The Possibility for Full Profile Incoherent Scatter Data Processing on the Base of the Simplex-processor Algorithm

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Abstract— For the processing of Irkutsk Incoherent Scatter Radar data we consider the problem of non-linear discrepancy functional minimization in long experimental data series processing when measured process is described by integro-differential equation. Considering the non-linear inverse problem for functions with two continuous derivatives determined on compact we present three main techniques parts: descent algorithm, genetic algorithm and solution regularization. The central part of the technique is the descent algorithm that works on n-dimensional net associated with some regular polytope. The regularization of solution is carried out by means of transition to Sobolev space. For deleting of ambiguities due to set of sub-optimal solution we use adaptation scheme on the base of genetic algorithms.

1. INTRODUCTION

The experimental data of Irkutsk Incoherent Scatter Radar (IISR) pose the non linear inverse problem where the ionospheric parameters (such as electron density, electron and ion temperature, as well as drift velocity and ion composition) can be derived from the experimental autocorrelation functions or energy spectrums [5].

The IISR inverse problems are described by integro-differential equations obtained via theoretical model of physical process, which is nonlinear and includes trigonometric function [12]. These problems usually have a set of solution and as a consequence are incorrect and require boundary conditions and special regularization techniques ([1, 4, 8, 9, 17]). The solutions can be obtained in the framework of least squares technique with some variants of descent algorithms in model functional space and special regularization methods. In addition to classical descent methods, one can use evolution techniques [6], (in particular, Monte-Carlo algorithms) and various genetic algorithms ([7, 18]).

In our recent research we suggest a new method for solving of multi-dimensional inverse problem for processes that are continuous in time and which have a pre-history of solutions. The typical IISR measurements have as a result the set of lag-products (or spectrums) which can be used for derivation of ionospheric parameters from the solution of a repeatable nonlinear inverse problem respectively to lag-product properties. When this problem is solving for full profiles of parameter, such as attitude profiles of electron density, electron and ion temperatures etc., we have a multidimensional problem which requires from $2N$ to N^2 calculations of objective function per dimension. This requires large computational resources and can be realized practically only on super computers.

In the present study we suggest a new approach to IISR data processing on the base of recently developed algorithm which solves the nonlinear inverse problem in the special projection space where any motion between the projection space noodles can be done along the multi-dimensional simplex grid. This kind of gradient conjugate algorithm has no significant limitations on problem dimension, it require only $(N + 1)$ calculations of objective function per dimension and it can be realized on any usual personal computer systems. The method called simplex-processor is realized on IISR for simultaneous fitting of few tens of spline coefficients that gives approximations for all parameters profiles mentioned above.

2. MATHEMATICAL FOUNDATION

We consider the inverse non-linear problem for functions f with two continuous derivatives f', f'' determined on compact $[a, b]$. Also we know N solutions $f_{-1}, f_{-2}, \dots, f_{-N}$, derived in previous processing sessions. The problems of that kind often arise in physical applications when some process is described by partial differential equation of second order. Natural definition domain for these equations is Sobolev space $W_2^{(2)}$ [16], that have very useful properties for regularization of inverse problem. By Sobolev's theorem we can guarantee that every generalized solution in fact equivalent to some ordinary function $f \in C_2$. Consider the operator equation that describes some physical process:

$$Af = u \quad A : W_2^{(2)} \to L_2 \tag{1}$$

The result of operator A action on function f is function $u \in L_2$ that can be measured in an experiment in the presence of noises and methodical inaccuracy, that are described by error $\varepsilon > 0$. The inverse problem is finding f when we have u. The parametric Tikhonov functional with regularization terms can be written as follows:

$$\Phi = \|Af - u\|^2_{L_2} + \alpha \left\|\Omega\left(A, f, u, f_{-1}, f_{-2}, \ldots, \varepsilon\right)\right\|^2_{L_2} \tag{2}$$

Here $\alpha > 0$ — regularization parameter and Ω — regularization operator which is defined by functional space type. In space $W_2^{(2)}$ we define this operator as follows:

$$\Omega = \sum_{k=0}^{2} \int_a^b p_k\left(A \cdot f, u, f_{-1}, f_{-2}, \ldots, \varepsilon\right) \left|f^{(k)}\right|^2 dx \tag{3}$$

Fitting procedure usually means equality of norms $\|Af\|_{L_2} = \|u\|_{L_2} = 1$. Unlike classical statement, [Tikhonov et al., 1990], the coefficients p_k at derivatives f', f'' are considered not equal. For determination of regularization coefficients $p_{1,2}$ in paper [14] the next expressions were obtained:

$$\alpha p_1 = \frac{3\sigma^2(b-a)^2}{\left\|f'_{-1}\right\|^2} \frac{1}{|u| + \sigma}; \quad \alpha p_2 = \frac{3\sigma^2(b-a)^2}{\left\|f''_{-1}\right\|^2} \frac{1}{|u| + \sigma} \tag{4}$$

where σ — dispersion of u calculated in previous processing sessions. These formulae are based on account of informational level below which the fitting procedure doesn't make sense.

In Sobolev space $W_2^{(2)}$ the solutions are ranged due to smoothness of function f, so that smooth functions are always closer to functional minima; for real physical system this feature corresponds to variational principle when function that describes real movement has minimal variation (integral derivative). The effectiveness of integral derivatives in experimental data processing is confirmed by theirs wide using in digital images processing ([2, 10, 15]). If f and are not equal to zero, then functional (2) always greater than zero. The dependence of regulation functions on u allows us to control the derivatives contribution in discrepancy regarding functional properties and experimental data quality.

3. DESCENT ALGORITHM

In alleged approach we use the n-dimensional analog of descent technique described in paper [13] with addition of mentioned above regularization technique and genetic algorithm for global convergence.

In our algorithm the discrepancy is calculated in regular polytope vertexes with center in current point of projection space. So as there are only three regular polytopes in dimensions $n \geq 4$ (namely, simplex, hypercube and orthoplex) we consider this technique only for these three polytopes. For movement on projection space grid the number of computation for each step is $Z = n + 1$ for simplex, $Z = 2^n$ for hypercube and $Z = 2 \cdot n$ for orthoplex.

It is obvious that minimal computation number is achieved for simplex and if parameter number n is sufficiently large, then in simplicial method almost two times lesser comparing with orthoplex method. Number of computations for hypercube method has exponential growth and so in high dimension it makes no sense to consider that case.

The general effectiveness of polytopes in the minimization problem also depends on n-dimensional space geometric properties. Let us consider the number of iteration that is necessary for minimum point reaching for simplicial and orthoplex method. For traditional coordinate descent technique base on orthoplex method (taxicab metric) the way from starting point to optimal point always equal to the sum of ways for all parameters. In simplicial method practically always we have simultaneous (diagonal) movement equivalent to parameters mixing. Due to this fact the distance in simplicial method is always less than distance in orthoplex method. The geometric distance characteristics also presented in Table 1. In two-dimensional case the distance in honeycomb structure (the net based of vertexes of regular triangles) can differ from distance in square-topped structure by ratio

$$K_{\text{dist}}^2 = \frac{s_{\text{sim}}}{s_{\text{ort}}} = \frac{4}{3} - \frac{\sqrt{3}}{18} \cong 0.9484 \tag{5}$$

At first sight this difference seems insignificant but in n-dimensional case this coefficient transforms to $K_{\text{dist}}^n = (0.9484)^{n-1}$. So the "mixing" parameters effect radically changes the computational characteristics of descent algorithm. On Fig. 1 we present the dependence for values of product $z \cdot K_{\text{dist}}^n$ on projection space dimension. For example, for problem with 50 parameters total number of computations for simplicial method is about ~ 25 times less than for orthoplex method. When projection space dimension is greater than 20 the mixing effect takes over dimension increase effect and number of computations on single iteration is diminished.

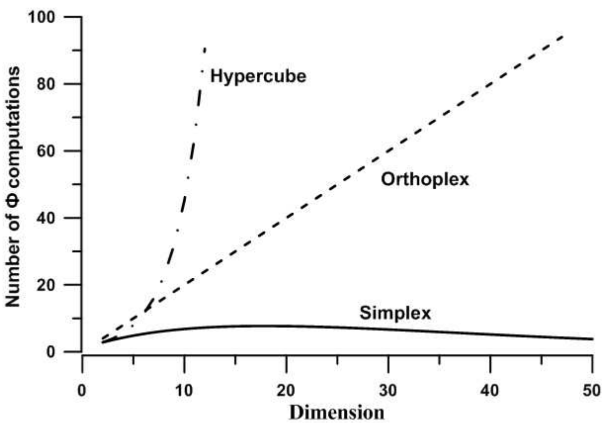

Figure 1: Comparison of computation numbers for different politopes.

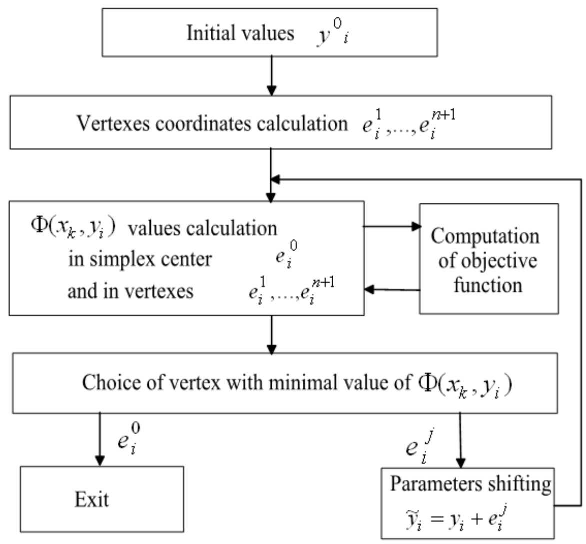

Figure 2: Flowchart of simplex-processor algorithm.

The simplicial method has a great advantage in descent speed when the number of discrepancy functional parameters is large. Thus the optimal algorithm for functional minimization from a start point in P_n is the algorithm that makes $n+1$ functional computation in vertexes of corresponding n-simplex Δ_n and after that moves the current point to vertex where the functional Φ value is minimal. The stopping criterion for this algorithm is the minimal value of functional Φ in simplex center comparing with functional values in all vertexes. The program code that realizes our algorithm we will name simplex-processor (SP).

4. BOUNDARY AND INITIAL CONDITIONS

All altitudinal distributions of ionospheric parameters are presented by means of local B-splines [3] and inverse problem reduces to minimization problem for functions of $n = k \times m$ parameters than can be considered as process image. All functions in process image are normalized so that step of their changing in projection space is approximately the same.

Boundary conditions for all functions f_i are set by means penalty functions Θ_k, that are built on the base of following principal. If $f_{\min}$ and $f_{\max}$ are low and upper limits of function f_i in the domain of its determination, then boundary conditions in projection space have the form:

$$\begin{aligned}
\Theta_{\min} &= \frac{\delta}{b-a} \int_a^b \exp\left(-\frac{f(h) - f_{\min}(h)}{\lambda \cdot f'_{-1,\max}(h)\Delta} \right) dh \\
\Theta_{\max} &= \frac{\delta}{b-a} \int_a^b \exp\left(\frac{f(h) - f_{\max}(h)}{\lambda \cdot f'_{-1,\max}(h)\Delta} \right) dh
\end{aligned} \tag{6}$$

Here δ is the penalty value (it should not be large then 5% of main discrepancy), denominator of the exponent determines the size of smooth border. Value Δ is the grid step in projection space, and the coefficient λ is assigned so that during the changing of Θ_k from zero to the unit the simplex processor could make few iterations.

In the same manner we set a bonus functions when external experimental or model data f_{ext} are available.

$$B = \frac{\delta}{b-a} \int_a^b \left(1 - \exp\left(-\frac{(f(h) - f_{ext}(h))^2}{(\sigma + \sigma_1)^2}\right)\right) dh \qquad (7)$$

Here δ is the bonus value. External data are used for low ionosphere, where IISR antenna side lobes give signal reflections from ground and in the topside ionosphere where GPS total electron content is used for estimation of plasmasphere parameters [12]. We should note that large amount of coefficients that are used in fitting procedure, as well as including of smooth penalty and bonus functions additionally to main discrepancy, makes the descent algorithm more stable.

Initial conditions for simplex-processor algorithm are assigned as set of starting images that are determined from previous sessions of data processing. For choosing of proper starting images we use genetic algorithm which leaves the most probable starting points as active set and has special algorithm for finding a start point in the case when process realization is non standard. In practical use a previous solution of inverse problem is almost always a good starting image and data stream processing can be easy implemented on usual personal computers.

5. CONCLUSION

The considered hybrid technique of inverse problem solving allows us to combine successfully the high speed of grid algorithm and adaptive properties of genetic algorithms. The main part of technique is simplex-processor that makes minimal number of computations for each iteration and geometrical properties of simplicial method allow sufficiently decrease total iteration number when we have a problem with large parameter number. The simplicity of this technique realization allows easily use parallel computations for multi-processors systems. These features of simplex-processor give the chance to use it for inverse problems with many parameters that cannot be solved by traditional techniques due to restricted computational resources.

The using of Sobolev space as a basic functional space of inverse problem realizes principal property of real physical systems consisting in variational principle when discrepancy functional minimum is searched in set of functions with minimal derivatives variation. Thus we get rid of complicated regularization schemes with penalty functions that are traditionally used for similar problems.

The computational possibilities of simplex-processor allow us to carry out global optimization with a help of genetic algorithm. For inverse problems with a priori existing solution the global optimization is reduced to SP starting point set selections and evolution technique for given starting point set.

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