Design and Analysis of Tunable Photonic Devices Based on the Co-integration of Graphene and Dielectric Waveguides

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Abstract— The co-integration between dielectric waveguides and graphene can open the way to the realization of photonic devices characterized by the well-established properties of mature technologies and by novel functionalities, e.g., the ultrafast tunability of the optical properties, that are typical of graphene-based devices. In this context, numerical modeling plays a key role. Graphene is often modeled as a thin layer with atomic thickness that is immersed in a micrometer-size dielectric structure, therefore the computational burden can become huge due to the size mismatch between the graphene layers and the environment. Here we demonstrate that the application of semi–analytical procedures based on the reformulation of techniques borrowed from classical optical waveguide theory allows to efficiently evaluate the modal properties of graphene–assisted dielectric waveguides. We consider two cases of study: first, we analyze an electro-absorption modulator based on the tunability of the conductivity of graphene. Then, a nonlinear waveguide where we exploit the huge third-order nonlinearity of graphene is studied. The excellent agreement between full-wave simulations and results obtained by applying the variation theorem for dielectric waveguides demonstrates the effectiveness of the approach.

1. INTRODUCTION

The peculiar properties of graphene have been attracting the ever increasing interest of the scientific community since the 2010 Nobel Prize in Physics, which was awarded to Novoselov and Geim for their groundbreaking studies. Scientists have already envisaged a plethora of innovative applications in electronics and photonics that exploit the unique characteristics stemming directly from the Dirac dispersion relation of electrons in graphene [1]. In particular, the ultrafast tunability of the electromagnetic parameters of this two-dimensional material can be a key feature for the realization of novel photonic devices. In this context, well-established analytical models are available nowadays to describe both the linear and the nonlinear optical properties of graphene in terms of its 2D complex surface conductivity σ_{2D} (see, e.g., [2–5]).

The interaction length between light and graphene is limited by the atomic thickness of the material; therefore, following the idea proposed by Liu et al. in a seminal paper [6], several structures based on the co-integration of conventional dielectric waveguides and graphene have been reported, with the goal of perturbing the propagation of a guided mode by means of localized (and tunable) variations of the electromagnetic parameters. This concept allowed to envisage, for instance, graphene-based amplitude [6, 7] and phase modulators [8], tunable directional couplers [9, 10] and highly nonlinear waveguides [11]. Most of these devices have been studied by resorting to full-wave simulations where graphene was modeled as a 3D brick with atomic thickness Δ, and volume conductivity $\sigma_{3D} = \sigma_{2D}/\Delta$. This approach is straightforward but it is characterized by a huge and often unacceptable computational burden, thus modeling techniques based on surface current boundary conditions [10, 11] and/or perturbation theory [8, 10] have recently been proposed.

In this work we illustrate the possibility of applying computationally efficient techniques such as perturbation theory for the analysis of graphene-based devices operating both in the linear and in the nonlinear regime. We demonstrate that the effects induced by the tunability of the conductivity of graphene, which is due to an electrical bias and/or to the Kerr effect, can be accurately predicted by exploiting the "concentrated" nature of the perturbation introduced by the graphene layers.

2. PERTURBATION THEORY FOR GRAPHENE-BASED WAVEGUIDES

The variation $\delta\beta_{eff}$ of the propagation constant of a guided mode propagating along z that is induced by a perturbation $\delta\epsilon$ of the xy transverse profile of the dielectric constant can be estimated by exploiting the variation theorem for dielectric waveguides, as described in [12]:

$$\delta\beta_{eff\,\nu} = \frac{\omega \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy\delta\epsilon \mathbf{E_\nu}\cdot\mathbf{E_\nu^*}}{4P}, \tag{1}$$

where ω is the angular frequency, P is the power carried by the mode ν and E_ν is the corresponding electric field profile.

This general expression can be reformulated by taking into account the peculiar characteristics of the problem at hand. In particular, our key issue is the analysis of the properties of conventional dielectric waveguides that are co-integrated with graphene layers in order to introduce a tunable perturbation of the mode propagation. The electromagnetic properties of graphene are anisotropic, since graphene interacts only with the component of the electric field that is parallel with respect to the 2D layers. As a matter of fact, analytical models typically provide the in-plane 2D conductivity of graphene as a function of ω and of the chemical potential μ_c (see e.g., [2] for the linear conductivity). Therefore, the perturbation of the volume dielectric constant can be calculated from the graphene surface conductivity by using the relation $\boldsymbol{\delta\epsilon} = -i\boldsymbol{\sigma}_{2D}/(\Delta\omega)$, which stems directly from Maxwell's equations. If we also reasonably assume that the electric field is invariant inside the graphene along the direction that is normal with respect to the layer itself we get

$$\delta n_{eff\,\nu} = -i\frac{1}{k_0}\int_g dg \mathbf{E}_{\boldsymbol{\nu}} \cdot \boldsymbol{\sigma}_{2D} \cdot \mathbf{E}^*_{\boldsymbol{\nu}}, \tag{2}$$

where k_0 is the free-space wave-number and the mode profiles are normalized for the sake of simplicity so that $P = 1/4\,\mathrm{W}$. Notice that by exploiting the two-dimensional nature of graphene the surface integral in Eq. (1) can be simplified to become a line integral running along the graphene layers. We labeled this curve as g. If the structure under exam includes M graphene layers, and the i-th layer is biased to a chemical potential μ_{ci}, Eq. (2) can be recast in the final form

$$\delta n_{eff\,\nu} = -i\frac{1}{k_0}\sum_{i=1}^{M}\int_{g_i} dg_i \mathbf{E}_{\boldsymbol{\nu}} \cdot \boldsymbol{\sigma}_{2D}(\mu_{ci}) \cdot \mathbf{E}^*_{\boldsymbol{\nu}}. \tag{3}$$

The same technique can be applied to estimate the nonlinear phase shift which is due to the graphene–induced Kerr effect. It is worth saying that several recent papers report different models for the 2D third-order conductivity σ_3 of graphene [3–5]. In the nonlinear regime the total conductivity of graphene can be written as $\boldsymbol{\sigma}_{nonlin} = \boldsymbol{\sigma}_{lin} + \boldsymbol{\sigma_3} \cdot \mathbf{E}_{\boldsymbol{\nu}} \cdot \boldsymbol{E}^*_{\boldsymbol{\nu}}$, and Eq. (2) can be reformulated as

$$\delta n_{eff\,\nu} = -i\frac{1}{4k_0 P}\int_g dg \mathbf{E}_{\boldsymbol{\nu}} \cdot \boldsymbol{\sigma}_{nonlin} \cdot \boldsymbol{E}^*_{\boldsymbol{\nu}}, \tag{4}$$

where the correct tensor σ_3 must be selected in order to model the Kerr effect ($\sigma_3 = \sigma_3(-\omega, \omega, \omega)$), and the mode profile is now normalized so that P corresponds to the real power which is carried by the propagating mode ν.

3. NUMERICAL RESULTS

The first case of study is the graphene-based optical modulator described in Ref. [6]. The structure is a silicon ridge waveguide on a silica substrate, and it is schematically depicted in Fig. 1(a). The width of the ridge is 600 nm, whereas its height is equal to 250 nm. Also the 50-nm-thick silicon layer that is necessary in order to bias the structure is considered. A thin layer of alumina (with thickness equal to 7 nm) is inserted between the ridge waveguide and three graphene layers (G_1 to G_3). In our simulations we have used a simple parallel-plate capacitor model in order to evaluate the chemical potential of G_1 and G_2 starting from the bias voltage, whereas the chemical potential of G_3 was assumed to be zero. In order to be consistent with Ref. [6] the offset voltage was fixed to $-0.8\,\mathrm{V}$, and the operating wavelength is $1.53\,\mu\mathrm{m}$. In Fig. 1(b) we show the graphene conductivity, calculated by using the analytical model described in Ref. [2], as a function of the bias voltage.

The propagating modes in this structure are hybrid, nevertheless in strict analogy with what is commonly done in silicon photonics we can label the first mode as a quasi-TE mode (the dominant component of the electric field is "horizontal"). In Fig. 2(a) we report the effective index of the quasi-TE mode as a function of the bias voltage. Notice the perfect agreement between the results from full-wave simulations and those from perturbation theory (Eq. (3)). It is also straightforward to see that, as expected, the shape of the curve is the same as the one that represents the dependence of the imaginary part of the conductivity on the applied voltage: when $\mathrm{Im}\{\sigma_{2D}\}$ is positive the effective index increases with respect to the unperturbed case, and the opposite happens when $\mathrm{Im}\{\sigma_{2D}\}$ is negative. In Fig. 2(b) we show the absorption (modulation depth) of the first mode in logarithmic scale. Also in this case the estimate obtained by using the semi-analytical model is rather good, indeed both the methods predict a maximum modulation depth equal to $0.06\,\mathrm{dB}/\mu\mathrm{m}$. Here the

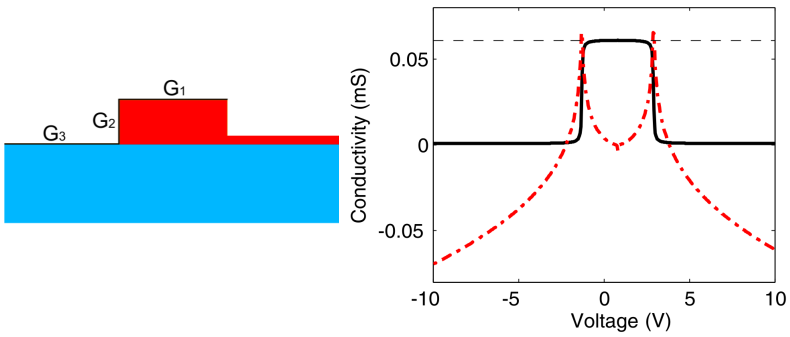

Figure 1: (a) Schematic view of the graphene-based modulator reported in Ref. [6]. The graphene layers are indicated in black. (b) Complex conductivity of graphene as a function of the bias voltage. Real part (solid black line), imaginary part (red dashed-dotted line), and universal conductivity (black dashed line).

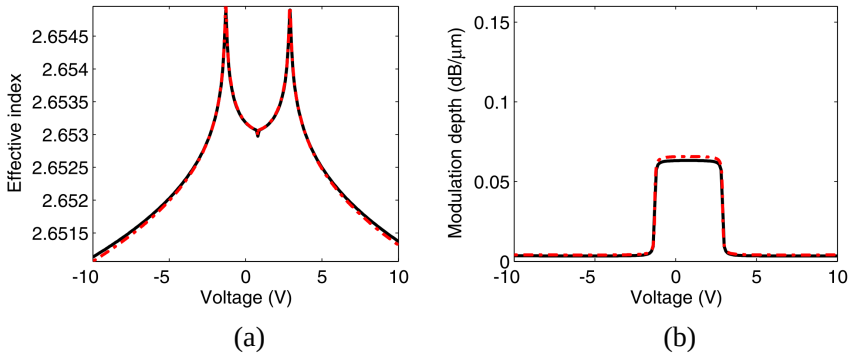

Figure 2: Full-wave simulations (black solid lines) and results from perturbation theory (red dashed-dotted lines) for the quasi-TE mode. (a) Effective index. (b) Modulation depth.

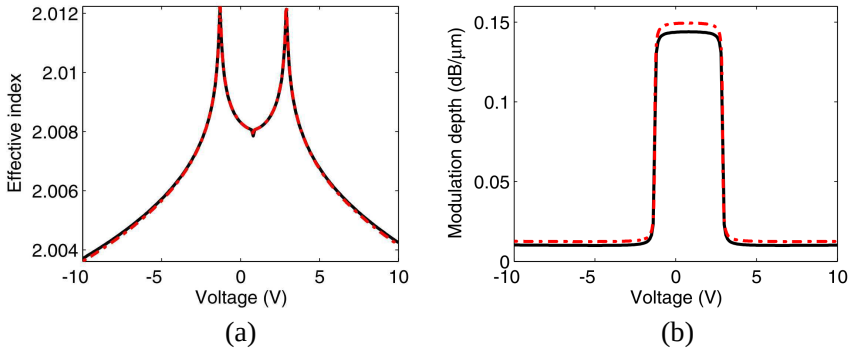

Figure 3: Full-wave simulations (black solid lines) and results from perturbation theory (red dashed-dotted lines) for the quasi-TM mode. (a) Effective index. (b) Modulation depth.

behavior of the curve is strictly related to the dependence of the real part of the conductivity on the bias voltage (see Fig. 1(b)).

The second propagating mode can be identified as a quasi-TM mode (the dominant component of the electric field is "vertical"). This part of the analysis is quite important because the experimental results reported in Ref. [6] exploit the propagation of this mode, as it is possible to infer from the analysis of Fig. 1 of that paper. In Figs. 3(a) and 3(b) we report effective index and modulation depth of the quasi-TM mode as a function of the bias voltage. We can see the same trend we have described for the previous case, in fact agreement between full–wave simulations and

perturbation theory is still very good. It is quite interesting to compare Fig. 3(b) with the corresponding experimental curve in Ref. [6]. The sharp transitions between high and low absorption are smoothed in the experimental curve with respect to the numerical one, nevertheless agreement is both qualitative and quantitative: experimental (numerical) results predict a maximum modulation depth of 0.1 (0.14) dB/μm, with high absorption for a range of voltages between −1 and 3.8 V (between −1.3 and 3 V). As a matter of fact, comparison between experimental and numerical results confirms that a single simulation of the structure without graphene is sufficient to determine the behavior of the structure with graphene, also as a function of the bias voltage. A thorough discussion about the peculiar behavior of this waveguide is well beyond the scope of the paper, anyway it is worth noting that the modulation depth of the quasi-TM mode is much larger with respect to the quasi-TE case, and this observation can be in some way counter-intuitive since the dominant electric field component of the second mode is normal with respect to the graphene layers. Nevertheless, we have found that in the quasi-TM case the modal field is strongly concentrated near the graphene layers and the longitudinal component of the electric field E_z, which is parallel to the graphene sheets, dominates with respect to the transverse components in that region.

The second case of study is the nonlinear waveguide described in Ref. [11]. A schematic view of the structure is shown in the inset of Fig. 4(a). It is a simple slab waveguide, with graphene layers that are placed at the interfaces between core ($\epsilon_{co} = 4.76$) and cladding ($\epsilon_{cl} = 2.25$). The width of the slab is $2s = 400$ nm, and the operating wavelength is 1.55 μm. For the sake of simplicity we considered only TE polarization. In Ref. [11] a model for the third-order nonlinearity, also including saturation of the nonlinear response, has been extrapolated from Z-scan measurements [13]. A detailed description of the model and the values of all the parameters can be found in the same paper. Here we have used the nonlinear model of graphene reported in [11] with the goal of performing a fair comparison, nevertheless the same analysis can be repeated by using analytical models such as those described in Refs. [3–5]. It is straightforward to demonstrate that Eq. (4), which is valid for the general nonlinear case, can be specialized for this structure so that we get

$$\delta n_{eff} = \Delta \frac{\delta \epsilon_{r\,Kerr} \left| E_y(x = \pm s) \right|^2}{n_{eff} \int_{-\infty}^{\infty} |E_y|^2 \, dx}, \tag{5}$$

where, by using again the same formulation as in Ref. [11], $\delta \epsilon_{r\,Kerr}$ is the variation of the volume dielectric constant of graphene which is due to the Kerr effect, and $E_y(x = \pm s)$ is the electric field on the graphene layers. Notice that it is possible to use the well-known closed-form expression for the mode profile E_y of the unperturbed slab (without graphene), by properly normalizing E_y in order to have the desired peak intensity. In Figs. 4(a) and 4(b) we plot real and imaginary part of the effective index of the slab waveguide as a function of the peak intensity, by comparing full-wave simulations and results obtained by using Eq. (5). It is possible to see that the curves are perfectly overlapped, thus the described technique can be considered as validated. Last, but not least, it is worth noting that the reported analysis in the nonlinear regime can be easily applied to more realistic structures such as the ridge waveguide in Fig 1(a).

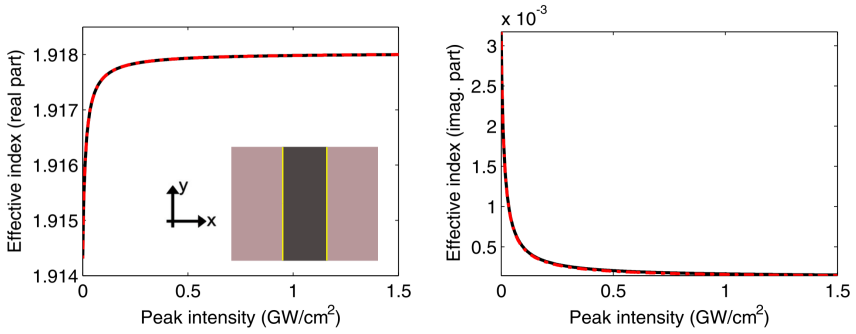

Figure 4: Effective index as a function of the peak intensity. (a) Real part. (b) Imaginary part. Full-wave simulations (black solid lines) and results from perturbation theory (red dashed-dotted lines) are reported. A schematic view of the slab is depicted in the inset, with the graphene layers which are indicated in yellow.

4. CONCLUSION

We have demonstrated that photonic devices characterized by the co-integration of conventional dielectric waveguides and graphene layers can be accurately and efficiently studied, both in the linear and in the nonlinear regime, by exploiting the "concentrated" nature of the perturbation introduced by graphene. In particular, we have shown that a clever and straightforward application of the variation theorem for dielectric waveguides allows to develop simple semi-analytical tools which allow to evaluate the propagation constants of the modes of complex guiding structures as a function of the bias voltage and of the optical power.

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