Analogy between the Ising Model and the Polarization Switching of Vertical-cavity Surface-emitting Lasers

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Abstract— Conventional vertical-cavity surface-emitting lasers (VCSELs) have a cylindrical laser cavity, which provides an intrinsic two-degree of freedom to the polarization direction of the emitted light. This isotropic symmetry of polarization is usually broken by the anisotropy of the physical structure of VCSELs; for example, the anisotropy of the lattice structure, residual stress, or thermal distribution. In some VCSELs, the broken polarization symmetry conducts a polarization switching (PS) phenomenon. During this process, the VCSEL remains in a phase of emitting linearly polarized light around the laser's threshold, and then switches to another phase of emitting orthogonally polarized light when the laser's current surpasses a switching current. The VCSEL's PS (VPS) can help to build some components for the high speed optical computing. Accordingly, the characteristic of VPS is an important research pursuit. Recently, VPS was reported to be a second-order phase transition (SOPT). That discovery was based on the criticalities observed in the experiments, which are the unique characteristics of SOPTs. This research attempts to explore the connection between VPS and the conventional Ising model. The said model employs the spin-up and spin-down spins to study the transition of magnetic materials from the ferromagnetic state to the paramagnetic state and identifies that transition as an SOPT. Therefore, both of the phase transitions of VPS and Ising model are SOPTs between two orthogonal states. Typical Ising model employs a two dimensional spin lattice to simulate the magnetic material. In this work, a Gaussian-distributed interaction was introduced into the center of the spin lattice to simulate the laser's beam profile and the spatial coherence. The investigation focused on the examination of the relaxation time required to transit from the ferromagnetic state to the paramagnetic state. The simulation results present an exponent of the relaxation time of about 2.23. This result approximates the experimental result of 2.3 well, revealing a success of this model. Some other results will also be reported.

1. INTRODUCTION

The laser cavity of conventional vertical-cavity surface-emitting lasers (VCSELs) is a cylinder with a relatively large cross section [1, 2]. This geometry provides a two-dimensional isotropic symmetry on the laser's facet. Therefore, the polarization direction of the laser's output intrinsically has a two-degree freedom. This symmetry is usually broken by the anisotropy of the physical structure of VCSELs [1, 2]. The broken symmetry causes the laser to emit a linearly polarized light with its polarization along a specific direction as the injection current becomes greater than the threshold current. In some VCSELs, when the injection current is increased further, that polarization will switch to the orthogonal direction at a current designated as the polarization switching (PS) current, exhibiting the PS phenomenon [1–6]. Many researches attributed the VCSEL's PS (VPS) to thermal effects induced by the self-heating of the laser's current. Therefore, a deep investigation of VPS from thermodynamic point of view is an important research pursuit. The investigations of the phase transition in the VCSEL's PS (VPS) were recently reported. That phase transition was identified as an SOPT according to four criticalities observed in the experiments [7].

A typically example of SOPT in thermodynamics is the conventional Ising model [8–10]. This model investigates a system which is described by a two-dimensional square lattice and consists of $N \times N$ spins, as schematically presented in Figure 1(a). Each spin can either be in a spin-up state or a spin-down state and they interact only with the nearest neighbors. For a given temperature, the thermal agitation is applied to this system to generate a lowest energy configuration. The latter is then employed to calculate the magnetization of the system at that given temperature. Figure 1(b) presents the variation of the magnetization with respect to the temperature, exhibiting an SOPT of the magnetic material when it transits from ferromagnetic to paramagnetic state. In phase transition terminology, the magnetization plays the role of the order parameter. A continuous or gradual variation of the order parameter across the transition region is a characteristic of the SOPT [11]. The temperature at which phase transition occurred is designated as the Curie temperature.

This research tries to introduce a Gaussian-distributed interaction into the conventional Ising model to simulate the VPS. That interaction is considered to represent the spatial coherence in the laser beam. Some simulation results will be reported. Among them, a success of this Gaussian-distribution embedded Ising model (GDIM) presents that an exponent of the relaxation time of about 2.23 is obtained, which is very close to the experimental result of 2.3.

2. EXPERIMENTS OF VPS

Figure 2 presents the polarization-resolved *L-I* curves of a VCSEL. The inset of Figure 2 depicts the experimental setup that is typical for the VPS researches. The laser was a single-transverse mode commercial VCSEL operated at 850 nm. The threshold current of this VCSEL was about 1.06 mA. Tuning the injection current from zero, the laser started to emit a linearly polarized light as the current becomes greater than the threshold current. That polarization direction was designated as X-polarization. Further increasing the current to 3.79 mA, the polarization of the laser's output abruptly switched to the orthogonal direction, Y-polarization, and remained that polarization thereafter. Whereas, in decreasing the current from 5 mA, the laser's polarization switched back to X-polarization at 3.06 mA, exhibiting a significant PS hysteresis loop in the *L-I* curves, as shown in Figure 2.

The laser's current driver was set to inject a quasi-step current into the VCSEL. Figure 3 displays the oscilloscope traces of the X- and Y-polarization power detected by the photodetectors at an

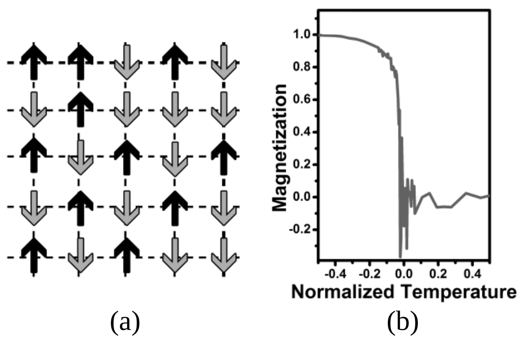

Figure 1: (a) A two-dimensional lattice consists of spins. (b) The variation of the magnetization with respect to the temperature, exhibiting a typical SOPT. The magnetization of 1.0 corresponds to a state with all spins align in the up state. The temperature is normalized with respect to the Curie temperature.

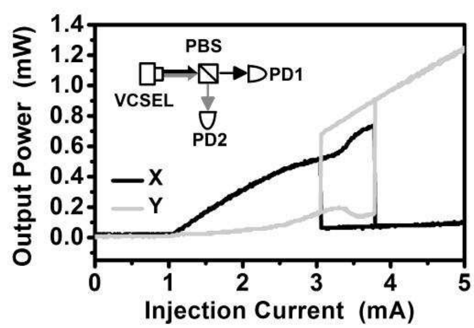

Figure 2: Polarization-resolved *L-I* curves of a VCSEL that can exhibit VPS. The curves were recorded at a current-modulation frequency of 10 Hz. The threshold current was 1.06 mA. PS occurred at the switching currents of 3.79 mA and 3.06 mA, respectively. The inset exhibits the experimental setup. PBS, polarized beam splitter; PD, photodectector.

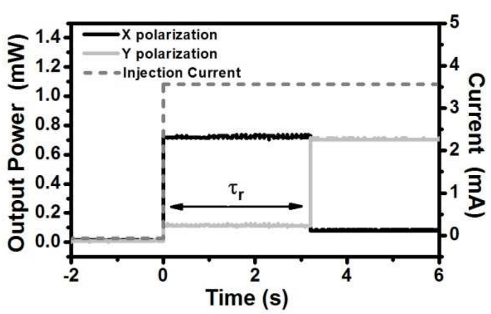

Figure 3: The oscilloscope traces of the injection current, X- and Y-polarization power in a step-current injection experiment at an injection current of 3.63 mA. The ambient temperature of the laser was stabilized at 22.4°C.

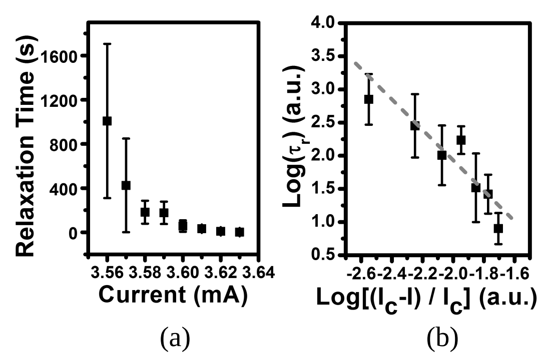

Figure 4: (a) The variation of the relaxation time with respect to the injection current obtained at 22.4°C. (b) Log-log plot of the experimental data shown in Figure 4(a). $I_c = 3.55$ mA. The plot exhibits a power law, and its slope gives the critical exponent, Δ, of about 2.3.

step injection current of 3.63 mA. This current was selected to be greater than the central current of the PS hysteresis loop.

During the leading edge of the step current, the laser was turned-on normally at the threshold current, with its output polarized in the X-direction. Then the injection current was kept constant. However, the laser still emitted stable power with X-polarization. After a significant long period, greater than 3 s in Figure 3, the VCSEL's polarization switched (or relaxed) to the Y-direction within about 20 nanoseconds and remained steady both in the power and polarization thereafter. The period that the laser idles in X-polarization is a measure of the relaxation time at that injection current.

Figure 4(a) presents the variation of the relaxation times with the step injection currents. Such an extraordinary long relaxation time was identified as a CSD phenomenon [7], which is a unique characteristic of the SOPT [11]. The log-log plot in Figure 4(b) gives a critical exponent of about 2.3 for this CSD.

3. GDIM SIMULATIONS

To provide a base to simulate the VPS, the conventional Ising model is a good candidate according to the following three reasons. First, in conventional Ising model, the geometry of the system is a two-dimensional lattice which can map to the VCSEL's facet. Second, the spins that constitute the system can only be in either of two states, spin-up or spin-down, corresponding to the X-polarization or Y-polarization. Both of the spin states and the polarizations are orthogonal in mathematics. Therefore, each spin can be imagined as an atom on the laser's facet and each of them can emit either X-polarized light or Y-polarized light. The polarization distribution on the lattice can assemble the laser's beam profile. Third, the conventional Ising model is a typical SOPT and many of its analysis methods or algorithms are available.

The simulated system is set to be a 101×101 lattice. The interaction between spins is presented in Eq. (1).

$$E = -\sum_{\langle i,j,mn\rangle} \left[(1-x) + x \cdot \exp\left(-\frac{i^2+j^2}{2\sigma^2}\right)\right] S_{ij} \cdot S_{mn}, \tag{1}$$

where E is the total energy of the system and i and j are the row and column indexes of a lattice site, $-50 \leqq i,\ j \leqq 50$ in this system. $S_{ij} = +1$ or -1 for spin-up or spin-down, respectively.

The summation is over pairs of adjacent spins and every pair is counted once. The notation $\langle ij, mn\rangle$ indicates that sites ij and mn are nearest neighbors. x is the strength of the Gaussian distributed interaction and $(1-x)$ is the strength of the spin-spin interaction in conventional Ising model. σ is the width of the Gaussian interaction, and is set to 25. Therefore, in conventional Ising model the interaction between spins is a short range interaction, interacting only with the nearest neighbor. Whereas, the Gaussian distributed interaction is a long range interaction, which represents the spatial coherence in the laser's beam. Numerically, the thermal agitation is introduced into the system by the Boltzmann distribution and Monte Carlo method. They are typical methods used in conventional Ising model.

Figure 5(a) presents a simulation results of GDIM with $x = 0.2$ at a temperature of 2.45. The black and white dots indicate the lattice sites with spin orientated in up and down, respectively. The

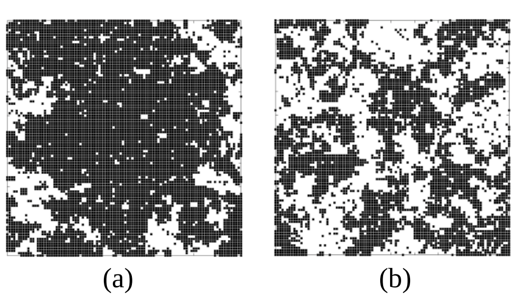

Figure 5: (a) The simulated configuration of the GDIM with $x = 0.2$ at a temperature of 2.45. (b) The simulated configuration of the conventional Ising model at a temperature of 2.47.

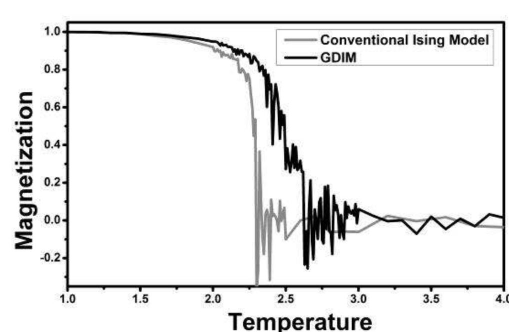

Figure 6: The dependence of the magnetization on the temperature for conventional Ising model (gray curve) and GDIM (black curve).

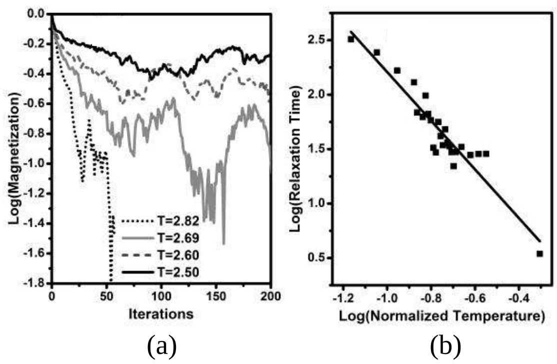

Figure 7: (a) Decay of the magnetization as a function of the iterations at various step temperatures in GDIM. (b) Log-log plot of the relaxation times versus the normalized temperatures. The later is defined by $(T - T_c)/T_c$, where T_c is the Curie temperature.

resulting configuration exhibits a significant domain of spin-up in the central region of the lattice, which reflects the effect of the Gaussian distributed interaction. For comparison, Figure 5(b) displays a similar simulation result of conventional Ising model at a temperature of 2.47. No any large structure is observed on that surface.

Figure 6 depicts the magnetization varied with the temperature both in conventional Ising model and in GDIM. Curie temperatures of about 2.27 for conventional Ising model and about 2.45 for GDIM are observed. The magnetization curve of the GDIM exhibits an SOPT-like continuous variation across the transition region.

Consider to prepare the system with all spins in the up state and at a temperature around zero. Therefore, its magnetization is 1 corresponding to a pure ferromagnetic state. Then, a step temperature profile with a temperature greater than the Curie temperature is applied to this system. Figure 7(a) depicts the variation of the magnetization with respect to the runs of the numerical iterations at various step temperatures. Here, the iteration plays the role of the time in simulations. Since the step temperatures are greater than the Curie temperature, the magnetization will decay approaching zero, which is the magnetization of the paramagnetic materials. The reciprocal of the slope of the semi-log curves in Figure 7(a) will give the time constants or relaxation time at those temperatures. The slope of the log-log plot of the relaxation times versus temperatures, as shown in Figure 7(b), will give the exponent of the relaxation time. From the analysis described above, the exponent of relaxation time of GDIM is about 2.23, which is very close to the experimental result of 2.3 presented in Figure 4(b).

4. CONCLUSIONS

In conclusion, the GDIM can present an SOPT-like characteristic of the magnetization variation with respect to the temperature in the simulation. However, in contrast to the sharp transitions observed in the VPS experiments, the simulated GDIM transition is relatively gradual. The simulations can also exhibit an exponent of the relaxation time that is very close to the experimental results, indicating a success of the GDIM.

There are many models about the physics behind the VPS; for example, the spin-flip model [12], the current-dependent gain model [13], and the stress-effect model [14]. But the question is still open. Constructing a model from the phase transition point of view for the VPS might help to explore more understanding of it. Based on some successes presented in this report, the GDIM is a good starting point. More investigations to verify the SOPT of the GDIM are required.

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