On the Characteristics of Spoof Surface Plasmons (SSP) in the High Frequency Limit

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Abstract— In this paper, we show that diffraction from the subwavelength features at the surface of a perfect electric conductor (PEC) plays a very important role in determining the near-field characteristics and dispersion of spoof surface plasmon (SSP) in the high-frequency limit. Using constraints posed by the boundary conditions, we show that in this limit the field profile of SSP consists of multiple evanescent orders. We have determined the corresponding decay constants using angular spectrum representation and have verified the multi-exponential nature of the field profile against data obtained from full vectorial 3-D finite difference time domain (FDTD) simulation. Further, we use tight-binding model to describe the inter-hole interaction and obtain a closed form expression for the SSP dispersion relation and analytic prediction for the self-collimation frequency with reasonable accuracy.

1. INTRODUCTION

In the low frequency limit (i.e., when operating wavelength is very large compared to the feature-size and periodicity), the field profile of spoof surface plasmon (SSP) is characterized by a single evanescent order and the corresponding dispersion relation is described by an expression analogous to the dispersion relation of surface plamon polaritons (SPP) [1]. However, in the high-frequency limit, the analogy between SSP and SPP breaks down [2, 3]. As the feature-size becomes comparable to the wavelength, the low frequency dispersion relation starts deviating from the dispersion calculated numerically [4].

In this paper, using arguments based on boundary conditions, we show that the field profile of SSP cannot have a single exponential decay. Instead, there should be multiple evanescent orders associated with it. These evanescent orders are determined using angular spectrum representation [5, 6]. Also, we describe how the presence of multiple evanescent orders leads to anisotropic propagation of this surface wave. Note that anisotropy of SSP propagation is very important in realization of several applications like self-collimation (SC), beam-splitting etc. [7, 8]. However, the SPP-type low frequency limit dispersion relation of SSP cannot predict this anisotropic propagation or self-collimation frequency. To this end, we note that the scattered field from each of the holes can interact with its neighboring holes in a manner analogous to the electron orbitals of different atoms in a crystal [9]. We show that a tight binding model which takes into account next nearest neighbor interaction can accurately describe the SSP dispersion relation in the high frequency limit and also can predict the SC frequency.

2. MULTIPLE EVANESCENT ORDERS

The structure under consideration has been shown in Fig. 1. It consists of a PEC block on which square holes have been drilled to form a 2D array in xy-plane. The side of each hole is 8.75 mm and the lattice constant is 10 mm. The depth of each hole is 40 mm. So, the plane $z = 40$ mm denotes the interface which supports the surface wave. Since, the holes on the PEC surface are of subwavelength dimension, the diffracted field from these features would be evanescent in the direction perpendicular to the interface (z-direction). The decay length (α) and the spatial frequencies (f_x, f_y) of the surface wave are related by: $\alpha^2 + k^2 = 4\pi^2(f_x^2 + f_y^2)$. Here, k is the wavenumber in the dielectric above the interface. This leaves us with two choices: either α would have a unique value (if this is the case, the near-filed would be exactly analogous to SPP and the propagation would be isotropic since, $(f_x^2+f_y^2)$ would be constant at a fixed frequency) or, α would have multiple values for a fixed frequency (in this case, the near field would be described by multiple evanescent orders in contrast to a single evanescent order for SPP and low-frequency SSP. Also, the propagation would be anisotropic). However, the boundary conditions suggest that the transition of the field across the interface should be smooth if there is no change in the dielectric constant below and above the interface. For single exponential decay, the transition of the field profile across the interface would not be smooth. However, superposition of multiple evanescent orders can make the transition smooth (for example, this phenomenon is found in fluorescence from a bulk material,

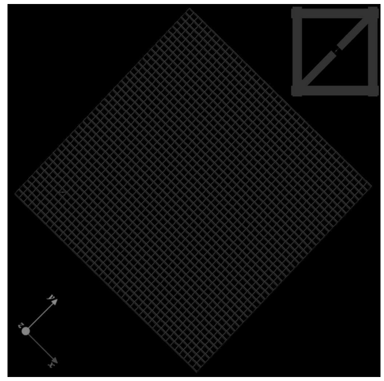

Figure 1: The 41×41 array of air-holes patterned on a PEC block. At the top right corner, the inset shows a magnified view of the cell with a pair of thin strips fed by a current source at 15.1 GHz.

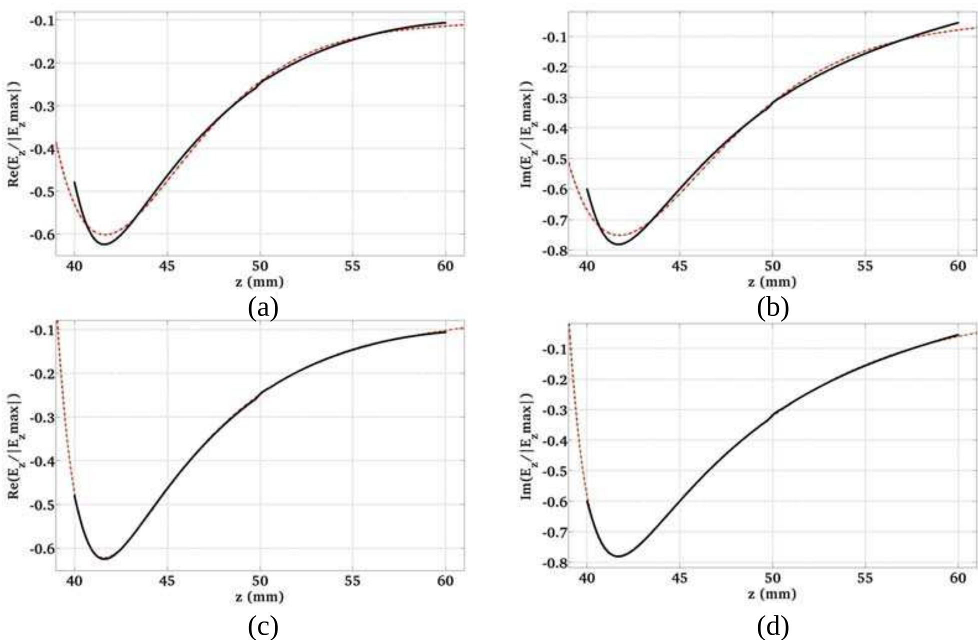

Figure 2: Fitting of (a), (c) real and (b), (d) imaginary parts of E_z above the interface corresponding to FDTD results are shown with black/solid line and fitted results are shown with red/dash-dotted line. The decay constants used are 0.162, 0.173, 0.153, 0.144, 0.90. (a) and (b) show degradation in the fitting when the decay constant 0.90 is excluded.

where stretched exponential decay [10] is used to describe the emission decay curves). In addition, the aperture of the hole occupies most part of the unit cell and constitutes a considerable fraction of the operating wavelength. So, in the high frequency limit, one expects the field profile of SSP to be described by superposition of multiple evanescent orders.

To show this, we have plotted the field profile obtained from FDTD simulation in Fig. 2 and compared this profile with multiple exponential fitting. The decay constants are obtained from [3].

3. HIGH FREQUENCY LIMIT DISPERSION RELATION

According to the tight binding model [11], the most general representation for the eigenenergies of the electrons in a crystal is given as:

$$E_k = \sum_{\vec{R}} e^{i\vec{k}.\vec{R}} H_{m,n}\left(\vec{R}\right) \tag{1}$$

where, $\vec{R}$ is a lattice vector and $H_{m,n}(\vec{R})$ denotes the hopping parameters or the coupling between the orbitals of atoms at the mth and nth lattice sites. In a similar way, the eigenfrequencies of the modes in a photonic crystal can be written as:

$$\omega_k = \sum_{\vec{R}} e^{i\vec{k}.\vec{R}} H_{m,n}\left(\vec{R}\right) \tag{2}$$

Under the next nearest neighbor approximation, Eq. (2) can be written as:

$$\begin{aligned}\omega_k &= t_0 - t_1 e^{ik_x a} - t_1 e^{ik_y a} - t_1 e^{-ik_x a} - t_1 e^{-ik_y a} \\ &\quad - t_2\left[e^{ik_x a}e^{ik_y a} + e^{ik_x a}e^{-ik_y a} + e^{-ik_x a}e^{ik_y a} + e^{-ik_x a}e^{-ik_y a}\right] \\ \Rightarrow \omega_k &= t_0 - 2t_1\left[\cos(k_x a) + \cos(k_y a)\right] - 4t_2\cos(k_x a)\cos(k_y a)\end{aligned} \tag{3}$$

Next, we illustrate a simple procedure to determine the hopping parameters based on a knowledge of the eigen frequencies at the high symmetry points. At Γ, X and M point Eq. (3) can be written as:

$$t_0 - \omega_\Gamma = 4(t_1 + t_2) \tag{4}$$

$$\omega_X = t_0 + 4t_2 \tag{5}$$

$$\omega_M = t_0 + 4t_1 - 4t_2 \tag{6}$$

Substituting the expression of t_0 from Eq. (4) into Eq. (6), t_1 can be evaluated as:

$$t_1 = \frac{1}{8}\left(\omega_M - \omega_\Gamma\right) \tag{7}$$

Using this expression for t_1 Eq. (4) and (5) can be solved for the other two hopping parameters as:

$$t_0 = \frac{1}{2}\left[\omega_X + \left(\frac{\omega_\Gamma + \omega_X}{2}\right)\right] \tag{8}$$

$$t_2 = \frac{1}{8}\left[\omega_X - \left(\frac{\omega_\Gamma + \omega_X}{2}\right)\right] \tag{9}$$

With the hopping parameters determined uniquely, we can now proceed to calculate the SC frequency using Eq. (3). At SC, the curvature of the EFC tends to infinity at $k_x = k_y$. In other words, the condition for having a flat contour amounts to $\frac{d^2k_y}{dk_x^2} = 0$ at $k_x = k_y$. From Eq. (3), the expression for the EFCs can be written as:

$$t_0 - 2t_1\left[\cos(k_x a) + \cos(k_y a)\right] - 4t_2\cos(k_x a)\cos(k_y a) = \text{constant} \tag{10}$$

For convenience of representation, we assume $(k_x a) = \chi$ and $(k_y a) = \xi$. Differentiating Eq. (10) w.r.t. χ, we get:

$$t_1\left(\sin\chi + \sin\xi\frac{d\xi}{d\chi}\right) + 2t_2\left(\sin\chi cos\xi + cos\chi\sin\xi\frac{d\xi}{d\chi}\right) = 0 \tag{11}$$

Thus,

$$\frac{d\xi}{d\chi} = -\frac{\sin\chi(t_1 + 2t_2\cos\xi)}{\sin\xi(t_1 + 2t_2\cos\chi)} \tag{12}$$

So, at $\chi = \xi$, $\frac{d\xi}{d\chi} = -1$.

Differentiation of Eq. (11) once again w.r.t. χ yields:

$$\frac{d^2\xi}{d\chi^2} + \left(\frac{d\xi}{d\chi}\right)^2\frac{\cos\xi}{\sin\xi} - \left(\frac{d\xi}{d\chi}\right)\frac{4t_2\sin\chi}{(t_1 + 2t_2\cos\chi)} + \frac{\cos\chi(t_1 + 2t_2\cos\xi)}{\sin\xi(t_1 + 2t_2\cos\chi)} = 0 \tag{13}$$

Now, using the the condition $\frac{d^2\xi}{d\chi^2} = 0$ at $\chi = \xi$ in the above equation, we obtain the condition for existence of a flat contour as:

$$\frac{2t_2\sin\chi}{(t_1 + 2t_2\cos\chi)} + \frac{\cos\chi}{\sin\chi} = 0 \quad\Rightarrow\quad 2t_2 + t_1\cos\chi = 0 \quad\Rightarrow\quad \cos(k_x a) = \cos(k_y a) = \left(-\frac{2t_2}{t_1}\right) \tag{14}$$

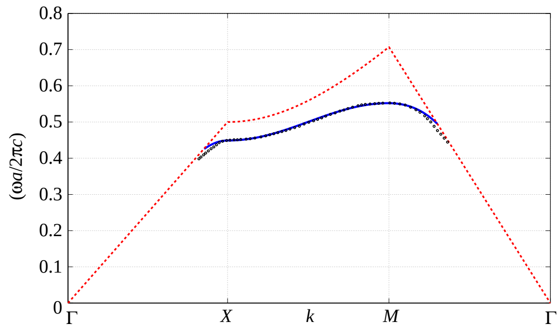

Figure 3: High frequency limit band diagram of SSP. Blue/solid line: dispersion diagram obtained using our theory, black circles: dispersion diagram obtained in [4] using FDTD, red/dashed line: light line.

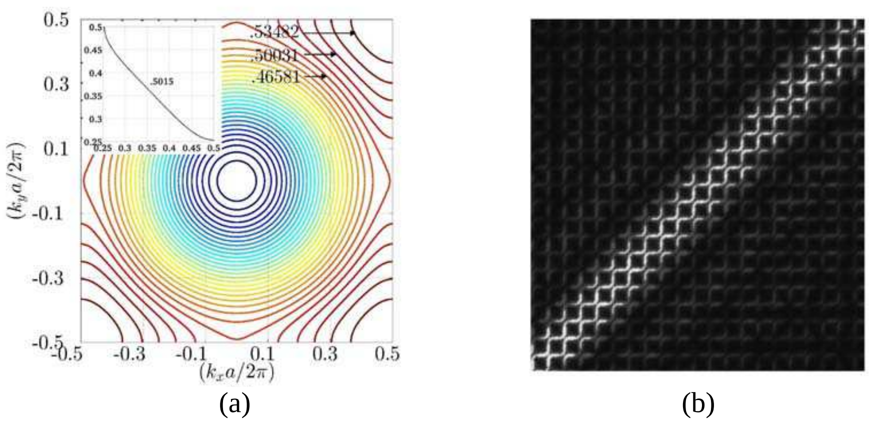

Figure 4: (a) Equi-frequency contour (EFC) plot for SSP, obtained using dispersion relation described by Eq. (3). The hopping parameters used in this plot are: $t_0 = 0.3625$, $t_1 = 0.0691$ and $t_2 = 0.0216$. The frequencies (f) corresponding to the level curves are normalized by (c/a). The inset shows the flat contour predicted by Eq. (15) at $f_{SC} = 0.5015(c/a)$. (b) Magnitude plot of the self-collimated SSP beam observed at $0.5033(c/a)$ in our FDTD simulation.

Inserting the above condition into Eq. (10), the SC frequency can be obtained as:

$$\omega_{SC} = t_0 + 8t_2 \left(1 - \frac{2t_2^2}{t_1^2}\right) \tag{15}$$

As an example, we take the values of the eigenfrequencies at 3 high-symmetry points from [4] as $\omega_\Gamma = 0$, $\omega_X = 0.45$ and $\omega_M = 0.545$. Using these values, we calculate the hopping parameters (normalized by $\frac{2\pi c}{a}$) as $t_0 = 0.3613$, $t_1 = 0.0681$ and $t_2 = 0.0222$. Figure 3 and Figure 4 show the corresponding band diagram and EFC plot, respectively, obtained by plugging in the hopping parameters into Eq. (3). The SC frequency predicted by Eq. (15) is $0.5015(c/a)$ and in our FDTD simulation we observed SC around $0.5033(c/a)$.

4. CONCLUSION

In summary, we have investigated the high frequency limit characteristics of SSP. Our analysis reveals that in this limit, the decay profile of SSP emerges from multiple diffracted evanescent orders. The existence of multiple evanescent orders play a significant role in determining the anisotropic propagation characteristic of SSP. Further, this anisotropy in the propagation and the corresponding dispersion relation can be analytically described by tight-binding model of atomic crystals.

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