nq}\mu_0}{R_{mpnq}} \mathbf{t}^{(l)}_{mp} \cdot \mathbf{e}_{nq}, \quad b^{(l)}_{mpnq} = \frac{w_{nq}}{\epsilon_0} \mathbf{t}^{(l)}_{mp} \cdot \nabla\nabla\left(\frac{1}{R}\right) \cdot \mathbf{e}_{nq}. \tag{7}$$

The above procedure could have a singularity problem which occurs when $m = n$ but we have developed a robust treatment technique [8].

In the temporal domain, we use the Laguerre function $\phi_j(t) = e^{-t/2} L_j(t)$ as a basis function to expand $e_n(t)$ [7], i.e.,

$$e_n(t) = \sum_{j=0}^{\infty} e_{n,j} \phi_j(st) \tag{8}$$

where s is the scaling factor to control the support of expansion. Substituting this representation to Eq. (6) and using the revelent formulations in the appendix in [7], we can obtain

$$\begin{aligned} &\sum_{n=1}^{N}\sum_{q=1}^{Q} \left[\left(0.25 s^2 a^{(l)}_{mpnq} + b^{(l)}_{mpnq}\right) e_{nj} I_{ii}(sR_{mpnq}/c)\right. \\ &= V_{mpi} - \sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{j=0}^{i-1} \left(0.25 s^2 a^{(l)}_{mpnq} + b^{(l)}_{mpnq}\right) e_{nj} I_{ij}(sR_{mpnq}/c) \\ &\quad - \sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{j=0}^{i} s^2 a^{(l)}_{mpnq} \sum_{k=0}^{j-1} (j-k) e_{nk} I_{ij}(sR_{mpnq}/c) \end{aligned} \tag{9}$$

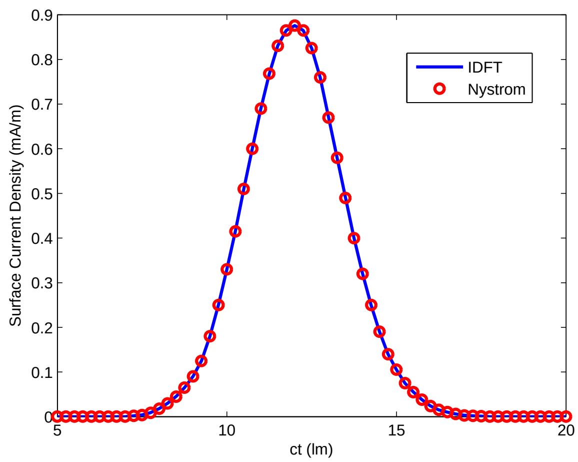

Figure 1: Solution of z-directed current density at $z = 0$ on a cube surface for transient scattering by a conducting cube.

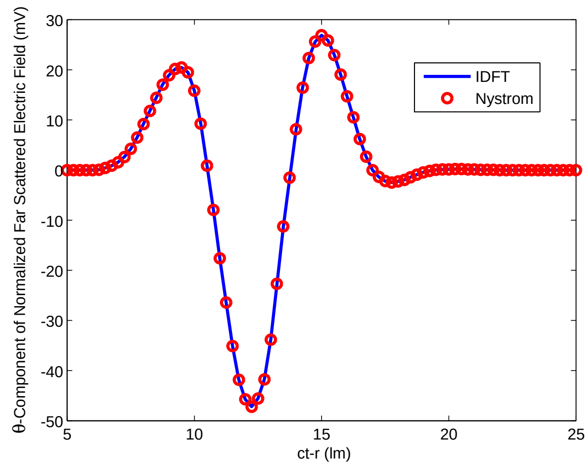

Figure 2: Solution of θ-component of normalized far-zone scattered electric field for transient scattering by a conducting cube.

where $I_{ij}(sR_{mpnq}/c)$ and V_{mpi} can be calculated according to the formulations in the appendix in [7]. Note that there are two spatial-domain coefficients in $a_{mpnq}^{(l)}$ and $a_{mpnq}^{(l)}$ which are the two independent components e_{nq}^u and e_{nq}^v of $\mathbf{e}_{nq}$ and they should be combined with the time-domain coefficient e_{ni} together to form two new unknown coefficients which can be determined by two groups of Eq. (9) when $l = 1, 2$.

4. NUMERICAL EXAMPLE

We consider the transient EM scattering by a conducting cube to demonstrate the approach. The cube is centered at the origin of a rectangular coordinate system and has a side length $s = 1.0$ m whose surface is discretized into 826 triangular patches. A Gaussian plane wave is illuminating the scatterer along the $-z$ direction and the electric field and magnetic field are defined by

$$\mathbf{E}^{inc}(\mathbf{r}, t) = \hat{x}\frac{4e^{-\gamma^2}}{\sqrt{\pi}T}, \quad \mathbf{H}^{inc}(\mathbf{r}, t) = -\frac{1}{\eta}\hat{z} \times \mathbf{E}^{inc}(\mathbf{r}', t) \tag{10}$$

where $\gamma = 4(ct - ct_0 + \mathbf{r} \cdot \hat{z})/T$, T is the width of Gaussian impulse, and t_0 is the time delay denoting the time when the pulse peaks at the origin. We choose a Gaussian pulse with $T = 8.0\,\text{lm}$ (light meter) and $ct_0 = 12.0\,\text{lm}$ as an incident wave. Figure 1 shows the solution of z-directed current density at $z = 0$ on the cube surface while Figure 2 plots the solution of the θ-component of normalized far-zone scattered electric field observed along the backward direction. The results agree well with the solutions obtained from the inverse discrete Fourier transform (IDFT) of frequency-domain solutions.

5. CONCLUSION

In this work, the transient EM scattering by conducting objects is solved by the TDEFIE. We develop a hybrid scheme by combining the Nyström method in spatial domain and the MoM with the Laguerre basis and testing functions in temporal domain. The benefits of the new scheme include the simple mechanism of implementation with a use of nonconforming meshes in spatial domain and the natural satisfaction of casuality with a full elimination of instability in temporal domain. A numerical example for EM scattering by a conducting cube is presented to demonstrate the scheme and good results have been observed.

ACKNOWLEDGMENT

This work was supported by the Specialized Research Fund for the Doctoral Program of Higher Education of China (Project No. 20120072110044).

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