Collapse of Nonlinear Terahertz Pulses in n-InSb

C. Castrejon-Martinez, V. Grimalsky, S. Koshevaya, and J. Escobedo-Alatorre
CIICAp, Autonomous University of State Morelos (UAEM)
Av. Universidad 1001, Cuernavaca, ZP 62209, Mor., Mexico

Abstract— There are investigated nonlinear electrodynamic phenomena in volume narrow-gap semiconductors n-InSb, n-InAs at frequencies 1–5 THz. The nonlinearity of the electron gas due to the Kane dispersion law is dominating there. The equation for the amplitude, which is slowly varying with respect to time only, has been derived. The saturation of nonlinearity is taken into account. Under a propagation of THz transverse limited electromagnetic pulses through the structure intrinsic InSb-n-InSb-intrinsic InSb both the longitudinal and transverse focusing occurs. This leads to the wave collapse, or the concentration of the wave energy near one point. The optimum conditions for realizing wave collapse are obtained. A comparison is given with the structures that include the graphene sheets.

1. INTRODUCTION

The radiation of the terahertz (THz) range $f = 100\,\text{GHz}$–$30\,\text{THz}$ is used in spectroscopy, medicine, scanning, and environmental science [1]. Nonlinear materials for THz range are: graphene and semimetals, paraelectrics, superlattices, and narrow gap semiconductors. The narrow-gap semiconductors like InSb, InAs, $\text{Cd}_x\text{Hg}_{1-x}\text{Te}$ possess high dynamic nonlinearity in THz range under the temperatures $T \geq 77\,\text{K}$ due to the nonparabolicity of the electron dispersion law [2, 3].

In this report the nonlinear electrodynamic phenomena in i-InSb-n-InSb-i-InSb structures are investigated, where the nonlinearity of the electric current due to the nonparabolicity of the dispersion law of conduction electrons is dominating. Generally this nonlinearity is not cubic and the general expression for the current density is used. Under a propagation of electromagnetic (EM) pulses through n-InSb both the longitudinal and transverse focusing occurs. This phenomenon results in the strong compression of the input pulses that leads to the wave collapse and concentration of the EM energy near one point.

2. BASIC EQUATIONS

Nonlinear propagation of transversely limited EM pulses in simple structures i-InSb-n-InSb-i-InSb is considered. The possibility of wave collapse of input pulses is of great interest. The structures have interfaces parallel to XY plane, while EM wave propagates in OZ direction. The almost linearly polarized EM wave $E = E_x$ is considered.

The n-InSb layer has the equilibrium electron concentration $n_0 \sim 10^{15}$–$10^{16}\,\text{cm}^{-3}$ under the temperatures $T \geq 77\,\text{K}$. The electrodynamic nonlinearity is due to the electron nonlinearity of the Kane dispersion law [2, 3]. The hydrodynamic equation of motion of the electron gas can be presented as [3]:

$$\frac{d}{dt}\left(\frac{m^* v}{(1-(v/v_n)^2)^{1/2}}\right) = eE, \quad \text{where} \quad v_n = \left(\frac{E_g}{2m^*}\right)^{1/2}; \tag{1}$$

This equation of motion is analogous to the relativistic equation of motion of the particle, when the velocity of light c is replaced by the characteristic velocity $v_n \approx 1.2 \cdot 10^8\,\text{cm/s}$. Here v is the electron velocity, $m^* \approx 0.014 \cdot m_e$ is the electron effective mass, $E_g \approx 0.23$ eV is the forbidden gap for InSb [4, 5]. From Eq. (1) it is possible to get the following expression for the density of the electron current:

$$j \equiv en_0 v = \frac{e^2 n_0}{m^*}\frac{A}{(1+(eA/m^* v_n)^2)^{1/2}}; \quad \text{where} \quad E \equiv \frac{\partial A}{\partial t} \tag{2}$$

Generally the electron nonlinearity is of the saturating type. For the electric field the wave equation is used:

$$c^2 \Delta E - \frac{\partial^2 D}{\partial t^2} - 4\pi \frac{\partial j}{\partial t} = 0 \tag{3}$$

Here D is the linear electric induction that includes the polarization of the crystalline lattice. The frequency-dependent linear dielectric permittivity of the semiconductor crystal is [4]:

$$\varepsilon(\omega) = \varepsilon(\infty) - \frac{\omega_T^2(\varepsilon(0) - \varepsilon(\infty))}{\omega(\omega - i\gamma) - \omega_T^2}; \tag{4}$$

Here $\varepsilon(0)$, $\varepsilon(\propto)$ are low- and high-frequency dielectric permittivities, ω_T is the frequency of transverse optical phonons, γ is the lattice dissipation [4].

The solution of Eq. (3) can be searched in the following form [6–9]:

$$E = \frac{1}{2}C(z, \rho, t)\exp(i\omega t) + c.c. \tag{5}$$

The amplitude C is slowly varying in time t, but the dependencies on z and the radial coordinate ρ are arbitrary. This is important for the nonlinear focusing of EM wave considered below.

From Eq. (5) it is possible to obtain the following expression for D [6, 9]:

$$D \approx \frac{1}{2}\left(\varepsilon C - i\frac{d\varepsilon}{d\omega}\frac{\partial C}{\partial t} - \frac{1}{2}\frac{d^2\varepsilon}{d\omega^2}\frac{\partial^2 C}{\partial t^2}\right)\exp(i\omega t) + c.c. \tag{6}$$

After substitution of Eqs. (6) and (2) into Eq. (3), the following nonlinear equation for the wave amplitude C has been derived:

$$\frac{\partial C}{\partial t} + \frac{ic^2}{2\omega\varepsilon^{(1)}}\left(\frac{\partial^2 C}{\partial z^2} + \Delta_\perp C\right) + \frac{i\omega\varepsilon}{2\varepsilon^{(1)}}\left(1 - \frac{\omega_p^2}{\omega^2\varepsilon}(1 + \frac{i\nu}{\omega})\right)C$$
$$+\frac{i\omega_p^2\omega}{2\omega^2\varepsilon^{(1)}}\left[1 - Q^{-1/2}\left(1 - \frac{e^2|C|^2}{8(m^*v_n\omega)^2}Q^{-1}\right)\right]C = 0; \quad \text{where} \quad Q = 1 + \frac{e^2|C|^2}{2(m^*v_n\omega)^2}. \tag{7}$$

Here $\varepsilon^{(1)} = \varepsilon + (\omega/2)(d\varepsilon/d\omega)$ is the correction of the permittivity due to the frequency dispersion, $\omega_p = (4\pi e^2 n_0/m^*)^{1/2}$ is the electron plasma frequency. The collision frequency ν of electrons is taken into account here. It is of about $\nu = 2\times10^{11} - 10^{12}\,\mathrm{s}^{-1}$ for InSb in the temperature $T = 77\,\mathrm{K}$–$300\,\mathrm{K}$. It follows from Eq. (7) that the nonlinearity in n-InSb is focusing both in the longitudinal direction z and in transverse one ρ. The volume nonlinearity of n-InSb results in the wave collapse of the pulses, as our simulations are demonstrated.

Equation (7) is added by boundary conditions that are obtained from the continuity of the tangential components E_x and H_y at the boundaries i-InSb-n-InSb. At $z < 0$ both the incident and reflected waves are present, at $z > L_z$ only the transmitted wave exists. In the parabolic approximation the boundary conditions are:

$$\frac{\partial C}{\partial z} - ikC = -2ikE_i(\rho, t), \quad z = 0; \quad \frac{\partial C}{\partial z} + ikC = 0, \quad z = L_z; \quad \text{where} \quad k = \frac{\omega}{c}\varepsilon^{1/2} \tag{8}$$

Here $E_i(\rho, t)$ is the amplitude of the incident wave in the input.

3. RESULTS OF SIMULATIONS

The incident pulse is:

$$E_i(\rho, t) = E_{i0}\exp\left(-\left(\frac{t - t_1}{t_0}\right)^6\right)\exp\left(-\left(\frac{\rho}{\rho_0}\right)^6\right) \tag{9}$$

The shape of the input pulse is almost rectangular both for time t and for the transverse radial coordinate ρ.

It is investigated a simple structure that includes n-InSb layer of the thickness $L_z = 0.12\,\mathrm{cm}$. At $z < 0$ and $z > L_z$ there is i-InSb. In n-InSb the equilibrium electron concentration is $n_0 = 10^{16}\,\mathrm{cm}^{-3}$, the electron collision frequency is $\nu = 6\cdot10^{11}$–$10^{12}\,\mathrm{s}^{-1}$ ($T \geq 77\,\mathrm{K}$), the low- and high-frequency permittivities are $\varepsilon(0) = 17.1$, $\varepsilon(\propto) = 15.15$; the frequency of transverse optical phonons is $\omega_T = 3.376\cdot10^{13}\,\mathrm{s}^{-1}$, the lattice dissipation is $\gamma = 0.01\omega_T$, see Eq. (4) [4, 5]. The THz pulses with the carrier frequency $\omega = 2 \cdot 10^{13}\,\mathrm{s}^{-1} < \omega_T$ are considered. The half-widths of the input pulses vary within the limits $\rho_0 = 0.01$–$0.03\,\mathrm{cm}$. The wave reflection at the input is small.

The results of simulations for n-InSb are presented in Figs. 1–3. The electric field is normalized to $E_n = 14.4\,\mathrm{kV/cm}$ there. For all cases the parameter t_1 is $t_1 = 12\,\mathrm{ps}$, see Eq. (9). The wave collapse occurs when the maximum amplitude of the input pulse exceeds some threshold that depends on the dissipation. Also the duration of the input pulse and the transverse width should be optimized, to produce the maximum compression.

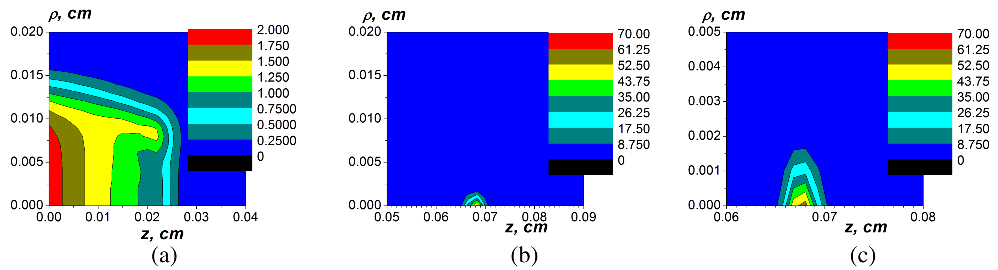


Figure 1: The collapse of the THz pulse in n-InSb with the maximum input amplitude $E_{i0} = 18.7\,\mathrm{kV/cm}$. The transverse width of the input pulse is $2\rho_0 = 0.031\,\mathrm{cm}$, the duration is $2t_0 = 11\,\mathrm{ps}$. The part (a) is distribution of $|C(z,\rho)|^2$ at $t = t_1 =$12 ps, the parts (b) and (c) are $|C|^2$ for the maximum compression $t = 21.5\,\mathrm{ps}$, general and detailed views. The electron dissipation is $\nu = 6 \cdot 10^{11}\,\mathrm{s}^{-1}$.

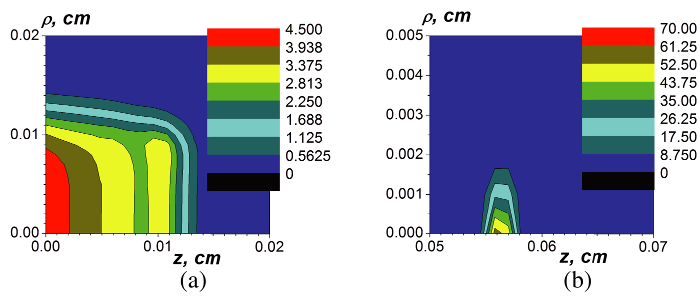


Figure 2: Spatial distribution $|C|^2$ of the pulse. The input amplitude is $E_{i0} = 28.8\,\mathrm{kV/cm}$. The transverse width of the input pulse is $2\rho_0 = 0.028\,\mathrm{cm}$, the duration is $2t_0 = 5.6\,\mathrm{ps}$. Part (a) is for the time moment $t = t_1 = 12\,\mathrm{ps}$, (b) is for $t = 23\,\mathrm{ps}$. A higher dissipation $\nu = 10^{12}\,\mathrm{s}^{-1}$.

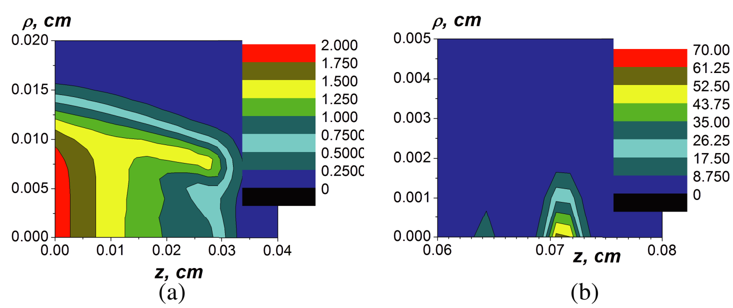
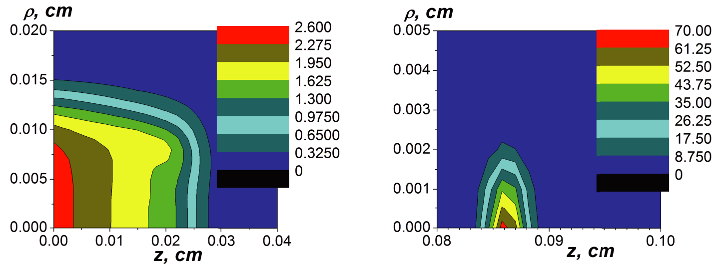


Figure 3: Spatial distribution $|C|^2$ of the pulse within the structure. The input amplitude is $E_{i0} = 18.7\,\mathrm{kV/cm}$. The transverse width of the input pulse is $2\rho_0 = 0.031\,\mathrm{cm}$, the duration is $2t_0 = 14\,\mathrm{ps}$ (a longer pulse, compared with Fig. 1). Part (a) is for the time moment $t = t_1 = 12\,\mathrm{ps}$, (b) is for $t = 22\,\mathrm{ps}$.

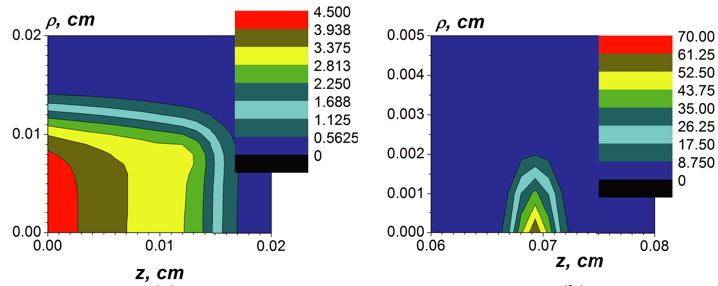
When there is some deviation from the optimum input parameters, EM concentrates near several points, as seen in Fig. 3(b). The input pulse is longer, when compared with Fig. 1.

The wave collapse also can be realized in the narrow-gap semiconductor n-InAs, where the structures i-InAs-n-InAs-i-InAs are simulated. The electron concentration is $n_0 = 10^{16}\,\mathrm{cm}^{-3}$, the effective mass is $m^* \approx 0.023 \cdot m_e$, the forbidden gap is $E_g \approx 0.31\,\mathrm{eV}$ [4, 5], $v_n \approx 1.1 \cdot 10^8\,\mathrm{cm/s}$; the low- and high-frequency permittivities are $\varepsilon(0) = 15.15$, $\varepsilon(\propto) = 12.3$; the frequency of transverse



(a) (b)

Figure 4: The wave collapse in n-InAs. The maximum input amplitude is $E_{i0} = 25\,\mathrm{kV/cm}$. The transverse width of the input pulse is $2\rho_0 = 0.03\,\mathrm{cm}$, the duration is $2t_0 = 9.6\,\mathrm{ps}$. Part (a) is for the time moment $t = 12\,\mathrm{ps}$, (b) is for $t = t_1 = 23.5\,\mathrm{ps}$. The electron dissipation is $\nu = 6 \cdot 10^{11}\,\mathrm{s}^{-1}$.



(a) (b)

Figure 5: The wave collapse in n-InAs. The maximum input amplitude is $E_{i0} = 33.4\,\mathrm{kV/cm}$. The transverse width of the input pulse is $2\rho_0 = 0.025\,\mathrm{cm}$, the duration is $2t_0 = 6\,\mathrm{ps}$. Part (a) is for the time moment $t = t_1$ =12 ps, (b) is for $t = 22\,\mathrm{ps}$. The higher dissipation $\nu = 10^{12}\,\mathrm{s}^{-1}$.

optical phonons is $\omega_T = 4.046 \cdot 10^{13}\,\mathrm{s}^{-1}$, the lattice dissipation is $\gamma = 0.01\omega_T$ [4, 5]. The results of simulations are given in Figs. 4, 5. Here the electric field is normalized to $E_n = 16.7\,\mathrm{kV/cm}$. The wave collapse occurs at higher input amplitudes of the pulses.

Therefore, the wave collapse of input THz pulses of the durations 5–20 ps and the transverse widths 0.02–0.04 cm in the structures that include n-InSb or n-InAs semiconductors can be used for the concentration of the electromagnetic energy near some point within the crystal. Because the electron dissipation is quite high $\nu > 5 \cdot 10^{11}\,\mathrm{s}^{-1}$, the realization of stable temporal or spatial solitons in THz range is doubtful, whereas the wave collapse can take place.

4. CONCLUSIONS

The principal effect reported in this paper is the nonlinear spatiotemporal focusing of THz pulses that leads to the wave collapse, or the concentration of EM energy near some point within the narrow-gap semiconductors n-InSb or n-InAs. The carrier frequency should be chosen below the frequency of transverse optical phonons. The shape of the input pulse should be optimized, to obtain the single nonlinear focus. The optimization should be realized for the input amplitudes of the pulses and on their durations and transverse widths.

In a distinction to the structures with n-InSb or n-InAs, in the structures with graphene layers [10] the wave collapse is absent [7, 8]. Other phenomena, like the transition to the transmission regime of short incident pulses due to nonlinearity and change of the transverse widths of the pulses [7, 8], can be realized both in the structures with the graphene layers and with the narrow-gap semiconductors. In THz range the nonlinearity manifests at similar levels of the input amplitudes both for the structures with graphene layers and with the n-InSb ones.

ACKNOWLEDGMENT

The authors are grateful to SEP-CONACyT (Mexico) for a partial support of our work.

REFERENCES

1. Perenzoni, M. and D. J. Paul (Eds.), *Physics and Applications of Terahertz Radiation*, Springer, N.Y., 2014.

2. Belyantsev, A. M., V. A. Valov, V. N. Genkin, A. M. Leonov, and B. A. Trifonov, "Dynamic mechanism of nonlinearity and frequency conversion of millimeter electromagnetic radiation in n-Type InSb," *Sov. Phys. JETP*, Vol. 34, No 3, 471–473, 1972.
3. Bass, F. G. and A. A. Bulgakov, *Kinetic and Electrodynamic Phenomena in Classical and Quantum Semiconductor Superlattices*, Nova Publ., N.Y., 1997.
4. Yu, P. Y., and M. Cardona, *Fundamentals of Semiconductors*, Springer, N.Y., 2010.
5. Levinshtein, M., S. Rumyantsev, and M. Shur, *Handbook Series on Semiconductor Parameters*, Vol. 1, World Scientific, Singapore, 1999.
6. Castrejon-M., C., V. Grimalsky, S. Koshevaya, and A. Zamudio-Lara, "Propagation of transversely bounded nonlinear electromagnetic pulses through periodic media," *Journal of Nonlinear Optical Physics and Materials*, Vol. 21, No 2, paper 1250022 (13 pp.), 2012.
7. Rapoport, Y., V. Grimalsky, and I. Iorsh, "Nonlinear reshaping of THz pulses with graphene metamaterials," *Pis'ma v ZhETF*, Vol. 98, No. 8, 561–564, 2013.
8. Rapoport, Y. G., V. V. Grimalsky, and C. Castrejon-M., "Nonlinear spatiotemporal focusing of terahertz pulses in the structures with graphene layers, *Proc. 2014 IEEE 34th Intl. Conf. on Electronics and Nanotechnology, ELNANO 2014*, 31–34, Kyiv, Ukraine, 15–18 April 2014.
9. Kivshar Y. S. and G. P. Agrawal, *Optical Solitons*, Academic Press, N.Y., 2003.
10. Mikhailov, S. A., "Non-linear electromagnetic response of graphene," *Europhys. Lett.*, Vol. 79, No. 2, paper 27002 (4 pp.), 2007.