Dyakonov-like Plasmonic Localized Waves on Graphene Metasurfaces

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Abstract— We study electromagnetic properties of a metasurface formed by array of coupled graphene nanoribbons. We show that surface conductivity tensor has principal components of different sign and the system supports Dyakonov-like plasmonic surface modes.

1. INTRODUCTION

Metasurfaces are known as a two-dimensional analogue of metamaterials, and they offer unprecedented control over the light propagation, reflection, and refraction [1, 2]. One of the main advantages of metasurfaces is that these structures are fully compatible with the modern planar fabrication technology and they can be readily integrated into the on-chip optical devices preserving the most of functionalities of three-dimensional metamaterials. Graphene, two-dimensional lattice of carbon atoms, exhibits a wide range of unique electronic and optical properties [3]. Graphene plasmonics [4] became a rapidly growing research field, both because plasmons in graphene exist in widely demanded THz field and because they can be effectively tuned with an external gate voltage. In [5] it has been shown that a wide range of metasurfaces can be constructed based on graphene sheets. In our work we show that nanostructuring the graphene sheet we can tailor the electronic properties directly and as consequence control the conductivity of these metasurfaces.

2. CONDUCTIVITY TENSOR

We study the system shown in Figure 1. We calculate the electronic band structure and eigenfunctions for the array of tunnel coupled armchair and zigzag nanoribbons. We show that the coupled edge states of zigzag nanoribbons form the additional electronic band characterizing by the hyperbolic Fermi surface and the band structure of coupled armchair ribbons is characterized by the overlap of electron and hole bands which results in the Fermi surfaces which simultaneously possess electron and hole pockets. Then, using the Kubo formula we calculate the AC conductivity tensors of these structures. In the the case of armchair nanoribbons (see Figure 1) the system can be described by an uniaxial conductivity tensor with principal components of different sign and low loss:

$$\widehat{\sigma}_0 = \begin{pmatrix} \sigma_\perp & 0 \\ 0 & \sigma_\parallel \end{pmatrix}. \tag{1}$$

Here $\sigma_\perp$ and $\sigma_\parallel$ are frequency dependent conductivities per unit length corresponding to the principal axes of the tensor.

Before analyzing dispersion equation let us discuss the form of conductivity tensor components $\sigma_\perp$ and $\sigma_\parallel$. Conductivity tensor describes a response of the layer on external electromagnetic field. In the common case, the response is not local in space and time that results in dependence of the conductivity on the frequency ω and wave vector k_z. Let us put forward the assumption that conductivity tensor components depend on the frequency according to the Drude-Lorentz model:

$$\sigma_s(\omega) = A\frac{ic}{4\pi}\frac{\omega}{\omega^2 - \Omega_s^2 + i\gamma\omega}, \quad s = \perp, \parallel. \tag{2}$$

Such dispersion shape is quite natural to many systems in optical, infrared, THz, and radio frequency ranges [6]. In Eq. (2) Ω_s are resonance frequencies along the principle axes of the conductivity tensor, γ is a relaxation constant which is supposed to be isotropic. Constant A has a

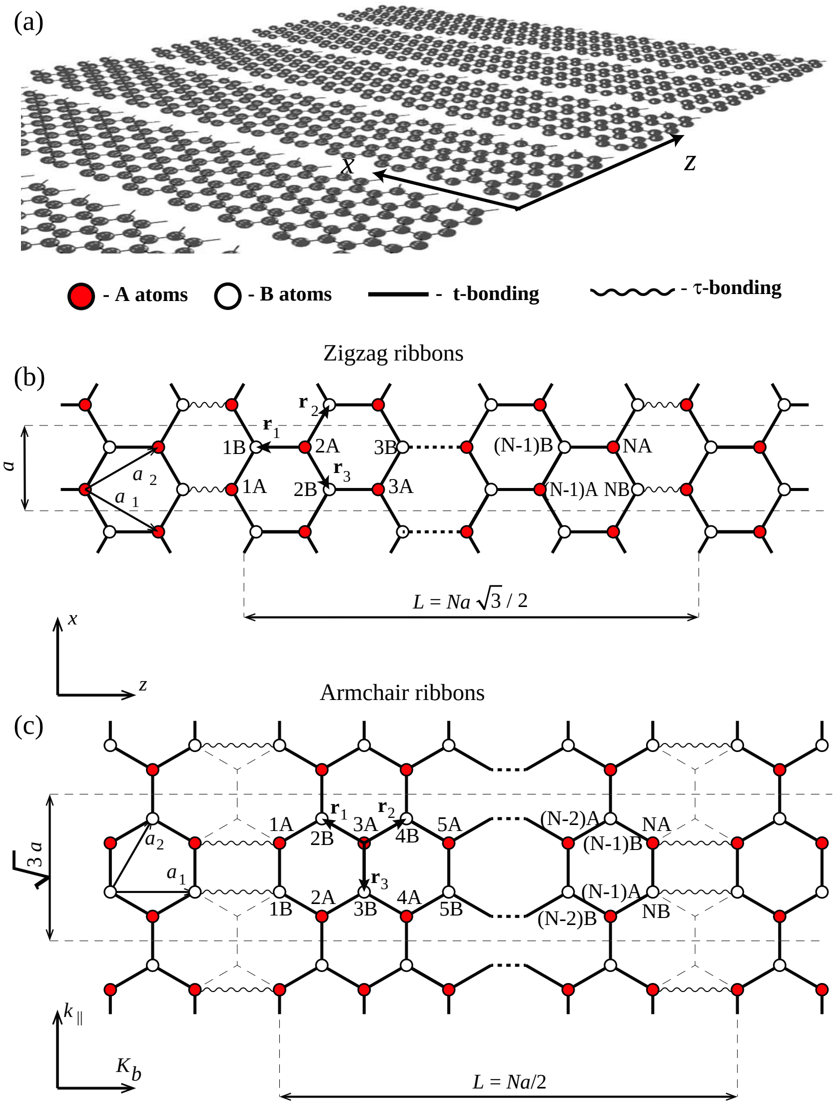

Figure 1: (a) Geometry of the problem. Surface wave propagates along z-direction. Atomic structure of the array of (b) zigzag nanoribbons (c) armchair nanoribbons.

dimension of rad/s. Explicit expression of A is defined by the metasurface design. This constant can be excluded from the analysis with the help of the following dimensionless variables:

$$\begin{gathered}\tilde{\sigma}_s = \frac{4\pi\sigma_s}{c}; \quad \tilde{\omega} = \frac{\omega}{A}; \quad \tilde{\gamma} = \frac{\gamma}{A}; \\ \tilde{\kappa} = \frac{c\kappa}{A\sqrt{\varepsilon}}; \quad \tilde{k}_z = \frac{ck_z}{A\sqrt{\varepsilon}}.\end{gathered} \tag{3}$$

Real part of $\tilde{\sigma}_\perp$ and $\tilde{\sigma}_\parallel$ is responsible for energy dissipation and imaginary part for the polarizability of the structure. Typical frequency dependence of $\mathrm{Im}(\tilde{\sigma}_\perp)$ and $\mathrm{Im}(\tilde{\sigma}_\parallel)$ is shown in Figure 2. One can see that signature of conductivity tensor (1) depends on the frequency. It is possible to distinguish three cases: (i) capacitive metasurface when both $\mathrm{Im}(\tilde{\sigma}_\perp)$ and $\mathrm{Im}(\tilde{\sigma}_\parallel)$ are negative; (ii) hyperbolic metasurface when $\mathrm{Im}(\tilde{\sigma}_\perp)\mathrm{Im}(\tilde{\sigma}_\parallel) < 0$; (iii) inductive metasurface when both $\mathrm{Im}(\tilde{\sigma}_\perp)$ and $\mathrm{Im}(\tilde{\sigma}_\parallel)$ are positive.

3. DISPERSION EQUATION

We consider two isotropic media with permittivities ε_1 and ε_2 separated by an anisotropic non-chiral metasurface or graphene sheet. In the figure we suggest one of the possible design of hyperbolic metasurface representing a two-dimensional array of asymmetric subwavelength resonators. Within

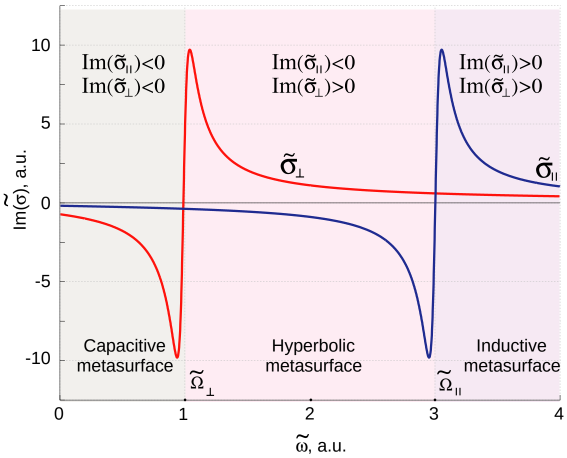

Figure 2: Frequency dependence of imaginary parts of dimensionless conductivity tensor components $\tilde{\sigma}_\perp$ and $\tilde{\sigma}_\parallel$. Parameters of conductivity tensor components are following: $\tilde{\Omega}_\perp = 1$, $\tilde{\Omega}_\parallel = 3$, $\tilde{\gamma} = 0.05$.

the local homogenization approach the electromagnetic properties of such structure can be described by a two dimensional conductivity tensor:

$$\widehat{\sigma}_0 = \begin{pmatrix} \sigma_\perp & 0 \\ 0 & \sigma_\parallel \end{pmatrix}. \tag{4}$$

Here $\sigma_\perp$ and $\sigma_\parallel$ are frequency dependent conductivities per unit length corresponding to the principal axes of the tensor.

We will seek solution of the Maxwell's equations in the form of a traveling wave propagating in the z-direction and localized in the y-direction. The both electric and magnetic fields depend on the z-coordinate and time t as $\exp(ik_z z - i\omega t)$. We assume that α is the angle between z-direction and one of principle axes of the tensor. Conductivity tensor is not diagonal in the chosen set of coordinates and can be written as

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{zx} & \sigma_{zz} \end{pmatrix}, \tag{5}$$

where

$$\sigma_{xx} = \sigma_\perp \cos^2\alpha + \sigma_\parallel \sin^2\alpha; \tag{6}$$

$$\sigma_{zz} = \sigma_\perp \sin^2\alpha + \sigma_\parallel \cos^2\alpha; \tag{7}$$

$$\sigma_{xz} = \sigma_{zy} = \frac{\sigma_\parallel - \sigma_\perp}{2} \sin 2\alpha. \tag{8}$$

Electric and magnetic fields ($\mathbf{E}$ and $\mathbf{H}$) obey the following boundary conditions on the metasurface:

$$[\mathbf{n}, \mathbf{H}_2] - [\mathbf{n}, \mathbf{H}_1] = \frac{4\pi}{c}\hat{\sigma}\mathbf{E}; \tag{9}$$

$$[\mathbf{n}, \mathbf{E}_2] - [\mathbf{n}, \mathbf{E}_1] = 0. \tag{10}$$

Index 1 (2) corresponds to the upper (lower) half-space, $\mathbf{n}$ is a unit vector normal to the interface.

Dispersion equation for surface waves can be obtained from Maxwell's equations using boundary conditions (9) and (10) and condition that electromagnetic field decays away from the interface

$$\left(\frac{c\kappa_1}{\omega} + \frac{c\kappa_2}{\omega} - \frac{4\pi i}{c}\sigma_{xx}\right)\left(\frac{\omega\varepsilon_1}{c\kappa_1} + \frac{\omega\varepsilon_2}{c\kappa_2} + \frac{4\pi i}{c}\sigma_{zz}\right) = \frac{16\pi^2}{c^2}\sigma_{xz}^2. \tag{11}$$

Here $\kappa_{1,2}^2 = k_z^2 - \varepsilon_{1,2}\omega^2/c^2$ is inversed penetration depth of the surface wave into upper and lower medium. Similar equation describes dispersion of magnetoplasmons, surface waves in a two-dimensional electron gas in the presence of a strong DC magnetic field [7, 8].

4. RESULTS AND DISCUSSION

In order to analyze dispersion of surface waves which is described by Eq. (11) let us neglect dissipation of the energy in the system and put $\tilde{\gamma} = 0$. For the sake of simplicity, further we will consider symmetric situation when $\varepsilon_1 = \varepsilon_2 = \varepsilon$. As an example, let us consider the structure with resonance frequencies of conductivity tensor components $\tilde{\Omega}_\perp = 1$ and $\tilde{\Omega}_\parallel = 3$. Dependence of wave vector k_z on frequency ω and propagation direction α can be obtained analytically from Eq. (11). This equation yields two solutions which correspond to hybrid polarized waves (quasi-TE and quasi-TM plasmons). Their dispersion for $\alpha = 60°$ is shown in Figure 3.

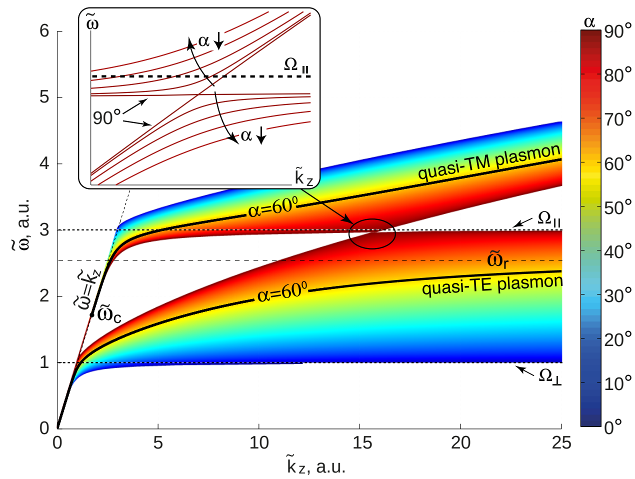

Figure 3: Dependence of $\tilde{k}_z$ on $\tilde{\omega}$ for the surface waves on hyperbolic metasurface for different propagation directions α. Two branches correspond to quasi-TM and quasi-TE surface plasmons. The inset shows the structure of dispersion curves at $\alpha \approx 90°$.

Surface waves of pure TE or TM polarization can propagate only along the principle axes of the conductivity tensor ($\alpha = 0°$ and $\alpha = 90°$). In this case, right part of Eq. (11) is equal to zero and the equation splits into two independent equations corresponding to two-dimensional TE and TM plasmons.

Frequency cutoff of quasi-TE plasmon is equal to zero and does not depend on α. Frequency cutoff ω_c of quasi-TM plasmon belongs to the interval $\Omega_\perp \leqslant \omega_c \leqslant \Omega_\parallel$ and depends on α. This dependence can be found from the equation:

$$\cot^2 \alpha = -\frac{\sigma_\parallel(\omega_c)}{\sigma_\perp(\omega_c)}. \tag{12}$$

Quasi TE-plasmon has a maximal frequency ω_r at which it can propagate. Dependence of ω_r on the α yields by the equation:

$$\tan^2 \alpha = -\frac{\sigma_\parallel(\omega_r)}{\sigma_\perp(\omega_r)}. \tag{13}$$

It follows from Eqs. (12) and (13) that simultaneous propagation of both surface waves at the same frequency is possible only if

$$\frac{\pi}{4} \leqslant |\alpha| \leqslant \frac{3\pi}{4}. \tag{14}$$

This condition does not depend on the specific dispersion of $\tilde{\sigma}_{\perp,\parallel}$ and can be fulfiled for any hyperbolic metasurface.

It was shown that an anisotropic interface separating a hyperbolic metamaterial and vacuum can support a certain class of plasmonic modes analogous to the Dyakonov surface states [9, 10]. Dyakonov surface states [11] are localized modes which can propagate in a narrow angle range along the interface of the anisotropic crystals. Despite the theoretical prediction back in the 1980s, these modes have been experimentally demonstrated only recently [12]. This is due to the fact that for the case of an interface of conventional anisotropic crystal these modes can propagate only in an extremely narrow range of angles, and thus it is hard to excite them experimentally. Nevertheless, these modes attract a significant scientific interest since they suggest a route for virtually lossless optical information transfer at the nanoscale, which is extremely important for the perspectives of on-chip optical data processing devices.

5. CONCLUSION

We have presented a comprehensive analysis of surface waves propagating along hyperbolic graphene surface. We have analysed dispersion of such waves in the most general form, not specifying a specific design of the metasurface and describing its properties using the effective conductivity approach. Within this approach, the problem does not acquire a specific scale and, therefore, the results can be applied to different frequencies ranging from microwaves to ultraviolet band.

We have shown that the spectrum of waves supported by hyperbolic metasurfaces consists of two branches of hybrid TE-TM polarized modes, that can be classified as quasi-TE and quasi-TM plasmons. Dispersion properties of these waves are strongly anisotropic, and they have some similar features with Dyakonov surface states, magnetoplasmons and two-dimensional TE and TM plasmons. Directivity of these plane waves can be controlled with the external voltage which can be used for the creation of graphene-based photonic integrated circuits.

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