Scattering of an Obliquely Incident Electromagnetic Plane Wave by an Array of Magnetized Plasma Cylinders

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Abstract— Scattering of a linearly polarized electromagnetic plane wave by an array of magnetized parallel plasma cylinders is studied in the case of oblique incidence. It is shown that the interaction of individual and collective mechanisms of scattering can lead to either reduction or enhancement of the scattered field compared with the case of scattering by a single cylinder. In the case of incidence of an H-polarized wave with a zero longitudinal electric-field component, conditions have been found under which the scattered field is characterized by a pronounced layered spatial structure of this component and a chessboard-type 2D-periodic structure of the longitudinal magnetic-field component.

1. INTRODUCTION

In recent years, the issues of scattering of the electromagnetic radiation by two-dimensional arrays, photonic crystals, and other periodic structures formed by identical elements have attracted increased interest. Such an interest is stimulated by numerous applications of the corresponding structures in, e.g., lithography, near-field microscopy, wave diagnostics of media, information processing and transmission, etc. [1–3]. Despite significant progress in studies of electromagnetic wave scattering by arrays of nongyrotropic elements [1], the results recently obtained for arrays of gyrotropic scatterers have revealed a variety of new interesting features [2, 3]. In particular, it has been found that the presence of gyrotropy in photonic crystals can significantly influence the reflection and transmission of electromagnetic waves [2]. For example, nonreciprocal properties of such crystals can be used for creating materials with one-path transmission of electromagnetic energy, as well as slow-wave systems and wideband isolators in the optical band. Moreover, it can be expected that the joint contribution of the individual and collective resonance-scattering mechanisms to the diffracted field of an array containing frequency-dependent resonance elements can lead to some interesting phenomena, which are yet to be determined.

In this work, we study the scattering of an obliquely incident electromagnetic plane wave by an equidistant array of axially magnetized parallel plasma cylinders.

2. BASIC FORMULATION

Consider an equidistant array consisting of identical, infinitely long, circular cylinders of radius a, which are filled with a magnetoplasma and located in free space (see Figure 1). The cylinders are aligned with the z axis of a Cartesian coordinate system (x, y, z). The axes of the cylinders lie in the xz plane and are separated by distance L, so that the coordinates of axes are specified by the relations $x = jL$ and $y = 0$, where $j = 0, \pm 1, \pm 2, \ldots$. An external static magnetic field $\mathbf{B}_0$ is parallel to the axes of the cylinders. The cold collisionless magnetoplasma is described by the permittivity tensor $\vec{\varepsilon}$ with the nonzero elements $\varepsilon_{\rho\rho} = \varepsilon_{\phi\phi} = \varepsilon$, $\varepsilon_{\rho\phi} = -\varepsilon_{\phi\rho} = -ig$, and $\varepsilon_{zz} = \eta$ in a cylindrical coordinate system (ρ, ϕ, z). Here, $\varepsilon = 1 - \omega_p^2/(\omega^2 - \omega_H^2)$, $g = \omega_p^2 \omega_H / [(\omega^2 - \omega_H^2)\omega]$, and $\eta = 1 - \omega_p^2/\omega^2$, where ω_p and ω_H are the plasma frequency and the gyrofrequency of electrons, respectively, and ω is the angular frequency. When writing the tensor elements, we neglected the contribution of ions. This is possible under the condition $\omega \gg \omega_{\mathrm{LH}}$, which is assumed throughout

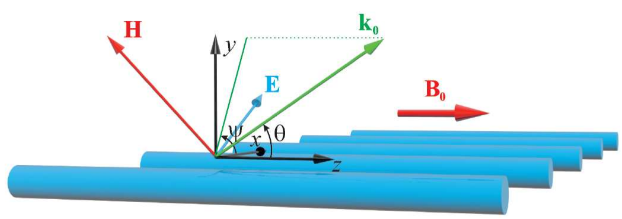

Figure 1: Geometry of the problem.

this work (here, $\omega_{\rm LH}$ is the lower hybrid resonance frequency). Note that Gaussian units are used in this paper.

Let a plane monochromatic wave with frequency ω (the time factor $\sim e^{i\omega t}$ is assumed) be incident on the array at an angle θ to the z axis such that the wave vector projection onto the xy plane makes an angle ψ with the x axis. It is assumed that the wavelength $\lambda = 2\pi c/\omega$, where c is the speed of light in free space, is much longer than the cylinder radius a. The field in the incident wave can be written as

$$\mathbf{E}^{(i)} = \mathbf{E}_0 \exp\left[-ik_0\left(qx\cos\psi + qy\sin\psi + pz\right)\right], \quad \mathbf{H}^{(i)} = \mathbf{H}_0 \exp\left[-ik_0\left(qx\cos\psi + qy\sin\psi + pz\right)\right]. \quad (1)$$

Hereafter, $k_0 = \omega/c$ is the wave number in free space, $p = \cos\theta$ and $q = \sin\theta$ are the normalized (to k_0) longitudinal and transverse components of the wave vector $\mathbf{k}_0$ in the incident wave, respectively, and the superscript (i) denotes the incident wave.

In the cylindrical coordinate system $(\rho_j,\ \phi_j,\ z)$ related to the jth cylinder, the total field can be represented in terms of the azimuthal harmonics with the indices $m = 0,\ \pm1,\ \pm2,\ \ldots$ as follows:

$$\mathbf{E} = \sum_{m=-\infty}^{\infty} \mathbf{E}_m\left(\rho_j\right) \exp\left[-i(m\phi_j + k_0 pz)\right], \quad \mathbf{H} = \sum_{m=-\infty}^{\infty} \mathbf{H}_m(\rho_j) \exp\left[-i(m\phi_j + k_0 pz)\right]. \quad (2)$$

In turn, the vector quantities $\mathbf{E}_m(\rho_j)$ and $\mathbf{H}_m(\rho_j)$ can be expressed via their longitudinal components $E_{z;j,m}$ and $H_{z;j,m}$, which satisfy the following equations in the plasma medium [4]:

$$\hat{L}_m E_{z;j,m} - k_0^2 \frac{\eta}{\varepsilon}(p^2 - \varepsilon) E_{z;j,m} = -ik_0^2 \frac{g}{\varepsilon} p H_{z;j,m}, \quad \hat{L}_m H_{z;j,m} - k_0^2\left(p^2 + \frac{g^2}{\varepsilon} - \varepsilon\right) H_{z;j,m} = ik_0^2 \frac{g}{\varepsilon} \eta p E_{z;j,m}, \quad (3)$$

where $\hat{L}_m = \frac{d^2}{d\rho_j^2} + \frac{1}{\rho_j}\frac{d}{d\rho_j} - \frac{m^2}{\rho_j^2}$. The transverse field components $E_{\rho;j,m}$, $E_{\phi;j,m}$, $H_{\rho;j,m}$, and $H_{\phi;j,m}$ are expressed via the longitudinal components $E_{z;j,m}$ and $H_{z;j,m}$ as

$$E_{\rho;j,m} = A\left\{ipg\frac{m}{\rho_j}E_{z;j,m} + ip\left(\varepsilon - p^2\right)\frac{dE_{z;j,m}}{d\rho_j} + \left(\varepsilon - p^2\right)\frac{m}{\rho_j}H_{z;j,m} + g\frac{dH_{z;j,m}}{d\rho_j}\right\}, \quad (4)$$

$$E_{\phi;j,m} = A\left\{p\left(\varepsilon - p^2\right)\frac{m}{\rho_j}E_{z;j,m} + pg\frac{dE_{z;j,m}}{d\rho_j} - ig\frac{m}{\rho_j}H_{z;j,m} - i\left(\varepsilon - p^2\right)\frac{dH_{z;j,m}}{d\rho_j}\right\}, \quad (5)$$

$$H_{\rho;j,m} = A\left\{\left[g^2 - \varepsilon\left(\varepsilon - p^2\right)\right]\frac{m}{\rho_j}E_{z;j,m} - p^2 g\frac{dE_{z;j,m}}{d\rho_j} + ipg\frac{m}{\rho_j}H_{z;j,m} + ip\left(\varepsilon - p^2\right)\frac{dH_{z;j,m}}{d\rho_j}\right\}, \quad (6)$$

$$H_{\phi;j,m} = A\left\{ip^2 g\frac{m}{\rho_j}E_{z;j,m} - i\left[g^2 - \varepsilon\left(\varepsilon - p^2\right)\right]\frac{dE_{z;j,m}}{d\rho_j} + p\left(\varepsilon - p^2\right)\frac{m}{\rho_j}H_{z;j,m} + pg\frac{dH_{z;j,m}}{d\rho_j}\right\}, \quad (7)$$

where $A = k_0^{-1}[g^2 - (p^2 - \varepsilon)^2]^{-1}$.

To obtain equations for the longitudinal field components and expressions for the corresponding transverse components outside the cylinder, one should put $\varepsilon = 1$, $g = 0$, and $\eta = 1$ in Eqs. (3)–(7).

The azimuthal harmonics of the longitudinal field components inside the jth cylinder are represented in the form [4]

$$E_{z;j,m}^{(t)} = \frac{i}{\eta}\sum_{k=1}^{2} B_{j,m}^{(k)} n_k J_m(k_0 q_k \rho_j), \quad H_{z;j,m}^{(t)} = -\sum_{k=1}^{2} B_{j,m}^{(k)} J_m(k_0 q_k \rho_j). \quad (8)$$

Here, the superscript (t) denotes the field transmitting to the cylinder, J_m is a Bessel function of the first kind of order m, $B_{j,m}^{(1)}$ and $B_{j,m}^{(2)}$ are the amplitude coefficients corresponding to the azimuthal index m for the field inside the jth cylinder, and

$$\begin{aligned} q_k^2(p) &= \frac{1}{2\varepsilon}\left\{\varepsilon^2 - g^2 + \varepsilon\eta - (\varepsilon+\eta)p^2 + (-1)^k\left[(\varepsilon-\eta)^2 p^4 + 2\left(g^2(\varepsilon+\eta) - \varepsilon(\varepsilon-\eta)^2\right)p^2 + \left(\varepsilon^2 - g^2 - \varepsilon\eta\right)^2\right]^{1/2}\right\}, \\ n_k &= -\frac{\varepsilon}{pg}\left(p^2 + q_k^2 + \frac{g^2}{\varepsilon} - \varepsilon\right), \quad k = 1, 2. \end{aligned} \quad (9)$$

The presence of two transverse wave numbers q_1 and q_2 in the magnetized plasma medium, which correspond to the same longitudinal wave number p, is related to anisotropic properties of a magnetoplasma, in which two normal waves, ordinary and extraordinary, are excited by an obliquely incident plane electromagnetic wave. This circumstance represents an essential difference of the problem discussed herein from that in [5], where an H-polarized plane wave incident normally on the array of gyrotropic cylinders was considered and only the extraordinary wave could be excited inside the cylinders.

The azimuthal harmonics of the longitudinal field components in the incident plane wave are written as

$$E^{(i)}_{z;j,m}=E_{0z}(-i)^m J_m(k_0q\rho_j)e^{i(m\psi-k_0qLj\cos\psi)}, \quad H^{(i)}_{z;j,m}=H_{0z}(-i)^m J_m(k_0q\rho_j)e^{i(m\psi-k_0qLj\cos\psi)}, \quad (10)$$

where E_{0z} and H_{0z} are constants.

The total scattered field, denoted by the superscript (s), is also written in terms of the cylindrical functions and has the following longitudinal components (with the $\exp(-ik_0pz)$ factor dropped):

$$E^{(s)}_z=\sum_{j=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} D^{(\mathrm{E})}_{j,m}H^{(2)}_m(k_0q\rho_j)e^{-im\phi_j}, \quad H^{(s)}_z=\sum_{j=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} D^{(\mathrm{H})}_{j,m}H^{(2)}_m(k_0q\rho_j)e^{-im\phi_j}, \quad (11)$$

where $H^{(2)}_m$ is a Hankel function of the second kind of order m, and $D^{(\mathrm{E})}_{j,m}$ and $D^{(\mathrm{H})}_{j,m}$ are the scattering coefficients of jth cylinder, which correspond to the azimuthal index m.

Satisfying the boundary conditions for the tangential field components on the surface of the jth cylinder and using the technique based on the scattering matrix method [6], we can exclude the coefficients $B^{(1,2)}_{j,m}$ and obtain a system of equations for the scattering coefficients $D^{(\mathrm{E})}_{j,m}$ and $D^{(\mathrm{H})}_{j,m}$ in the form

$$\begin{aligned}
\tilde{S}^{\mathrm{HE}}_m D^{(\mathrm{E})}_{j,m}+\tilde{S}^{\mathrm{HH}}_m D^{(\mathrm{H})}_{j,m}=&\sum_{n=-\infty}^{\infty}\left[\sum_{l<j}(-1)^{m-n}D^{(\mathrm{H})}_{l,n}H^{(2)}_{m-n}(k_0qL|j-l|)+\sum_{l>j}D^{(\mathrm{H})}_{l,n}H^{(2)}_{m-n}(k_0qL|j-l|)\right]\\
&+H_{0z}(-i)^m\exp\left[i(m\psi-k_0qLj\cos\psi)\right],\\
\tilde{S}^{\mathrm{EE}}_m D^{(\mathrm{E})}_{j,m}+\tilde{S}^{\mathrm{EH}}_m D^{(\mathrm{H})}_{j,m}=&\sum_{n=-\infty}^{\infty}\left[\sum_{l<j}(-1)^{m-n}D^{(\mathrm{E})}_{l,n}H^{(2)}_{m-n}(k_0qL|j-l|)+\sum_{l>j}D^{(\mathrm{E})}_{l,n}H^{(2)}_{m-n}(k_0qL|j-l|)\right]\\
&+E_{0z}(-i)^m\exp\left[i(m\psi-k_0qLj\cos\psi)\right],
\end{aligned} \quad (12)$$

where $\tilde{S}^{\mathrm{HH}}_m$, $\tilde{S}^{\mathrm{HE}}_m$, $\tilde{S}^{\mathrm{EH}}_m$, and $\tilde{S}^{\mathrm{EE}}_m$ are the elements of the matrix that is inverse of the single-cylinder scattering matrix. To find these elements, we consider the system of equations

$$\begin{pmatrix}
\frac{in_1}{\eta}J^{(1)}_m & \frac{in_2}{\eta}J^{(2)}_m & -H^{(2)}_m & 0\\
i\hat{J}^{(1)}_m & i\hat{J}^{(2)}_m & \frac{pm}{Q^2}H^{(2)}_m & -\frac{i}{Q}H^{(2)\prime}_m\\
-J^{(1)}_m & -J^{(2)}_m & 0 & -H^{(2)}_m\\
-n_1\tilde{J}^{(1)}_m & -n_2\tilde{J}^{(2)}_m & \frac{i}{Q}H^{(2)\prime}_m & \frac{pm}{Q^2}H^{(2)}_m
\end{pmatrix}
\begin{pmatrix} R^{(1)}_m\\ R^{(2)}_m\\ S^{(1)}_m\\ S^{(2)}_m\end{pmatrix}
=\begin{pmatrix} J_m\\ -\frac{pm}{Q^2}J_m\\ 0\\ -\frac{i}{Q}J'_m\end{pmatrix}C^{(\mathrm{E})}_m+
\begin{pmatrix} 0\\ \frac{i}{Q}J'_m\\ J_m\\ -\frac{pm}{Q^2}J_m\end{pmatrix}C^{(\mathrm{H})}_m, \quad (13)$$

where

$$J^{(k)}_m=J_m(Q_k), \quad \hat{J}^{(k)}_m=\frac{J^{(k)}_{m+1}}{Q_k}+\alpha_k m\frac{J^{(k)}_m}{Q_k^2}, \quad \tilde{J}^{(k)}_m=\frac{J^{(k)}_{m+1}}{Q_k}-\beta_k m\frac{J^{(k)}_m}{Q_k^2},$$

$$Q_k=k_0q_ka, \quad Q=k_0qa, \quad \alpha_k=-1+\frac{p^2+q_k^2-\varepsilon}{g}, \quad \beta_k=1+\frac{p}{n_k}.$$

In system (13), the prime indicates the derivative with respect to the argument, and the arguments of the functions $J_m(Q)$ and $H^{(2)}_m(Q)$ are omitted for brevity. Upon finding the solutions $R^{(1,2)}_m$ and $S^{(1,2)}_m$ of system (13), the desired quantities $\tilde{S}^{\mathrm{HH}}_m$, $\tilde{S}^{\mathrm{HE}}_m$, $\tilde{S}^{\mathrm{EH}}_m$, and $\tilde{S}^{\mathrm{EE}}_m$ are represented as

$$\tilde{S}^{\mathrm{HH}}_m=S_{11}/\Delta, \quad \tilde{S}^{\mathrm{HE}}_m=-S_{12}/\Delta, \quad \tilde{S}^{\mathrm{EH}}_m=-S_{21}/\Delta, \quad \tilde{S}^{\mathrm{EE}}_m=S_{22}/\Delta, \quad \Delta=S_{11}S_{22}-S_{12}S_{21}, \quad (14)$$

where $S_{11} = S_m^{(1)}$ and $S_{12} = S_m^{(2)}$ under the conditions $C_m^{(\mathrm{H})} = 0$ and $C_m^{(\mathrm{E})} = 1$, while $S_{21} = S_m^{(1)}$ and $S_{22} = S_m^{(2)}$ under the conditions $C_m^{(\mathrm{H})} = 1$ and $C_m^{(\mathrm{E})} = 0$.

Using the translational symmetry of the problem, i.e., the identity $\mathbf{H}^{(s)}(x+L, y, z) = \mathbf{H}^{(s)}(x, y, z) \times e^{-ik_0qL\cos\psi}$, we can establish the relation between the terms $D_{j+1,m}^{(\mathrm{E,H})}$ and $D_{j,m}^{(\mathrm{E,H})}$ in the form $D_{j+1,m}^{(\mathrm{E,H})} = D_{j,m}^{(\mathrm{E,H})} e^{-ik_0qL\cos\psi}$. Hence, we may put $D_{j,m}^{(\mathrm{E,H})} = \hat{D}_m^{(\mathrm{E,H})} e^{-ik_0qLj\cos\psi}$, where $\hat{D}_m^{(\mathrm{E,H})} \equiv D_{0,m}^{(\mathrm{E,H})}$. As a result, we arrive at the following system of equations for $\hat{D}_m^{(\mathrm{E,H})}$:

$$\begin{aligned} \tilde{S}_m^{\mathrm{HE}} \hat{D}_m^{(\mathrm{E})} + \tilde{S}_m^{\mathrm{HH}} \hat{D}_m^{(\mathrm{H})} &= H_{0z}(-i)^m e^{im\psi} + \sum_{n=-\infty}^{\infty} \hat{D}_n^{(\mathrm{H})} G_{n-m}, \\ \tilde{S}_m^{\mathrm{EE}} \hat{D}_m^{(\mathrm{E})} + \tilde{S}_m^{\mathrm{EH}} \hat{D}_m^{(\mathrm{H})} &= E_{0z}(-i)^m e^{im\psi} + \sum_{n=-\infty}^{\infty} \hat{D}_n^{(\mathrm{E})} G_{n-m}, \end{aligned} \tag{15}$$

where

$$G_m = \sum_{l=1}^{\infty} H_m^{(2)}(k_0qLl) \left[e^{ik_0qLl\cos\psi} + (-1)^m e^{-ik_0qLl\cos\psi} \right]. \tag{16}$$

The above expressions allow us to proceed to numerical calculations of the scattered field, the results of which are discussed in what follows.

3. NUMERICAL RESULTS

We now consider the scattering of a monochromatic H-polarized plane wave whose magnetic field has a nonzero H_z component, but a zero E_z component. All field components are normalized to the magnetic-field amplitude in the incident wave.

Numerical calculations were performed for the following values of parameters: $\omega_p/\omega_H = 8$, $\omega_p a/c = 0.188$, and $\psi = \pi/2$. Figures 2(a) and 2(b) show the absolute values of the scattered and total magnetic fields at the resonant frequency $\omega = 6.043\omega_H$ for normal incidence on the array when $\theta = \pi/2$. In this case, the scattered field has a layered structure [Figure 2(a)] and is almost periodic along the x axis. Note that the maximum value of the scattered field at the indicated frequency, which is reached near the array plane, exceeds the magnetic-field amplitude in the incident wave. Because of this, the total field [Figure 2(b)], which is formed due to interference of the incident and scattered waves, has a cell-like spatial structure with quasi-periodically alternating maxima and minima in the direction of the y axis. In this case, the magnetic-field structure is almost symmetric with respect to the array plane $y = 0$.

In the case of oblique incidence where $\theta \neq \pi/2$, the scattered field is hybrid, i.e., the longitudinal components of both the electric and magnetic fields are nonzero. Let us dwell on the case where the H wave is incident on the array at the angle $\theta = \pi/4$ to the axes of the cylinders. Analysis of the frequency dependences of the scattering coefficients of the electric- and magnetic-field components shows that near the resonant frequencies of the array, frequency ranges exist in which the absolute values of these coefficients are much smaller than the field amplitude in the incident wave.

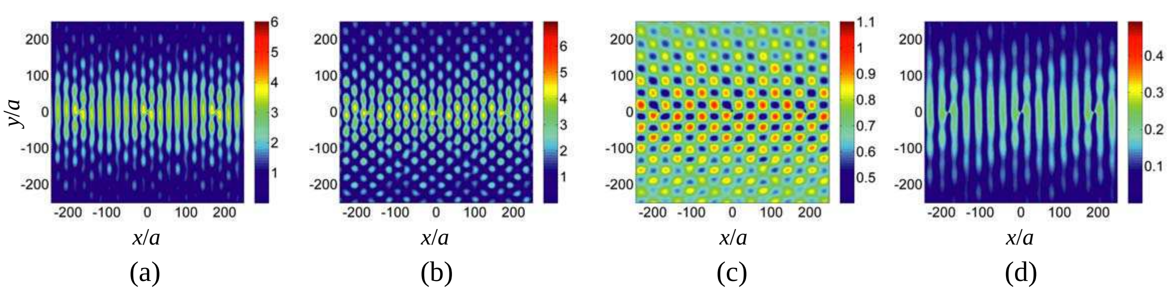

Figure 2: Absolute values of the longitudinal component of (a) the scattered and (b) total magnetic fields for normal incidence of the H wave on the array at the resonant frequency $\omega = 6.043\omega_H$ for $k_0L/2\pi = 3.9909$, $L/a = 177$, and $\theta = \pi/2$, and the absolute values of the longitudinal component of (c) the total magnetic and (d) electric fields for oblique incidence of the H wave on the array at the angle $\theta = \pi/4$ to the axes of the cylinders for $\omega = 6.023\omega_H$, $k_0qL/2\pi = 4.2465$, and $L/a = 189$.

Moreover, at the resonant frequencies of the array, these coefficients vanish. Thus, the interaction of the individual and collective scattering mechanisms can lead to a significant weakening and even suppression of the scattered field.

Note that in the case of oblique incidence of the H wave, the longitudinal component of the total magnetic field comprises the corresponding components of the incident and scattered waves, whereas the longitudinal component of the total electric field is determined only by the corresponding component of the scattered field. This feature is illustrated by Figures 2(c) and 2(d), which show the spatial structures of the magnitudes of the longitudinal magnetic- and electric-field components at the frequency $\omega = 6.023\omega_H$, which is slightly higher than the nearest resonant frequency of the array. In this case, the electric field has the layered structure and its amplitude is comparable with the amplitude in the incident wave. The corresponding component of the magnetic field has the 2D-periodic chessboard-type structure.

4. CONCLUSION

In this work, we have studied the scattering of an H-polarized electromagnetic plane wave by an array of equidistant parallel cylinders filled with a cold collisionless magnetoplasma. The solution for the field in the case of incidence at an arbitrary angle to the axes of the cylinders has been obtained. It is shown that the interaction of the individual and collective scattering mechanisms can lead to either enhancement or weakening of scattering compared with the case of a single cylinder. Conditions have been determined under which this or that scattering mechanisms gives a dominant contribution to the scattering by the array. It is shown that in the case of oblique incidence of the H wave on the array, the longitudinal component of the electric field can have, under certain conditions, a layered structure elongated in the direction perpendicular to the array, whereas the longitudinal component of the magnetic field can have a 2D-periodic chessboard-type structure.

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