Total Internal Reflection as a Technique for Study of Surface Optical Characteristics of Left-handed Materials

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Abstract— In this work we consider an effect of surface inhomogeneities of left-handed materials on light reflection and refraction. It is shown that total internal reflection could be useful for investigation of surface optical characteristics of metamaterial. Scattering of refracted wave in the near-surface layer leads to existence of radiation propagating from the boundary to the point of observation. The angular distribution of the resulting radiation is derived and the opportunity of getting information about optical properties of metamaterial surface with help of this method is discussed.

1. INTRODUCTION

Real material surface is known to differ from ideal plane surface because of the presence of non-uniform near-surface layer. The properties of the layer may influence considerably various processes of macroscopic electrodynamics, such as reflection and refraction of light, propagation of surface waves, radiation from fast charged particle beams [1–3]. As the thickness of the layer is much smaller than the light wavelength, all inhomogeneities caused by natural surface anisotropy [4–6], impurity atom adsorption, surface relaxation and reconstruction, fluctuations in surface electronic states, — all these can be taken into account via introduction of surface polarization current into boundary conditions [3]. This surface current is linearly dependent on the electrical field, and proportionality factor is the phenomenological characteristic of dielectric and magnetic surface features.

Information about surface properties can be of great use for developing of metamaterial optics. Actually the so-called gradient mediums with variable electromagnetic parameters are of interest for optics of photonic crystals, waveguides and compact antennas [7]. Also a large number of works has been focused on materials with simultaneously negative permittivity and permeability, which are called "left-handed", because of its extraordinary features such as inverse Snell's law, reverse Doppler effect and Cherenkov radiation [8]. This materials find the applications in near-field optics, so considerable attention is paid to the investigation of surface waves on the left-handed material border, that play an important role, for example, in multilayered structures [9], perfect lens [10], terahertz optics [11], cloaking and so on.

It is known that in case of total internal reflection there is no energy transfer from the first medium where incident and reflected waves exist to the second medium. Therefore let us consider the interface between the left-handed material and the vacuum. The presence of inhomogeneities on it gives rise to the scattering of the refracted wave in the near-surface layer so that radiation propagating from the boundary in the vacuum appears. It is convenient to describe this radiation as a result of excitation of the surface current that can be included into boundary conditions. Let us discuss the origin of this current more detailed.

2. MEDIUM RESPONSE TO ELECTROMAGNETIC FIELDS

Electromagnetic field affecting on bound electron of matter excites forced oscillations of this electron. Thus the electron deflects from its initial point $\mathbf{r}$ and moves in the vicinity of this point. As a result average micro-current density of the bound electrons and therefore the electric induction appear:

$$D_i(\mathbf{r},\omega) = E_i(\mathbf{r},\omega) + \frac{4\pi i}{\omega} j_i(\mathbf{r},\omega) \tag{1}$$

On the other hand medium properties are characterized by the constitutive equation between $\mathbf{D}$, $\mathbf{E}$ and $\operatorname{rot}\mathbf{B}$ (note that magnetic induction $\mathbf{B}$ is an axial vector), that can be written in the most general form [12]:

$$D_i(\mathbf{r},t) = \int d^3r' \int dt' \left\{ \alpha_{is}(\mathbf{r},t,\mathbf{r}',t') E_s(\mathbf{r}-\mathbf{r}',t-t') + \beta_{is}(\mathbf{r},t,\mathbf{r}',t') \left(\operatorname{rot}\mathbf{B}(\mathbf{r}-\mathbf{r}',t-t')\right)_s \right\} \tag{2}$$

Coefficients $\alpha_{is}(\mathbf{r},t,\mathbf{r}',t')$ and $\beta_{is}(\mathbf{r},t,\mathbf{r}',t')$ describe dielectric and magnetic medium properties correspondingly. Dependence of α_{is} and β_{is} on $\mathbf{r}$ and $\mathbf{r}'$ define local and nonlocal dielectric responses, dependence on t — nonstationarity of the medium. So, for example, in case of the stationary medium in the absence of spatial dispersion and macroscopic inhomogeneity one can obtain:

$$D_i\left(\mathbf{r},\omega\right)=\alpha_{is}\left(\omega\right)E_s\left(\mathbf{r},\omega\right)+\beta_{is}(\omega)\left(\operatorname{rot}\mathbf{B}\left(\mathbf{r},\omega\right)\right)_s \tag{3}$$

Substitution of Eq. (3) in Maxwell's equation $\operatorname{rot}\mathbf{B}(\mathbf{r},\omega)=-i\omega/c\mathbf{D}(\mathbf{r},\omega)+4\pi/c\mathbf{j}_0(\mathbf{r},\omega)$ makes it possible to introduce the macroscopic magnetic field

$$H_i\left(\mathbf{r},\omega\right)=\mu_{is}^{-1}(\omega)B_s\left(\mathbf{r},\omega\right),\quad \text{where}\quad \mu_{is}(\omega)\equiv\frac{1}{1+i\frac{\omega}{c}\beta_{is}(\omega)}\quad \text{— the permittivity,}$$

and electric displacement

$$\tilde{D}_i\left(\mathbf{r},\omega\right)=\varepsilon_{is}(\omega)E_s\left(\mathbf{r},\omega\right),\quad \text{where}\quad \varepsilon_{is}(\omega)\equiv\alpha_{is}(\omega)\quad \text{— the permittivity.}$$

Nevertheless, there is another approach to consideration of the matter response to applied electromagnetic field. Indeed, using Maxwell's equation $\operatorname{rot}\mathbf{E}(\mathbf{r},t)=-\frac{1}{c}\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}$ it can be possible to move from Eq. (2) to the connection between $\mathbf{D}$ and $\mathbf{E}$ directly:

$$D_i\left(\mathbf{r},t\right)=\int d^3r'\int dt'\zeta_{is}\left(\mathbf{r},t,\mathbf{r}',t'\right)E_s\left(\mathbf{r}-\mathbf{r}',t-t'\right), \tag{4}$$

where tensor $\zeta_{is}(\mathbf{r},t,\mathbf{r}',t')$ includes both dielectric and magnetic properties of the matter. For a number of materials this approach can be more useful in case of quick-changing fields, when the permeability loses its physical meaning [13]. Thus there are two ways of description of field-matter interaction: in terms of $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$ and in terms of $\mathbf{E}$, $\tilde{\mathbf{D}}$, $\mathbf{B}$ and $\mathbf{H}$. Below we shall proceed from Eq. (4), just because of its more simple form.

3. SCATTERING OF REFRACTING WAVE ON SURFACE INHOMOGENEITIES OF LEFT-HANDED MATERIAL

Let us consider the left-handed isotropic medium placed in semi-infinite space $z<0$, where plane electromagnetic wave polarized arbitrarily towards the incident plane xz propagates. Under interaction of incident wave with the interface $z=0$ reflected and refracted waves appear. In case of total internal reflection, when incident angle $\gamma\geq\gamma\equiv\arcsin|\varepsilon(\omega)\mu(\omega)|^{-1/2}$, the refracted wave propagates along the boundary. Incident and reflected fields produce volume polarization currents in the metamaterial that are usually taken into account via permittivity $\varepsilon(\omega)<0$ and permeability $\mu(\omega)<0$. Due to the existence of surface inhomogeneities there is an additional current in the near-surface layer, average density of which can be obtained with help of Eqs. (1) and (4):

$$\Delta\mathbf{j}\left(\mathbf{r},\omega\right)=\frac{\omega}{4\pi i}\xi\left(\mathbf{r},\omega\right)\mathbf{E}\left(\mathbf{r},\omega\right)\theta(-z). \tag{5}$$

Here $\xi(\mathbf{r},\omega)$ is a small deflection from ζ in Eq. (4), which contains additions $\varepsilon(\mathbf{r},\omega)$ to $\varepsilon(\omega)$ and $\mu(\mathbf{r},\omega)$ to $\mu(\omega)$, raised by surface inhomogeneities.

Then boundary conditions on the interface between the metamaterial and the vacuum can be written in a form:

$$\left[\mathbf{e}\left\{\mathbf{H}_1\left(\mathbf{R},0,\omega\right)-\mathbf{H}_2\left(\mathbf{R},0,\omega\right)\right\}\right]=\left(4\pi/c\right)\mathbf{J}\left(\mathbf{R},\omega\right), \tag{6}$$
$$\left[\mathbf{e}\left\{\mathbf{E}_1\left(\mathbf{R},0,\omega\right)-\mathbf{E}_2\left(\mathbf{R},0,\omega\right)\right\}\right]=0, \tag{7}$$
$$\left(\mathbf{e}\left\{\varepsilon\left(\omega\right)\mathbf{E}_1\left(\mathbf{R},0,\omega\right)-\mathbf{E}_2\left(\mathbf{R},0,\omega\right)\right\}\right)=\left(i/\omega\right)\left\{\partial J_x/\partial x+\partial J_y/\partial y\right\}, \tag{8}$$
$$\left(\mathbf{e}\left\{\mu\left(\omega\right)\mathbf{H}_1\left(\mathbf{R},0,\omega\right)-\mathbf{H}_2\left(\mathbf{R},0,\omega\right)\right\}\right)=0, \tag{9}$$

where

$$\mathbf{J}\left(\mathbf{R},\omega\right)=\int_{-\infty}^{\infty}dz\boldsymbol{\tau}\Delta\mathbf{j}\left(\mathbf{R},z,\omega\right),$$

$\mathbf{R}=\mathbf{r}-\mathbf{e}(\mathbf{er})$ — radius-vector in the surface plane, $\mathbf{e}$ — unit director vector of $\mathbf{z}$ axis, $\boldsymbol{\tau}$ — unit director vector of $\mathbf{R}$. Here $\mathbf{E}_1$, $\mathbf{H}_1$-fields in metamaterial, $\mathbf{E}_2$, $\mathbf{H}_2$-fields in the vacuum.

The small thickness of near-surface layer makes it possible to consider this layer as small perturbation, which lets us solve Eqs. (6)–(9) with the method of stepwise approximation, representing the fields as

$$\mathbf{E}_{1(2)} = \mathbf{E}^{(0)}_{1(2)} + \mathbf{E}^{(1)}_{1(2)}, \mathbf{H}_{1(2)} = \mathbf{H}^{(0)}_{1(2)} + \mathbf{H}^{(1)}_{1(2)}.$$

For the fields of zero approximation the surface current is absent in the boundary condition and the problem reduces to finding of reflected and refracted fields $\mathbf{E}^{(0)}_{1(2)}$, $\mathbf{H}^{(0)}_{1(2)}$ in case of ideal plane surface, which are well-known [13]. For the fields of the first approximation, which rise out of the scattering of the fields of zero approximation, so the surface current in Eqs. (6)–(9) is assumed to be set:

$$\mathbf{J}\left(\mathbf{R},\omega\right) \to \mathbf{J}^{(0)}\left(\mathbf{R},\omega\right) = \frac{\omega}{4\pi i}\xi\left(\mathbf{R},\omega\right)\mathbf{E}^{(0)}_{\tau}\left(\mathbf{R},0,\omega\right), \tag{10}$$

where

$$\xi(\mathbf{R},\omega) = \int\limits_{-\infty}^{0} dz\xi(\mathbf{R},z,\omega), \quad \mathbf{E}^{(0)}_{1\tau} = \mathbf{E}^{(0)}_{2\tau} \equiv \mathbf{E}^{(0)}_{\tau}.$$

Let us find the energy flux of the scattered field, propagating out of the interface in the vacuum (similarly to the fields in the metamaterial) along the unit normal vector $\mathbf{e}$ through the plane $z = z_0$:

$$W_z = \frac{c}{4\pi}\iiint dxdydt\left(\mathbf{e}\left[\mathbf{E}^{(1)}_2\left(x,y,z_0,t\right)\mathbf{H}^{(1)}_2\left(x,y,z_0,t\right)\right]\right), \tag{11}$$

Applying Fourier transform in Eq. (11) and taking into account that all components of electric and magnetic field vectors can be expressed in terms of normal components by means of Maxwell's equations, we obtain:

$$W_z = 2\pi^2\iint dq_xdq_y\int d\omega\frac{g_2\omega}{q_x^2+q_y^2}\left(\left|E^{(1)}_{2z}\right|^2 + \left|H^{(1)}_{2z}\right|^2\right). \tag{12}$$

The normal components $E^{(1)}_{2z}$, $H^{(1)}_{2z}$ can easily be obtained from Eqs. (6)–(9):

$$E^{(1)}_{2z}(q_x,q_y,z,\omega) = -\frac{4\pi}{\omega}\frac{g_1\left(q_xJ^{(0)}_x\left(q_x,q_y,\omega\right)+q_yJ^{(0)}_y\left(q_x,q_y,\omega\right)\right)}{\varepsilon_1(\omega)g_2+\varepsilon_2(\omega)g_1}e^{ig_2z}, \tag{13}$$

$$H^{(1)}_{2z}\left(q_x,q_y,z,\omega\right) = -\frac{4\pi\mu_1}{c}\frac{q_xJ^{(0)}_y\left(q_x,q_y,\omega\right)-q_yJ^{(0)}_x\left(q_x,q_y,\omega\right)}{\mu_1g_2+\mu_2g_1}e^{ig_2z}, \tag{14}$$

$$g_1 = \sqrt{\frac{\omega^2}{c^2}\varepsilon(\omega)\mu(\omega)-q_x^2-q_y^2}, g_2 = \sqrt{\frac{\omega^2}{c^2}-q_x^2-q_y^2}. \tag{15}$$

The surface current $\mathbf{J}^{(0)}$ is specified by tangential component of the electric field of the refracted wave $\mathbf{E}^{(0)}_{\tau}(x,\,y,\,z,\,\omega) = \mathbf{E}^{(0)}_{\tau}e^{-ik_xx+ik_zz}\delta(\omega-\omega_0)$, where $k_x = (\omega/c)\sqrt{\varepsilon\mu}\sin\gamma$, $k_z = i(\omega/c)\sqrt{\varepsilon\mu\sin^2\gamma - 1}$:

$$\mathbf{J}^{(0)}\left(q_x,q_y,\omega\right) = \frac{\omega}{4\pi i}\mathbf{E}^{(0)}_{\tau}\xi\left(q_x+k_x,q_y,\omega\right)\delta\left(\omega-\omega_0\right) \tag{16}$$

Let us introduce substitution $q = (\omega/c)\sin\theta$. Thus, the distribution of energy $W = W_z/\cos\theta$ along unit vector $\mathbf{n} = \mathbf{r}/r$ ($\mathbf{r}$ — point of observation) over the frequency $d\omega$ and solid angle $d\Omega = \sin\theta d\theta d\varphi$ is:

$$\begin{aligned}\frac{d^2W\left(\mathbf{n},\omega\right)}{d\Omega d\omega} &= \frac{T\pi}{4}\frac{\omega^4}{3}\cos\theta|\xi\left(q_x+k_x,q_y,\omega\right)|^2\delta\left(\omega-\omega_0\right)\\ &\times\left(\frac{\varepsilon\mu-\sin^2\theta}{\left(\sqrt{\varepsilon\mu-\sin^2\theta}+\varepsilon\cos\theta\right)^2}\left|E^{(0)}_x\cos\varphi+E^{(0)}_y\sin\varphi\right|^2\right.\\ &\left.+\frac{\mu^2}{\left(\sqrt{\varepsilon\mu-\sin^2\theta}+\mu\cos\theta\right)^2}\left|E^{(0)}_y\cos\varphi-E^{(0)}_x\sin\varphi\right|^2\right),\end{aligned} \tag{17}$$

where T is the total observation time. Equation (17) gives us the dependence of spectral-angular distribution of the radiation energy on dielectric and magnetic characteristics of the magnetic medium surface that are described by the parameter ξ. We would like to notice that Equation (17) describes the radiation at positive $\cos\theta > 0$.

4. DISCUSSION

In this paper the problem of light reflection and refraction on the inhomogeneous surface of magnetic medium, in particular, on the surface of left-handed material is solved using the method of given surface current. It should be also noted that all results of this paper are obtained on the assumption of that the layer thickness is much smaller than light wavelength. Measuring of the spectral-angular radiation density resulting from scattering of refracted field can be employed for investigation surface response on electromagnetic field. The function $d^2W(\theta,\varphi,\omega)/d\Omega d\omega$ obtained from the experiment can be of use in finding $\xi(x,y,\omega)$:

$$\xi\left(x,y,\omega\right)=\int\frac{\omega^2}{c^2}\cos\theta d\Omega\sqrt{\frac{1}{f\left(\theta,\varphi,\omega\right)}\frac{d^2W\left(\theta,\varphi,\omega\right)}{d\Omega d\omega}}e^{i\frac{\omega}{c}\left(\cos\varphi\sin\theta+\left|\sqrt{\varepsilon\mu}\right|\sin\gamma\right)+i\frac{\omega}{c}\sin\varphi\sin\theta}, \tag{18}$$

where

$$\begin{aligned}f\left(\theta,\varphi,\omega\right)\ =\ &\frac{T\pi}{4}\frac{\omega^4}{3}\delta\left(\omega-\omega_0\right)\cos\theta\left(\frac{\varepsilon\mu-\sin^2\theta}{\left(\sqrt{\varepsilon\mu-\sin^2\theta}+\varepsilon\cos\theta\right)^2}\left|E_x^{(0)}\cos\varphi+E_y^{(0)}\sin\varphi\right|^2\right.\\&\left.+\frac{\mu^2}{\left(\sqrt{\varepsilon\mu-\sin^2\theta}+\mu\cos\theta\right)^2}\left|E_y^{(0)}\cos\varphi-E_x^{(0)}\sin\varphi\right|^2\right).\end{aligned} \tag{19}$$

Thus, information of parameter ξ as a function of coordinates can give clear vision of different phenomena and processes on the surface of magnetic materials.

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