Efficient Imaging of Dielectric Targets Based on Contrast Source Inversion Method

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Abstract— An efficient reconstruction scheme for dielectric targets is proposed based on the contrast source inversion method (CSIM). The CSIM could be preferred to the traditional Born iterative method (BIM) or its variations because it updates the contrast source and contrast simultaneously and avoids the direct solution of forward scattering integral equation (FSIE). The CSIM is similar to the modified gradient method (MGM) but it is more computationally efficient and also requires less measurement data. This is very desirable for the reconstruction when the observation is limited or measurement condition is poor. Two numerical examples with a limited observation are presented to demonstrate the approach and good images have been observed.

1. INTRODUCTION

Reconstructing targets by microwave illumination requires an efficient solution of inverse electromagnetic (EM) scattering equations. This could be very challenging because the involved governing equations are nonlinear and the solutions are inherently nonunique [1]. In particular, the problems become more difficult when measuring conditions are unfavorable and diverse scattering data are not available, which will be encountered in many applications but has been seldom considered [2]. Usually, the inverse problems are solved by linearizing governing equations and gradually minimizing the mismatch between calculated data and measured data in an iterative scheme. In the integral equation approach (IEA) for reconstructing dielectric targets, there are forward scattering integral equation (FSIE) and inverse scattering integral equation (ISIE) and traditionally we need to alternatively solve them in the context of Born iterative method (BIM) or its variations [3]. The solution of permittivity from the ISIE can reveal the profile of unknown target in the imaging domain.

In this work, we use the contrast source inversion method (CSIM) [4] to reconstruct the dielectric targets for the application in target recognition and atmosphere or space sensing. In such an application, the details of target are not important but obtaining a sufficient observation or measurement to the target may be difficult. The CSIM is motivated by and similar to the modified gradient method (MGM) [5], but it updates the contrast source and contrast simultaneously instead of the field and contrast. The CSIM can avoid the direct solution of FSIE because it defines a cost functional including mismatches or errors from both the FSIE and ISIE and update them together [6]. Compared with the MGM, the CSIM is more computationally efficient and also requires less measurement data [7]. This is particularly desirable when we consider the reconstruction with a poor measurement condition which may be encountered in many applications. Two numerical examples for reconstructing typical dielectric targets under a limited view is presented and good images can be observed.

2. GOVERNING EQUATIONS

We consider the reconstruction for a 3D dielectric target enclosed by a cubic imaging domain V in the free space which is the background medium with a wavenumber k_b). The observation surface with transmitting and receiving antennas surrounding the imaging domain is denoted with S. In the traditional BIM or its variations, we have the following forward scattering integral equation (FSIE) which is used to update the total electric field [3]

$$\mathbf{E}_j(\mathbf{r}) = \mathbf{E}_j^{inc}(\mathbf{r}) + k_b^2 \int_V \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \xi(\mathbf{r}')\mathbf{E}_j(\mathbf{r}')dV', \quad \mathbf{r} \in V \tag{1}$$

and inverse scattering integral equation (ISIE) which is used to update the contrast of permittivity $\xi(\mathbf{r}') = k^2(\mathbf{r}')/k_b^2 - 1$ between the imaging domain and surrounding medium

$$\mathbf{E}_j^{sca}(\mathbf{r}) = \mathbf{E}_j(\mathbf{r}) - \mathbf{E}_j^{inc}(\mathbf{r}) = k_b^2 \int_V \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \xi(\mathbf{r}')\mathbf{E}_j(\mathbf{r}')dV', \quad \mathbf{r} \in S. \tag{2}$$

In the above, $\mathbf{E}_j^{inc}(\mathbf{r})$ is the incident electric field from a transmitter where j represents the jth incident electric field, $\mathbf{E}_j(\mathbf{r}')$ is the jth total electric field at a source point $\mathbf{r}'$ inside the imaging domain V, and $\mathbf{E}_j^{sca}(\mathbf{r})$ denotes the jth scattered electric field obtained by measurement at an observation point $\mathbf{r}$ located at an observation or measurement surface S. Also, $\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the dyadic Green's function defined by

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_b^2}\right) g(\mathbf{r},\mathbf{r}') \tag{3}$$

where $\bar{\mathbf{I}}$ is an identity dyad and $g(\mathbf{r},\mathbf{r}') = e^{ik_bR}/(4\pi R)$ is the 3D scalar Green's function in which $R = |\mathbf{r}-\mathbf{r}'|$ is the distance between an observation point $\mathbf{r}$ and a source point $\mathbf{r}'$. In addition, $k(\mathbf{r}')$ is the wavenumber related to the contrast inside the imaging domain. In the CSIM, the equivalent contrast source is defined [4]

$$\mathbf{S}_j(\mathbf{r}) = \xi(\mathbf{r})\mathbf{E}_j(\mathbf{r}) \tag{4}$$

and the FSIE becomes an target or state equation

$$\mathbf{S}_j(\mathbf{r}) = \xi(\mathbf{r})\mathbf{E}_j^{inc}(\mathbf{r}) + \xi(\mathbf{r})\mathbf{F}_V[\mathbf{S}_j(\mathbf{r}')], \quad \mathbf{r} \in V \tag{5}$$

while the ISIE becomes a data equation

$$\mathbf{E}_j^{sca}(\mathbf{r}) = \mathbf{F}_V[\mathbf{S}_j(\mathbf{r}')], \quad \mathbf{r} \in V. \tag{6}$$

3. RECONSTRUCTION BY CONTRAST SOURCE INVERSION METHOD (CSIM)

In the CSIM, the following cost functional is defined [7]

$$C(\mathbf{S}_j,\xi) = \eta_S \sum_{j=1}^{J} \left|\left|\mathbf{E}_j^{sca}(\mathbf{r}) - \mathbf{F}_S(\mathbf{S}_j)\right|\right|_S^2 + \eta_V \sum_{j=1}^{J} \left|\left|\xi\mathbf{E}_j^{inc}(\mathbf{r}) + \xi\mathbf{F}_V(\mathbf{S}_j) - \mathbf{S}_j\right|\right|_V^2 \tag{7}$$

and it is minimized by updating the contrast source and contrast simultaneously. The contrast source is updated for the nth iteration based on the $n-1$ iteration by

$$\mathbf{S}_{j,n} = \mathbf{S}_{j,n-1} + \alpha_n^s \mathbf{v}_{j,n} \tag{8}$$

where

$$\mathbf{v}_{j,0} = 0, \quad \mathbf{v}_{j,n} = \mathbf{t}_{j,n} + \mathbf{v}_{j,n-1} \frac{\mathrm{Re}\left\{\sum_l \langle \mathbf{t}_{l,n}, \mathbf{t}_{l,n} - \mathbf{t}_{l,n-1}\rangle\right\}_V}{\sum_l \langle \mathbf{t}_{l,n-1}, \mathbf{t}_{l,n} - \mathbf{t}_{l,n-1}\rangle_V} \tag{9}$$

and

$$\alpha_n^s = \frac{-\mathrm{Re}\left\{\sum_{j=1}^{J} \langle \mathbf{t}_{j,n}, \mathbf{v}_{j,n}\rangle\right\}_V}{\eta_S \sum_{j=1}^{J} \left|\left|\mathbf{F}_S(\mathbf{v}_{j,n})\right|\right|_S^2 + \eta_{V,n} \sum_{j=1}^{J} ||\mathbf{b}||_V^2} \tag{10}$$

where $\mathbf{b} = \mathbf{v}_{j,n} - \xi_{n-1}\mathbf{F}_V(\mathbf{v}_{j,n})$. On the other hand, the contrast is updated through

$$\xi_n = \xi_{n-1} + \alpha_n^{\xi} d_n \tag{11}$$

where

$$\alpha_n^{\xi} = \frac{1}{\eta_{V,n}}, \quad d_n = \eta_{V,n} \frac{\sum_{j=1}^{J} (\mathbf{S}_{j,n} - \xi_{n-1}\mathbf{E}_{j,n}) \cdot \mathbf{E}_{j,n}^*}{\sum_{j=1}^{J} |\mathbf{S}_{j,n}|^2}. \tag{12}$$

4. NUMERICAL EXAMPLES

The reconstruction for two typical dielectric targets is considered to demonstrate the approach. It is assumed that the measurement surface is located at one side (top) of the reconstructed target so that the observation is limited and more practical. Such an observation is quite different from the standard and widely-applied experimental setup of Institut Fresnel [8]. The vertical distance between the measurement surface and the center of the imaging domain is $r = 5.0\,\mathrm{m}$ and the transmitting or receiving antennas are equally distributed on the surface in terms of its center. We choose the transmitter-receiver combination as $M_T \times M_R = 81 \times 25$. The transmitters are uniformly located at a $1.6\,\mathrm{m} \times 1.6\,\mathrm{m}$ square with a 9×9 array while the receivers are evenly located at the same square with a 5×5 array. The distance between two neighboring antennas in both x and y directions is $0.2\,\mathrm{m}$ for transmitters and $0.4\,\mathrm{m}$ for receivers. The incident wave has a single frequency $f = 3\,\mathrm{GHz}$ and is propagating toward the center of the imaging domain. For numerical simulations, the measured data for scattered electric fields are replaced with calculated data by assuming that the target to be reconstructed is known. The imaging domain is chosen as a cubic box with a dimension $L \times L \times L$ where $L = 140\,\mathrm{mm}$. In the first example, we reconstruct a dielectric sphere with a radius $a = 25\,\mathrm{mm}$ and relative permittivity $\epsilon_r = 3.5$. Fig. 1 shows the reconstructed image in a 3D view and the image can clearly reflect the real target. In the second example, we image a dielectric cylinder with a radius $a = 30\,\mathrm{mm}$ in its cross section, a height $h = 60\,\mathrm{mm}$, and a relative permittivity $\epsilon_r = 3.5$. Fig. 2 sketches its 3D-view image which is distinguishable as a cylinder.

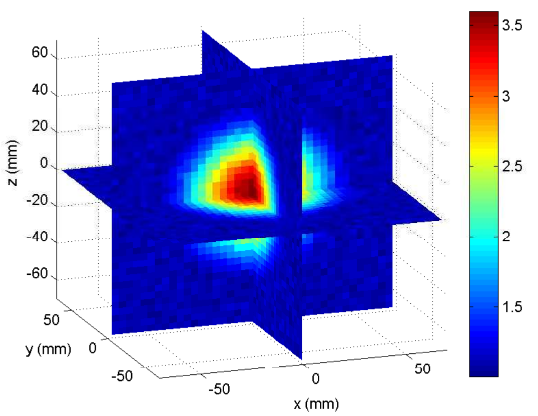

Figure 1: Reconstruction of a dielectric sphere with a radius $a = 25$ mm and a relative permittivity $\epsilon_r = 3.5$.

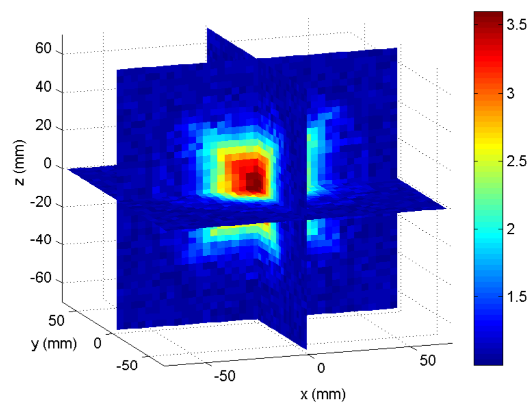

Figure 2: Reconstruction of a dielectric cylinder which has a radius $a = 30\,\mathrm{mm}$, a height $h = 60\,\mathrm{mm}$, and a relative permittivity $\epsilon_r = 3.5$.

5. CONCLUSION

In this work, we use the CSIM to reconstruct dielectric targets for target recognition and atmosphere or space sensing in which the observation to the targets may be very difficult. The CSIM updates the contrast source and contrast simultaneously in the iteration of reconstruction so that the direct solution of FSIE can be avoided, yielding much convenience in numerical implementation. Although the method is somewhat similar to the MGM, it is more computationally efficient and more importantly, it requires less density of measurement data which is particularly desirable for the applications with unfavor measurement or observation condition. We present two numerical examples with a limited observation to demonstrate the approach and good imaging results have been obtained.

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