

Multicomponent Rogue Waves

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Abstract— We overview theoretical and experimental advances in the field of rogue wave solutions of multi-component optical wave systems. In these systems, the transfer of energy among the coupled waves may lead to novel and complex extreme wave phenomena. We focus our attention on the case of vector field co-propagation in randomly birefringent optical fibers, and on the coupling among counter-propagating waves in a periodic nonlinear waveguide.

1. INTRODUCTION

Rogue and extreme waves occur in many scientific and social contexts, ranging from hydrodynamics and oceanography to geophysics, plasma physics, Bose-Einstein condensates, financial markets and nonlinear optics. A typical example of rogue wave is given by the sudden appearance in the open sea of an isolated giant wave, whose height and steepness are much larger than the average sea values, that subsequently disappears without a trace. A universal model for the dynamics of rogue waves is provided by the one-dimensional nonlinear Schrödinger (NLS) equation in the self-focusing regime [1, 2]. Here the mechanism that leads to the generation of rogue waves is nonlinear wave mixing, that generates modulation instability (MI) of the continuous wave (CW) background. The nonlinear development of MI past the initial stage of exponential sideband amplification is described by families of exact solutions such as the Akhmediev breathers. A special member of this solution family is the Peregrine soliton [3], which represents a wave that is localized both in its space and time dimensions. The Peregrine soliton was only recently experimentally observed in optical fibers [4].

A new frontier in the study of rogue waves is provided by multi-component wave systems, where the transfer of energy among the coupled modes may lead to novel and unexpected complex phenomena: consider for example parametric three-wave interactions in quadratic media [5]. Here we provide an overview of our recent advances in the theory and experiments on multi-component rogue waves, involving either the co-propagation of two orthogonal polarization modes in randomly birefringent optical fibers [6, 7], or the counter-propagation of two linearly polarized waves in a periodic nonlinear Bragg grating [8]. Polarization coupling in randomly birefringent telecommunication fibers is described by the vector NLS equation (VNLSE) or Manakov system. In both the anomalous and in the normal dispersion regime of the fiber, we found a new class of coupled wave rogue wave solutions. In the normal dispersion regime, where MI is absent for scalar waves, we experimentally demonstrated the generation of black vector rogue waves by means of standard telecom components. On the other hand, pulse propagation in periodic fiber Bragg gratings is described by a variant of the massive Thirring model. We found the rational rogue wave solution of the massive Thirring model, which opens the way for the observation of rogue waves in periodic optical media.

2. POLARIZATION ROGUE WAVES

Let us consider first the VNLSE (also known as Manakov system), that we write in a dimensionless form as

$$\begin{cases} iu_t^{(1)} + u_{xx}^{(1)} - 2s \left(|u^{(1)}|^2 + |u^{(2)}|^2 \right) u^{(1)} = 0 \\ iu_t^{(2)} + u_{xx}^{(2)} - 2s \left(|u^{(1)}|^2 + |u^{(2)}|^2 \right) u^{(2)} = 0, \end{cases} \quad (1)$$

where $u^{(1)}(x, t)$, $u^{(2)}(x, t)$ represent field envelopes and x , t are the transverse and longitudinal coordinates, respectively. Subscripted variables in Eq. (1) stand for partial differentiation. Here,

we have normalized the equations in a way such that $s = \pm 1$. Note that in the case $s = -1$, Eq. (1) refer to the focusing (or anomalous dispersion) regime; in the case $s = 1$, Eq. (1) refer to the defocusing (or normal dispersion) regime.

As shown in [6, 7], both semi-rational and rational solutions of the VNLSE exist, with the property of representing amplitude peaks that are localized in both x and t coordinates. These solutions are constructed by means of the standard Darboux dressing method and, for Eq. (1) with $s = -1$ (anomalous dispersion regime), semi-rational solutions be expressed as [6]

$$\begin{pmatrix} u^{(1)}(x, t) \\ u^{(2)}(x, t) \end{pmatrix} = e^{2i\omega t} \left[\frac{L}{B} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \frac{M}{B} \begin{pmatrix} a_2 \\ -a_1 \end{pmatrix} \right], \quad (2)$$

with the following notation: $L = \frac{3}{2} - 8\omega^2 t^2 - 2a^2 x^2 + 8i\omega t + |f|^2 e^{2ax}$, $M = 4f(ax - 2i\omega t - \frac{1}{2})e^{(ax+i\omega t)}$, and $B = \frac{1}{2} + 8\omega^2 t^2 + 2a^2 x^2 + |f|^2 e^{2ax}$, where f is a complex arbitrary constant. The dressing construction of the vector rogue wave (2) leads to the arbitrary complex parameter f , and two real parameters a_1, a_2 associated with the background plane wave. We note also that the dependence of L, M and B (see (2)) on x, t is both polynomial and exponential only through the dimensionless variables ax and $\omega t = a^2 t$. Moreover the vector solution (2) turns out to be a combination of the two constant orthogonal vectors $(a_1, a_2)^T$ and $(a_2, -a_1)^T$ [6].

The superposition of the dark and bright contributions in each of the two wave components $|u^{(j)}|$ may lead to complicated breather — like pulses. The single contributions of the dark shape L/B and bright shape M/B are better displayed when f.i. $a_2 = 0$. In this case typical distributions $|u^{(1)}(x, t)|, |u^{(2)}(x, t)|$ are displayed in Fig. 1. Here we show a vector dark-bright soliton together with a single Peregrine soliton. By decreasing the value of $|f|$, Peregrine and dark-bright solitons separate. By increasing $|f|$, Peregrine and dark-bright solitons merge and the Peregrine bump cannot be identified while the resulting dark-bright pulse appears as a boomeron-type soliton, i.e., a soliton solution with a time-dependent velocity.

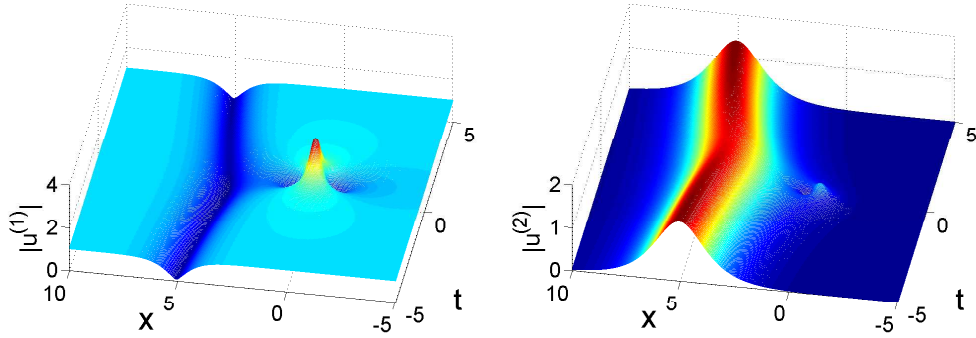


Figure 1: Deterministic vector freak wave envelope distributions $|u^{(1)}(x, t)|$ and $|u^{(2)}(x, t)|$ of (2). Here, $f = 0.1, a_1 = 1, a_2 = 0$.

On the other hand, in the normal dispersion or self-defocusing regime Eq. (1) with $s = 1$ have the rational solutions [7]

$$u^{(j)} = u_0^{(j)} \left[\frac{p^2 x^2 + p^4 t^2 + px(\alpha_j + \beta\theta_j) - i\alpha_j p^2 t + \beta\theta_j}{p^2 x^2 + p^4 t^2 + \beta(px + 1)} \right] \quad (3)$$

where

$$u_0^{(j)} = a_j e^{i(q_j x - \nu_j t)}, \quad \nu_j = q_j^2 + 2(a_1^2 + a_2^2), \quad j = 1, 2; \quad (4)$$

represent the backgrounds of expressions (3),

$$\begin{aligned} \alpha_j &= 4p^2 / (p^2 + 4q_j^2), \quad \theta_j = (2q_j + ip)/(2q_j - ip), \quad j = 1, 2; \\ \beta &= p^3 / \chi (p^2 + 4q_1 q_2), \quad p = 2\text{Im}(\lambda + k), \\ q_1 + q_2 &= 2\text{Re}(\lambda + k), \quad q_1 - q_2 = 2q, \quad \chi = \text{Im}k. \end{aligned}$$

As for the computation of the complex value of k and λ , k is either one of the complex solutions of a fourth order polynomial, and λ is the double solution of a cubic polynomial [7].

Figure 2 shows a typical dark-dark solution (3), that will be the object of the experimental study reported in the last section of this article. The family of solutions (3) in the defocusing regime, exhibits a novel feature with respect to the focusing regime solutions. In fact, here threshold conditions for the parameters a_1 , a_2 , q exist, due to the requirement that the parameter k be strictly complex, and that λ be a double solution of its polynomial equation. Quite remarkably, these rogue wave existence conditions are the same conditions that also lead to the presence of baseband modulation instability (MI), that is, to MI gain for arbitrarily small frequency shifts of the sidebands from the orthogonally polarized pumps [7].

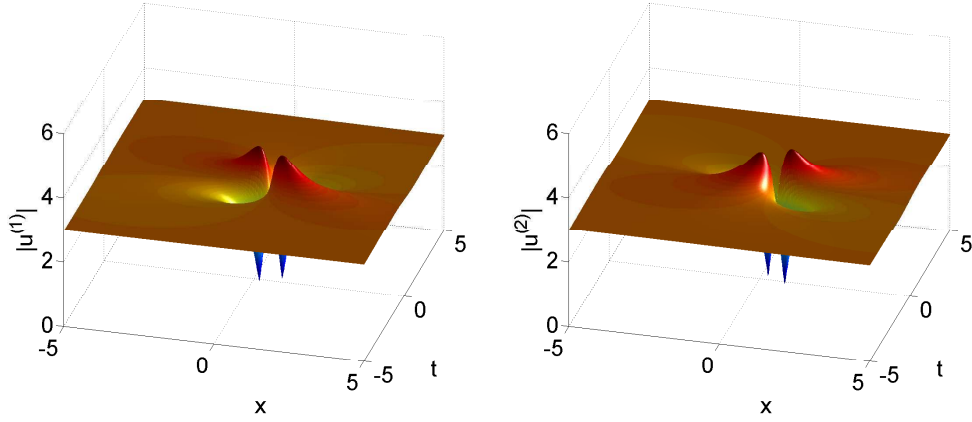


Figure 2: Rogue waves envelope distributions $|u^{(1)}(x, t)|$ and $|u^{(2)}(x, t)|$ of (3). Here, $a_1 = 3$, $a_2 = 3$, $q = 1$. $k = 4.02518i$ and $\lambda = -4.92887i$.

3. ROGUE WAVES IN PERIODIC MEDIA

We discuss now the rogue wave solution of the so-called classical massive Thirring model (MTM), a two-component nonlinear wave evolution equation that is completely integrable by the inverse scattering transform technique [9]. The MTM is a particular case of the coupled mode equations (CMEs) that describe pulse propagation in periodic or Bragg nonlinear optical media [10]. Note that soliton solutions of the MTM can be mapped into Bragg or gap solitons, that enable pulse reshaping and dispersion-less slow light generation in nonlinear Bragg gratings [11].

Let us express the MTM equations for the forward and backward waves with envelopes U and V , respectively, as

$$\begin{aligned} U_\xi &= -i\nu V - \frac{i}{\nu}|V|^2 U \\ V_\eta &= -i\nu U - \frac{i}{\nu}|U|^2 V. \end{aligned} \quad (5)$$

Here the light-cone coordinates ξ , η are related to the space coordinate z and time variable t by the relations $\partial_\xi = \partial_t + c\partial_z$ and $\partial_\eta = \partial_t - c\partial_z$, where $c > 0$ is the linear group velocity. By developing a novel form of the Darboux transform method [12], we obtained the MTM rogue soliton solution

$$U = ae^{-i\omega t} \frac{\mu^*}{\mu} \left[1 - \frac{4}{\mu^*} q^*(q+i) \right], \quad V = -ae^{-i\omega t} \frac{\mu}{\mu^*} \left[1 - \frac{4}{\mu} q^*(q+i) \right] \quad (6)$$

where: $\omega = -\nu(1 - \frac{a^2}{\nu^2})$, $q = -\frac{a^2}{\nu c}(ip(z - z_0) - c(t - t_0))$, $p = \sqrt{\frac{\nu^2}{a^2} - 1}$, $\mu = 2|q|^2 - iq + iq^* + 1 - \frac{1}{p}(q - q^* + i)$, and z_0 and t_0 are arbitrary space and time shifts.

Figure 3 shows an example of the analytical rogue wave solution (6). Here we have set $\nu = -1$, $c = 1$, $a = 0.9$, $t_0 = 2$ and $z_0 = 3.5$. As can be seen, the initial spatial modulation at $t = 0$ evolves into an isolated peak with a maximum intensity of about *nine times* larger than the CW background intensity. We numerically confirm the stability of the analytical solution (6), and show that it may also be applied to describe the generation of extreme waves in the more general context of nonlinear grating propagation as described by the CMEs. Finally, we discuss the physical implementation

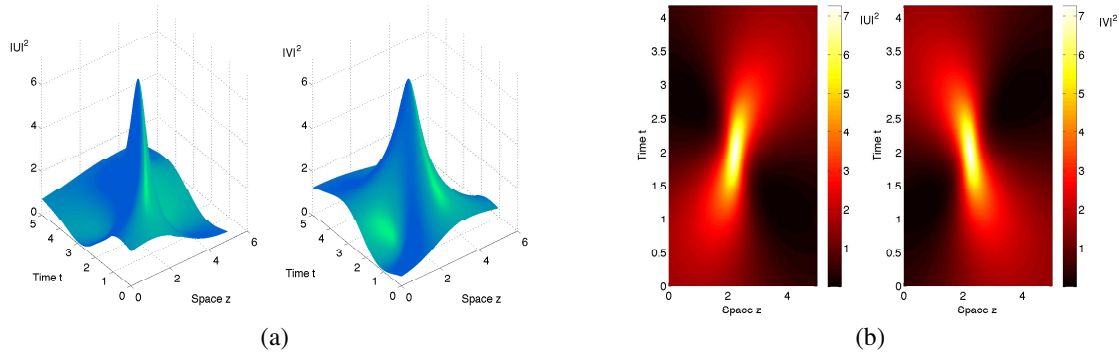


Figure 3: Space-time evolution of intensities in forward and backward rogue components: (a) surface or (b) contour plots.

of MTM rogue waves by using electromagnetically induced transparency, which leads to giant enhancement of cross-phase modulation, and suppression of self-phase modulation [13].

4. EXPERIMENTAL RESULTS

Let us finally discuss the experimental observation of vector dark-dark rogue wave solution of the VNLSE (3), resulting from the nonlinear coupling of two orthogonally polarized pump waves, propagating at different carrier frequencies in the normal dispersion regime of the randomly birefringent optical fiber. Thanks to cross-phase modulation and dispersive group-velocity walk-off, the two coupled pumps experience baseband MI [7]. Next, the nonlinear evolution of MI leads to the generation of spatio-temporal localized black rogue waves, exhibiting a hole of the optical intensity in each of the waves. In our experiments, MI was induced by the initial intensity modulation of the two orthogonal pumps with the frequency shift Ω . Nonlinear propagation happens in a reverse True Wave fiber, with relatively large normal chromatic dispersion of $-14 \text{ ps nm}^{-1} \text{ km}^{-1}$, the nonlinear coefficient $\gamma = 2.4 \text{ W}^{-1} \text{ km}^{-1}$ and the linear loss coefficient of 0.25 dB/km at $\lambda_0 = 1554.7 \text{ nm}$. This fiber has a very low PMD value ($0.017 \text{ ps km}^{-1/2}$). Panels (e), (f) of Fig. 4 compare the experimentally observed output intensity after propagation in 3 km of fiber (red curves), with the intensity

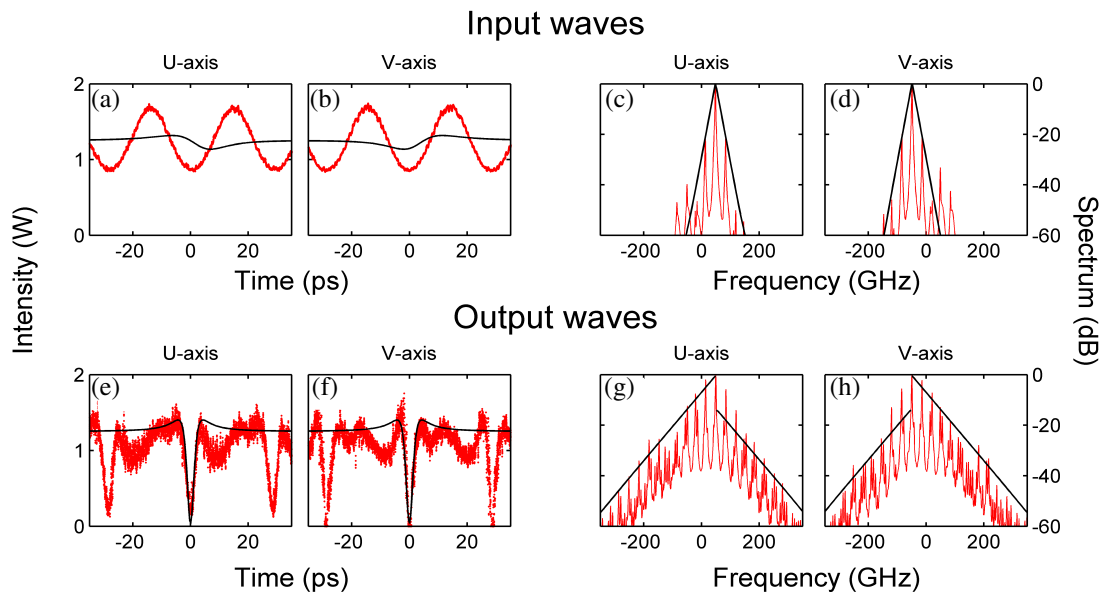


Figure 4: Observation of black vector rogue wave. (a), (b) temporal profile of power in the U and V polarization modes at the fiber input; (c), (d) power spectra at the fiber input; (e), (f) output intensities after 3 km of optical fiber; (g), (h) power spectra at the fiber output. Red solid traces are experimental results and black solid curves represent the analytical black vector rogue wave.

profile of the analytical dark-dark rogue solution (black curves). As can be seen, an excellent agreement is achieved, which is surprising since the experimentally imposed initial modulation is much deeper than the exact solution (see Figs. 4(a), (b)). Note that a temporal periodic experimental waveform (and not a single dark dip) is obtained, because of practical experimental constraints: in principle, an isolated rogue dip could be observed by indefinitely decreasing the initial modulation frequency Ω . Note that the signature of the rogue dip in the frequency domain is the development of a significant spectral asymmetry (see Figs. 4(g), (h)).

5. CONCLUSION

We presented an overview of our recent theoretical and experimental progress on rogue wave solutions of multi-component optical wave systems. In particular, we described rogue waves in both the anomalous and the normal dispersion regime for the co-propagation of two orthogonally polarized modes in a randomly birefringent optical fiber. We further obtained the space-time localized rogue wave solution resulting from the coupling of counter-propagating waves in a periodic nonlinear medium. We have also report the first experimental observation of a multi-component rogue wave, in the form of a dark-dark vector rogue light hole in a telecommunication optical fiber.

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