# Identification of Abnormal Blood Cells Using Scattering of a Focused Laser Beam by a Cluster

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**Abstract—**

## 1. INTRODUCTION

Shapes deviations of red blood cells (RBCs) from normal ones are sensitive remarks for various blood disorders and diseases [1]. Normal mature red blood cells (RBCs) are shaped as biconcave oblate discs. These shapes can be deformed by several inherited disorders into other shapes such as spherical, crescent, oval, or any other abnormal shapes [2]. Identifying abnormal RBCs in a sample contains normal and abnormal cells using scattering of a focused laser Gaussian beam is illustrated in this paper. A chain cluster consists of normal and/or abnormal RBCs is chosen as a sample in the analysis. The modified cluster $T$-matrix method, as introduced in a previous work [3], is used to calculate the angular scattering intensity of a cluster illuminated with a focused Gaussian beam. The beam is modeled by the plane wave spectrum method [4]. From the results of the angular scattering intensity the abnormal RBCs in the sample can be identified. We think that this technique is a powerful one to define the diseased RBCs for more accurate diagnostics purposes.

## 2. ANALYSIS AND RESULTS

The problem that investigated in this paper is the scattering of a chain cluster of normal and/or abnormal RBCs illuminated with a lowest order ($TEM_{00}$) monochromatic Gaussian beam propagating in the $z$-direction of a right handed Cartesian coordinate system ($x$, $y$, $z$) as shown in Fig. 1. The beam is polarized in the $xz$-plane and its focal point is at the origin of the coordinate system. The beam can be focused into an arbitrarily spot size $w_o$. The plane wave spectrum method is used to model such physically realizable beam, whether or not the beam can be represented by analytical formula [4].

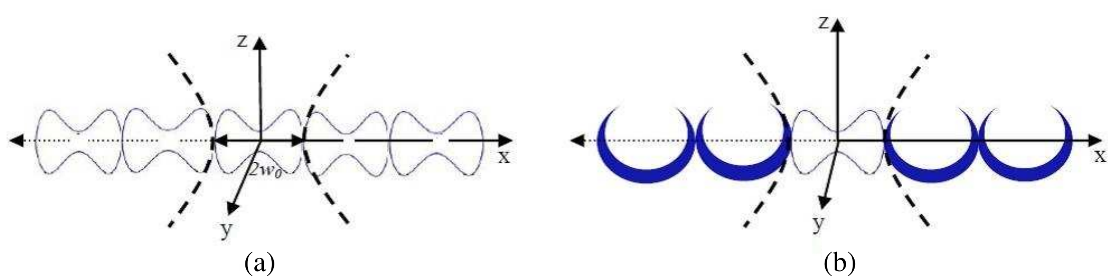


Figure 1: A chain cluster of different cells centered at the origin of a Cartesian coordinate system ($x$, $y$, $z$). The cluster is illuminated with a focused Gaussian beam of a spot size $w_0$, propagating along $z$-direction, and polarized in the $xz$-plane. The cluster consists of (a) five identical biconcave normal red blood cells, (b) four identical abnormal red blood cells and one centered biconcave cell.

The total incident electric field vector $E^{inc}$ polarized in $xz$-plane can be expressed by

$$\mathbf{E}^{inc}(x,y,z) = E_x^{inc}(x,y,z)\mathbf{i}_x + E_z^{inc}(x,y,z)\mathbf{i}_z \tag{1}$$

where $\mathbf{i}_x$ and $\mathbf{i}_z$ are unit vectors in the $x$- and $z$-directions respectively. The time variation $e^{-j\omega t}$ is omitted. In terms of vector spherical harmonics (VSH) the incident Gaussian beam can be expressed as [4],

$$\mathbf{E}^{inc}(k\mathbf{r}) = H\sum_m\sum_n D_{mn}\left[a^t_{emn}\mathbf{M}^1_{emn}(k\mathbf{r}) + a^t_{omn}\mathbf{M}^1_{omn}(k\mathbf{r}) + b^t_{emn}\mathbf{N}^1_{emn}(k\mathbf{r}) + b^t_{omn}\mathbf{N}^1_{omn}(k\mathbf{r})\right] \tag{2}$$

where $H$, and $D_{mn}$ are normalization factors depend on the incident beam. The $a^t_{emn}$, $a^t_{omn}$, $b^t_{emn}$, and $b^t_{omn}$ are the incident field expansion coefficients, and $M^1_{emn}$, $M^1_{omn}$, $N^1_{emn}$, and $N^1_{omn}$ are VSH of the first kind. $m$ (in italic) is the azimuthal mode index and $n$ is the mode number. The symbols $e$ and $o$ stand for even and odd respectively. The wave number is $k = 2 * \pi/\lambda$; $\lambda$ is the wavelength. In the case of a unit amplitude incident plane wave, $H$ is unity and the azimuthal mode index $m = 1$.

The scattered field external to the cluster is represented by the superposition of the incident field and scattered fields consist of components radiated from each cell in the cluster [5], i.e.,

$$\mathbf{E}^{ext} = \mathbf{E}^{inc} + \mathbf{E}^{sca} = \mathbf{E}^{inc} + \sum_{i=1}^{Ns} \mathbf{E}^{sca,i} \tag{3}$$

where $N_s$ is the number of the cells in the cluster. Each partial field $\mathbf{E}^{sca,i}$ represented by an expansion of VSH of the $i$th cell to the origin is [4]

$$\mathbf{E}^{sca,i} = H \sum_m \sum_n D_{mn} \left[ f^i_{emn} \mathbf{M}^3_{emn}(k\mathbf{r}) + f^i_{omn} \mathbf{M}^3_{omn}(k\mathbf{r}) + g^i_{emn} \mathbf{N}^3_{emn}(k\mathbf{r}) + g^i_{omn} \mathbf{N}^3_{omn}(k\mathbf{r}) \right] \tag{4}$$

where $\mathbf{M}^3(k\mathbf{r})$ and $\mathbf{N}^3(k\mathbf{r})$ are the VSH of the third kind (outgoing wave functions) obtained from the VSH of the first kind [4]. The coefficients $f^i_{emn}$, $f^i_{omn}$, $g^i_{emn}$ and $g^i_{omn}$ are the scattered field expansion coefficients for the $i$th cell. Note that the maximum expansion of $n$ is $N_{o,i}$ orders which satisfy the convergence requirement for each cell. This parameter, $N_{o,i}$, is proportional to the size parameter $x_i$ of the cell $i$ in the cluster.

The incident field at the $i$th cell consists of the incident field of the beam plus scattered fields originated from all other cells in the cluster. Therefore the expansions of the fields at the surface of the $i$th cell take the following form [6],

$$\begin{bmatrix} f^i \\ g^i \end{bmatrix} = \mathbf{T}^i \left( \begin{bmatrix} \mathbf{a}^{i0} \\ \mathbf{b}^{i0} \end{bmatrix} + \sum_{l \neq i} \begin{bmatrix} \mathbf{A}(k\mathbf{r}_{li}) & \mathbf{B}(k\mathbf{r}_{li}) \\ \mathbf{B}(k\mathbf{r}_{li}) & \mathbf{A}(k\mathbf{r}_{li}) \end{bmatrix} \begin{bmatrix} f^l \\ g^l \end{bmatrix} \right), \quad i = 1, \ldots, Ns. \tag{5}$$

The way to calculate the expansion coefficients of the incident field $a^{i0}_{mn}$ and $b^{i0}_{mn}$ and the translation coefficients $A_{mn\mu\nu}(k\mathbf{r}_{li})$ and $B_{mn\mu\nu}(k\mathbf{r}_{li})$ is given in detail in [3] and [4]. The expansion coefficients of the individual scattered fields $f^i_{mn}$ and $g^i_{mn}$ can be computed for each of the cluster's cell using Eq. (5). Consequently these coefficients are directly introduced into Eq. (4) to compute the angular scattering field intensity of each cell. Finally Eq. (3) can be used to calculate total angular scattering intensity of the cluster.

Inversion of the system in Eq. (5) identifies the particle-centered $\mathbf{T}^{ij}$ matrix with respect to particle $j$ in the cluster which yields to [3],

$$\begin{bmatrix} f^i_{emn} \\ f^i_{omn} \\ g^i_{emn} \\ g^i_{omn} \end{bmatrix} = \sum_{j=1}^{N_s} \sum_{n'=1}^{N_{o,i}} \sum_{m'=-n}^{n'} T^{ij}_{mnm'n'} \begin{bmatrix} a^{tj}_{em'n'} \\ a^{tj}_{om'n'} \\ b^{tj}_{em'n'} \\ b^{tj}_{on'm'} \end{bmatrix}, \quad i = 1, 2, \ldots, N_s \tag{6}$$

where $T^{ij}$ transforms the incident field expansion coefficients to the scattered field coefficients of the $i$th particle. Total cross sections in both fixed and random orientations can be obtained from $T^{ij}$. To obtain the differential scattering cross sections (i.e., the scattering matrix), it is advantageous to transform the particle-centered $T^{ij}$ matrix to each origin of the particle $i$ in the cluster. This transformation takes the form:

$$T_{nl} = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \sum_{n'=1}^{N_{o,i}} \sum_{l'=1}^{N_{o,i}} j^{oi}_{nn'} T^{ij}_{n'l'} j^{jo}_{l'l} \tag{7}$$

where $J^{0i}$ and $J^{j0}$ matrices are formed from the scattering coefficients resulted from the coupling effect of particle $i$ with each other particle in the cluster as mentioned above [5]. All other parameters are given in [6]. Note that, the orientation averaged scattering matrix elements can be analytically obtained from the $T$-matrix of the scatterers using the procedures developed in [7], and [8]. Since the $T^{ij}$-matrix can be calculated, then total cross sections in both fixed and random orientation can be obtained.

Here the presented modified technique is applied to real life applications such as clusters consist of normal and abnormal RBCs. The computed angular scattering of a cluster composed from different cells as shown in Fig. 1(b) is computed and compared with those of a cluster of normal biconcave RBCs as shown in Fig. 1(a). Each biconcave particle is of size parameter $x = 10$, refractive index $m = 1.4$ and deformation parameter $\xi = 0.2$. Each abnormal cell presented in Fig. 1(b) (crescent shape) is formulated by a coated spherical cell of an offset core $l = 0.3\,\mu\text{m}$, coating parameter $R = a_c/a_s = 0.7$ (where $a_c$ and $a_s$ are the radius of the core and the shell respectively) [3]. The refractive indices for the shell and the core are $m_s = 1.4$, $m_c = 1.0$ respectively. The size parameter of each cell is $x = 10$. Both clusters shown in Fig. 1 are illuminated with a Gaussian beam propagating in the $z$-direction and polarized in the $xz$-plane. The beam's wavelength is $\lambda = 0.6328\,\mu\text{m}$ and waist $w_0 = 2\,\mu\text{m}$. The computed angular scattering is illustrated in Fig. 2 for different cases of the clusters. Case 1 is when the cluster consists of 5 biconcave cells(normal cells). Case 2 is when the cluster consists of 4 biconcave cells and one abnormal cell (crescent shape) and so on to the 6th case when the cluster contains 5 abnormal cells.

The results in Fig. 2 illustrate that the angular scattering intensity of a cluster contains more than 50% of abnormal cells (for example cases 5 and 6) deviate from the intensity distributions of the cluster contains normal cells (case 1). As the number of abnormal cells increases in the cluster of the sample compared with the normal RBCs the angular scattering distributions get more deviation and its ripples get smaller compared to those of the sample of normal cells. For the cluster of five abnormal cells, the angular scattering intensity has greater front scattering amplitudes compared with those for the cluster of normal RBCs as shown in Fig. 2 (yellow curve).

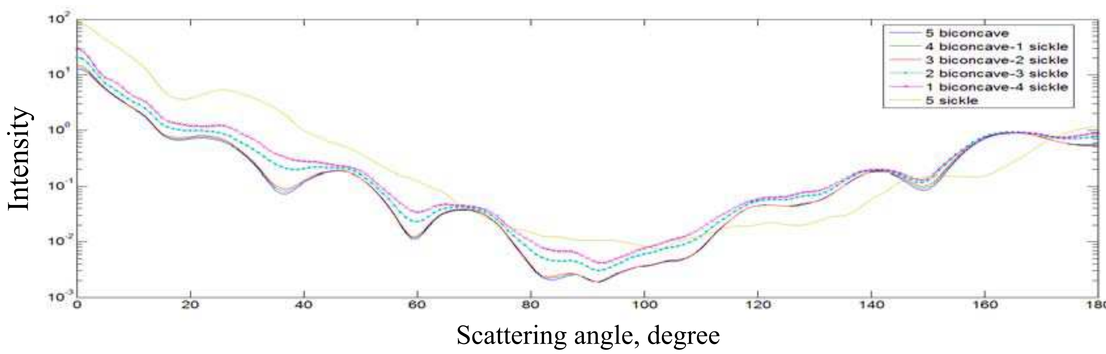


Figure 2: Angular scattering intensities as a function of the angle $\theta$ measured from the $z$-axis for a cluster consists of a linear chain of, (a) Case 1: five biconcave cells as illustrated in Fig. 1(a) (blue curve). Case 2: one abnormal cell in one side beside four biconcave cells (green curve). Case 3: two abnormal cells in one side beside three biconcave cells (red curve). Case 4: three abnormal cells in one side beside two biconcave cells (aqua curve). Case 5: two abnormal cells in both side of a centered biconcave cell as illustrated in Fig. 1(b) (purple curve). Case 6: five abnormal cells (yellow curve).

## 3. CONCLUSION

The scattering by a shain cluster consists of normal and abnormal blood cells illuminated with a focused Gaussian beam is studied analytically and numerically. The cluster $T$-matrix method is modified to combine the plane wave spectrum method to model the incident beam. It is shown that the angular scattering intensity distributions of a cluster of normal cells get more deviations as the number of the abnormal cells increases in the cluster. The illustrated numerical results are important in several branches of science and industry, such as nanotechnology, pharmaceuticals, chemistry, biology and moreover to improve the properties of polymers.

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