A Nyström-based Approach for Solving Time-domain Magnetic Field Integral Equation

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Abstract— Transient electromagnetic (EM) scattering by conducting objects is formulated by time-domain magnetic field integral equation (TDMFIE). Traditionally, the TDMFIE is solved by combining the method of moments (MoM) in spatial domain and a march-on in time (MOT) scheme in temporal domain. We propose a different scheme in which the Nyström method is used in spatial domain while the MoM with Laguerre function as basis and testing functions is employed in temporal domain. The Nyström method is simple in implementation and does not require a basis or testing function and conforming meshes. The time-domain MoM can naturally enforce the causality and fully eliminate the numerical instability. A numerical example for EM scattering by a conducting cylinder is presented to demonstrate the approach and its merits can be observed.

1. INTRODUCTION

Transient electromagnetic (EM) problems can be formulated by a time-domain integral equation approach. For conducting or homogeneously penetrable objects, the time-domain surface integral equations (TDSIEs) including the time-domain electric field integral equation (TDEFIE), time-domain magnetic field integral equation (TDMFIE), or their combination can be applied. Traditionally, the TDSIEs are solved by combining the method of moments (MoM) in spatial domain and a march-on in time (MOT) scheme in temporal domain [1]. The MoM requires a well-designed basis function like the Rao-Wilton-Glisson (RWG) basis function [2] which needs conforming triangular meshes in geometric discretization and double surface integrations in evaluating matrix elements. The MOT scheme, on the other hand, suffers from a late-time instability problem although some improved versions have been proposed in recent years [3]. The instability and accuracy are still dependent upon the choice of time step [4].

In this work, we propose a different scheme to solve the TDMFIE for transient scattering problems by conducting objects. Compared with the TDEFIE, the TDMFIE is thought of little more difficult to solve due to the nature of its integral kernel. We use the Nyström method [5] to discretize the spatial domain while employ the MoM with Laguerre basis and testing functions to discretize the temporal domain for the TDMFIE [6]. The merits of Nyström method in spatial domain includes the simple mechanism of implementation, removal of basis and testing functions, and allowance of using nonconforming meshes. The Nyström method requires an efficient treatment for singular integrals, but we have developed a robust technique to handle them [7] and it can be applied to the TDSIEs by a minor revision. The time-domain MoM (TDMoM) with Laguerre basis and testing functions may be superior to the MOT because the Laguerre function can naturally enforce the causality and also the Galerkin's testing scheme can fully eliminate the numerical instability with a march-on-in-degree (MOD) procedure [8]. A numerical example for EM scattering by a conducting cylinder is presented to illustrate the scheme and its robustness has been verified.

2. TIME-DOMAIN MAGNETIC FIELD INTEGRAL EQUATION (TDMFIE)

Consider the scattering of transient EM wave by a conducting object with a surface S in free space. The problem can be formulated by the TDMFIE, i.e.,

$$\hat{n} \times \left[\mathbf{H}^{inc}(\mathbf{r}, t) + \mathbf{H}^{sca}(\mathbf{r}, t)\right] = \mathbf{J}(\mathbf{r}, t), \quad \mathbf{r} \in S \tag{1}$$

where $\mathbf{H}^{inc}(\mathbf{r}, t)$ and $\mathbf{H}^{sca}(\mathbf{r}, t)$ are the incident magnetic field and scattered magnetic field, respectively, $\hat{n}$ is the unit normal vector at an observation point $\mathbf{r}$, and $\mathbf{J}(\mathbf{r}, t)$ is the induced electric current density on the object. The scattered magnetic field is related to the vector potential $\mathbf{A}(\mathbf{r}, t)$ by

$$\mathbf{H}^{sca}(\mathbf{r}, t) = \frac{1}{\mu_0} \nabla \times \mathbf{A}(\mathbf{r}, t) \tag{2}$$

and the tangential component of the magnetic field can be written as

$$\hat{n} \times \mathbf{H}^{sca}(\mathbf{r}, t) = \frac{1}{4\pi} \nabla \times \int_S \frac{\mathbf{J}(\mathbf{r}', \tau)}{R} \, dS' = \frac{1}{2} \mathbf{J}(\mathbf{r}, t) + \hat{n} \times \frac{1}{4\pi} \fint_S \nabla \times \frac{\mathbf{J}(\mathbf{r}', \tau)}{R} \, dS' \tag{3}$$

where the integral with a dash indicates that it is a Cauchy-principal-value (CPV) integral. With the above derivation, the MFIE can be formulated as follows

$$\frac{1}{2} \mathbf{J}(\mathbf{r}, t) - \hat{n} \times \frac{1}{4\pi} \fint_S \nabla \times \frac{\mathbf{J}(\mathbf{r}', \tau)}{R} \, dS' = \hat{n} \times \mathbf{H}^{inc}(\mathbf{r}, t) \tag{4}$$

from which the unknown current density can be solved.

3. NYSTRÖM-BASED APPROACH FOR SOLVING THE TDMFIE

We mesh a conductor surface into N small triangular patches and ΔS_n is the area of the nth patch. We introduce the unknown source vector $\mathbf{e}(\mathbf{r}', t)$ which is related to the unknown current density by

$$\mathbf{J}(\mathbf{r}', t) = \frac{\partial}{\partial t} \mathbf{e}(\mathbf{r}', t) \tag{5}$$

and it can be expanded as

$$\mathbf{e}(\mathbf{r}', t) = \sum_{n=1}^{N} e_n(t) \mathbf{J}_n(\mathbf{r}'). \tag{6}$$

The unknown current density can then be expanded as

$$\mathbf{J}(\mathbf{r}', t) = \sum_{n=1}^{N} \mathbf{J}_n(\mathbf{r}') \frac{\partial}{\partial t} e_n(t). \tag{7}$$

If we apply the Nyström method in spatial domain, i.e., the integration over a small triangular patch is replaced with a summation under a quadrature rule provided that the integrand is regular, then we have

$$\begin{aligned} &\frac{1}{2} \frac{\partial}{\partial t} e_m(t) \hat{t}_{mp}^{(l)} \cdot \mathbf{J}_{mp} - \frac{1}{4\pi} \hat{t}_{mp}^{(l)} \cdot \hat{n}_{mp} \times \sum_{n=1}^{N} \sum_{q=1}^{Q} w_q \left[\frac{1}{c} \frac{\partial^2 e_n(\tau_{mpnq})}{\partial t^2} \mathbf{J}_{nq} \times \frac{\hat{R}_{mpnq}}{R_{mpnq}} \right. \\ &\left. + \frac{\partial e_n(\tau_{mpnq})}{\partial t} \mathbf{J}_{nq} \times \frac{\hat{R}_{mpnq}}{R_{mpnq}^2} \right] = V_{mp}^{(l)}(t), \quad m = 1, 2, \ldots, N; \;\; p = 1, 2, \ldots, Q \end{aligned} \tag{8}$$

where $\mathbf{J}_{nq} = \mathbf{J}(\mathbf{r}'_{nq}) = J_{nq}^u \hat{u} + J_{nq}^v \hat{v}$ is the value of spatial component of unknown current density at the source point $\mathbf{r}'_{nq}$ or the qth quadrature point in the nth patch and it only has two independent components J_{nq}^u and J_{nq}^v in a local coordinate system $\{u, v, w\}$ established over the patch plane because it is a surface current density. Also, $\mathbf{J}_{mp} = \mathbf{J}(\mathbf{r}_{mp}) = J_{mp}^u \hat{u} + J_{mp}^v \hat{v}$ has an analogous meaning but for the observation point $\mathbf{r}_{mp}$ located at the pth quadrature point in the mth patch. These spatial components of current densities should be transformed into the components in the global coordinate system $\{x, y, z\}$ in the implementation. We have chosen two orthogonal unit tangential vectors $\hat{t}_{mp}^{(l)} = \hat{t}^{(l)}(\mathbf{r}_{mp}) = t_{mp}^{x(l)} \hat{x} + t_{mp}^{y(l)} \hat{y} + t_{mp}^{z(l)} \hat{z}$ at the observation point ($l = 1, 2$) to test the above equation so that the TDMFIE can be transformed into a matrix equation in space domain. The right-hand side of the equation is given by

$$V_{mp}^{(l)}(t) = \hat{t}_{mp}^{(l)} \cdot \hat{n}_{mp} \times \mathbf{H}^{inc}(\mathbf{r}_{mp}, t) \tag{9}$$

and $\hat{n}_{mp}$ is the unit normal vector at the observation point. The above procedure can only be used for a far interaction between the observation point and a source patch or $m \neq n$ which allows the direct application of a quadrature rule. If the observation point is inside the source patch or $m = n$, we have a singularity problem and we have to employ a singularity treatment technique to handle it [7].

In the temporal domain, we use the Laguerre function $\phi_j(t) = e^{-t/2}L_j(t)$ as a basis function to expand $e_n(t)$ [6], i.e.,

$$e_n(t) = \sum_{j=0}^{\infty} e_{nj}\phi_j(st) \tag{10}$$

where s is the scaling factor to control the support of expansion. Substituting this representation to Eq. (8) and using $\phi_i(st)$ as a testing function to test Eq. (8), we can obtain

$$\begin{aligned}&\hat{t}_{mp}^{(l)} \cdot \frac{s}{2}\sum_{j=0}^{i}\left(\frac{1}{2}e_{mj} + \sum_{k=0}^{j-1} e_{mk}\right) - \sum_{n=1}^{N}\sum_{q=1}^{Q}\left\{\frac{s^2}{c}I_{1nq}^{(l)}\sum_{j=0}^{i}\left[\frac{1}{4}e_{nj} + \sum_{k=0}^{j-1}(j-k)e_{nk}\right] I_{ij}(sR_{mpnq}/c)\right.\\ &\left. + sI_{2nq}^{(l)}\sum_{j=0}^{i}\left(\frac{1}{2}e_{nj} + \sum_{k=0}^{j-1} e_{nk}\right) I_{ij}(sR_{mpnq}/c)\right\} = V_{mpi}^{(l)}, \quad i = 0, 1, \ldots, \infty\end{aligned} \tag{11}$$

where

$$I_{1nq}^{(l)} = \frac{w_q}{4\pi}\hat{t}_{mp}^{(l)} \cdot \hat{n}_{mp} \times \mathbf{J}_{nq} \times \frac{\hat{R}_{mpnq}}{R_{mpnq}} \tag{12}$$

$$I_{2nq}^{(l)} = \frac{I_{1nq}^{(l)}}{R_{mpnq}} \tag{13}$$

$$I_{ij}(sR_{mpnq}/c) = \int_0^{\infty} \phi_i(st)\phi_j(st - sR_{mpnq}/c)\, d(st) \tag{14}$$

$$V_{mpi}^{(l)} = \int_0^{\infty} \phi_i(st)V_{mp}^{(l)}(t)\, d(st) \tag{15}$$

and $I_{ij}(sR_{mpnq}/c)$ and $V_{mpi}^{(l)}$ can be calculated according to the formulations in the appendix in [6]. Note that there are two spatial-domain coefficients in $I_{1nq}^{(l)}$ and $I_{2nq}^{(l)}$ which are the two independent components J_{nq}^u and J_{nq}^v of $\mathbf{J}_{nq}$ and they should be combined with the time-domain coefficient e_{nj} together to form two new unknown coefficients which can be determined by two groups of Eq. (11) when $l = 1, 2$.

4. NUMERICAL EXAMPLE

We consider the transient EM scattering by a conducting cylinder to demonstrate the approach. The cylinder is centered at the origin of a rectangular coordinate system and it has a cross-section

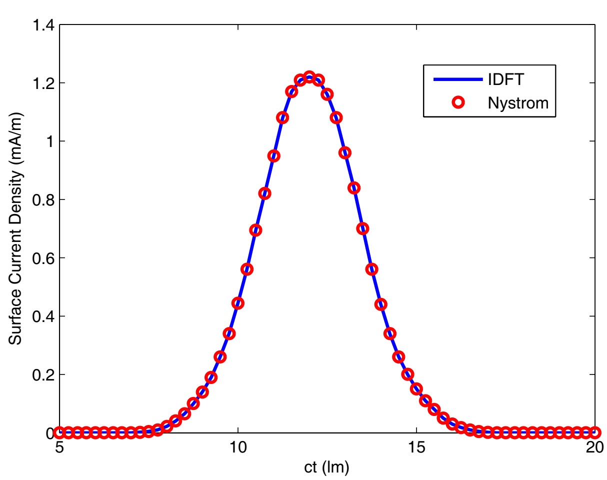

Figure 1: Solution of z-directed current density at $z = 0$ on a cylinder surface for transient scattering by a conducting cylinder.

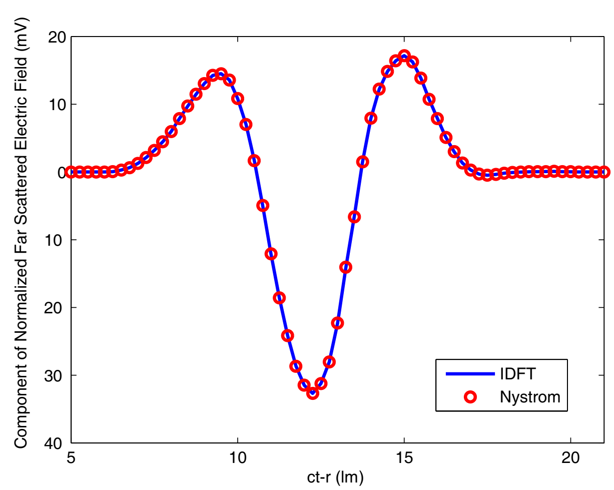

Figure 2: Solution of θ-component of normalized far-zone scattered electric field for transient scattering by a conducting cylinder.

radius $a = 0.5\,\mathrm{m}$ and a height $h = 1.0\,\mathrm{m}$. The surface of the cylinder is discretized into 826 triangular patches. A Gaussian plane wave is illuminating the scatterer along the $-z$ direction and the electric field and magnetic field are defined by

$$\mathbf{E}^{inc}(\mathbf{r}, t) = \hat{x}\frac{4e^{-\gamma^2}}{\sqrt{\pi}T}, \quad \mathbf{H}^{inc}(\mathbf{r}, t) = -\frac{1}{\eta}\hat{z} \times \mathbf{E}^{inc}(\mathbf{r}', t) \tag{16}$$

where $\gamma = 4(ct - ct_0 + \mathbf{r}\cdot\hat{z})/T$, T is the width of Gaussian impulse, and t_0 is the time delay denoting the time when the pulse peaks at the origin. We choose a Gaussian pulse with $T = 8.0\,\mathrm{lm}$ (light meter) and $ct_0 = 12.0\,\mathrm{lm}$ as an incident wave. Figure 1 shows the solution of z-directed current density at $z = 0$ on the cylinder surface while Figure 2 plots the solution of the θ-component of normalized far-zone scattered electric field observed along the backward direction. The results agree well with the solutions obtained from the inverse discrete Fourier transform (IDFT) of frequency-domain solutions.

5. CONCLUSION

In this work, the transient EM scattering by conducting objects is formulated through the TDMFIE. We develop a different scheme to solve the TDMFIE by combining the Nyström method in spatial domain and the MoM with the Laguerre basis and testing functions in temporal domain. The benefits of the new scheme include the simple mechanism of implementation with a use of non-conforming and low-quality meshes in spatial domain and the natural satisfaction of causality with a full elimination of instability in temporal domain. A numerical example for EM scattering by a conducting cylinder is presented to demonstrate the scheme and good results have been observed.

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REFERENCES

1. Rao, S. M., *Time Domain Electromagnetics*, Academic Press, San Diego, 1999.
2. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, Vol. 30, No. 3, 409–418, May 1982.
3. Weile, D. S., G. Pisharody, N.-W. Chen, B. Shanker, and E. Michielssen, "A novel scheme for the solution of the time-domain integral equations of electromagnetics" *IEEE Trans. Antennas Propagat.*, Vol. 52, No. 1, 283–294, Jan. 2004.
4. Shi, Y. and J.-M. Jin, "A time-domain volume integral equation and its marching-on-in-degree solution for analysis of dispersive dielectric objects," *IEEE Trans. Antennas Propagat.*, Vol. 59, No. 3, 969–978, Mar. 2011.
5. Canino, L. S., J. J. Ottusch, M. A. Stalzer, J. L. Visher, and S. Wandzura, "Numerical solution of the Helmholtz equation in 2D and 3D using a high-order Nyström discretization," *J. Comput. Phys.*, Vol. 146, 627–663, 1998.
6. Jung, B. H., Y.-S. Chung, and T. K. Sarkar, "Time-domain EFIE, MFIE, and CFIE formulations using Laguerre polynomials as temporal basis functions for the analysis of transient scattering from arbitrary shaped conducting structures," *Progress In Electromagnetics Research*, Vol. 39, 1–45, 2003.
7. Tong, M. S. and W. C. Chew, "A novel approach for evaluating hypersingular and strongly singular surface integrals in electromagnetics," *IEEE Trans. Antennas Propagat.*, Vol. 58, No. 11, 3593–3601, Nov. 2010.
8. Jung, B.-H., Z. Ji, T. K. Sarkar, M. Salazar-Palma, and M. Yuan, "A comparison of marching-on in time method with marching-on in degree method for the TDIE solver," *Progress In Electromagnetics Research*, Vol. 70, 281–296, 2007.