A Comparison of SAR Imaging Performance between Matching Filter and Compressed Sensing

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Abstract— This paper is devoted to propose some new efficient indicators in comparison of synthetic aperture radar (SAR) imagine performance between matching filter (MF) and compressed sensing (CS). Generally, SAR utilizes matched filtering (MF) technique to improve resolution, whereas CS is a newly developed technology for sparse or compressible signal which can be applied to SAR imaging to achieve super-resolution images. Based on the theory derivation, the paper compares CS with MF in several aspects, such as resolution and SNR, several new indicators were also given to evaluate imaging performance because of new features caused by CS processing, such as microscopic and macroscopic resolution and group spread function (GSF). The comparison results are discussed and verified by data simulation.

1. INTRODUCTION

SAR utilizes the relative movement between target and radar to synthesize a large virtual aperture to improve azimuth resolution, and adopts MF technique to improve range resolution [1]. In 2006, Donoho proposed compressed sensing (CS) theory [2], which takes full advantage of the sparsity or compressibility of signal to reconstruct original signal while the sampling rate can be reduced considerably. In recent years, some scholars have discussed CS in SAR imaging and published some related articles [3], and it is proved that CS has the ability to achieve super resolution and decrease sampling rate.

2. THEORETICAL ANALYSIS

2.1. CS Theory

CS theory shows that when the signal is sparse in some transform domain, it can be recovered from a small amount of measurements with high probability by solving an optimization problem. A signal $\mathbf{x}$ with a length of $\mathbf{N}$ can be sparsely represented in the basis $\psi = \mathbf{C}^{M\times N}$ as $\mathbf{x} = \mathbf{\Psi s}$. After $\mathbf{x}$ is projected onto a measurement matrix $\mathbf{\Phi} \in \mathbf{C}^{M\times N}$ with $M < N$, the measured values can be expressed as $\mathbf{y} = \mathbf{\Phi x}$. The signal can be recovered by solving a convex optimization problem as

$$\min \ \|\mathbf{s}\|_1, \text{ subject to } \mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi \Psi s} = \mathbf{\Theta s} \tag{1}$$

Candes and Tao pointed out that in order to ensure convergence of the algorithm, matrix $\mathbf{\Theta}$ in (1) must satisfy the restricted isometry property (RIP) [4]. If a matrix $\mathbf{\Theta}$ satisfies (2) for all vectors $\mathbf{v}$ with at most k non-zero elements, the matrix $\mathbf{\Theta}$ has RIP

$$(1-\delta_K)\|\mathbf{v}\|_2^2 \leq \|\mathbf{\Theta v}\|_2^2 \leq (1+\delta_K)\|\mathbf{v}\|_2^2 \tag{2}$$

where $\delta_K \in (0,1)$ is the smallest quantity for matrix $\mathbf{\Theta}$ to satisfy (2).

2.2. SAR Imaging Based on CS Theory

According to CS theory, sparse dictionary and recovery algorithm play crucial role in signal reconstruction. At high frequencies, target can be modeled as the sum of echoes from strong scattering centers. Assuming the target contains$N$strong scattering centers and radar transmits chirp signal, the target echo can be expressed as

$$s_r(t) = \sum_{n=1}^{N} rect\left(\frac{t-\tau_n}{T}\right) e^{j\pi K(t-\tau_n)^2} \tag{3}$$

where T is the pulse width, K is the chirp rate, τ_n is time delay caused by the round-trip distance between radar and each scattering center, and c is the speed of light.

According to above analysis, sparse dictionary $\mathbf{\Theta}$ can be constructed by the set of echoes from target at various ranges. The construction steps are as follows: [5]

1) Assuming that $[R_0, R_0 + \Delta R]$ is the distance range of the target. Divide the distance range at intervals of Δr as $Range = \{R_0, R_0 + \Delta r, R_0 + 2\Delta r, \cdots, R_0 + \lfloor \Delta R/\Delta r \rfloor \times \Delta r\}$.

2) The echo from target at different range is treated as the column vector $\mathbf{\Psi_n}$ of basis matrix. Let f_s be sampling rate, I be the number of samples. After discretization, the element is $\Psi_{i,n} = rect(\frac{i/f_s - 2(R_0+(n-1)\Delta r)/c}{T}) \exp\{j\pi K(i/f_s - 2(R_0 + (n-1)\Delta r)/c)^2\}$.

3) Let $\mathbf{\Theta_n}$ be the column vector of $\mathbf{\Theta}$, and it can be obtained by extracting data from $\mathbf{\Psi_n}$ at intervals of d, that means $\Theta_{m,n} = \Psi_{1+(m-1)d,n}$, $m = 1, 2, \cdots, \lfloor I/d \rfloor$. Each column vector $\mathbf{\Theta_n}$ is called an atom of sparse dictionary.

3. SIMULATION AND DISCUSSIONS

Experiments are carried out and simulation results are presented to illustrate imaging performance based on MF and CS theory.

3.1. Microscopic and Macroscopic Resolution

To prove microcosmic resolution and the effect of rank of $\mathbf{\Theta}$ on CS reconstruction, we test with 1-D both range echoes and azimuth echoes from 10 scattering centers which have uniform amplitude and are regularly spaced in the range [3000, 3020]. By changing the value of Δr, the rank of $\mathbf{\Theta}$ varies from less than the number of columns to greater than it.

Table 1: Simulation parameters.

Parameters	Values
Band Width B (MHz)	60
Wavelength λ (m)	0.032
Slant Range $R0$ (km)	30
Sampling Rate/fs (MHz)	150
Pulse Width T (μs)	10
Radar Velocity v (m/s)	250
Pulse Repeat Frequency (PRF)/fa (Hz)	600

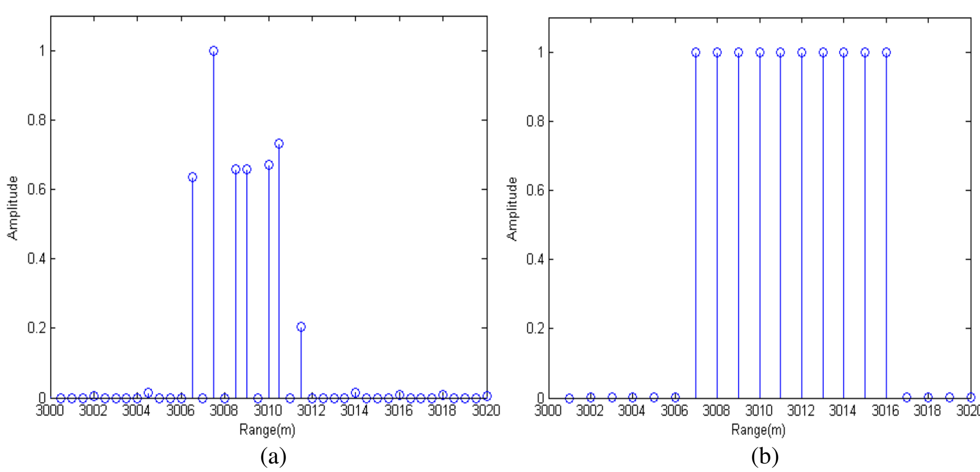

Figure 1: Reconstruction results of 1-D range signal. (a) $\Delta r < c/2f_s$. (b) $\Delta r \geq c/2f_s$.

From Fig. 1(a) and Fig. 2(b), when interval Δr is less than microcosmic resolution, or distance range R is bigger than macroscopic resolution, the signal could not be recovered correctly by utilizing CS theory as signal amplitude is not uniform in Fig. 1(a) and some signals appear in unexpected positions in Fig. 2(b) because there are some columns in sparse dictionary whose coherence is large. However, when $\Delta r \geq c/2f_s$ and $\Delta R < 2\pi V f_a/d\,|K_a|$, CS method can recover signal perfectly. Furthermore, the distance between two scattering centers in Fig. 1(b) is $c/2f_s = 1\,\text{m}$, and the range resolution after MF should be $c/2B = 2.5\,\text{m}$, which proves that the resolution can be greatly improved by using CS reconstruction.

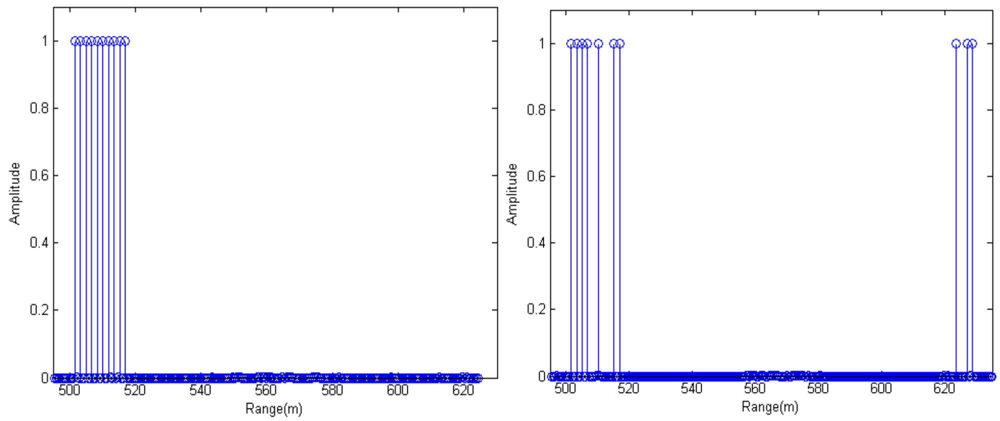

Figure 2: Reconstruction results of 1-D azimuth signal. (a) $\Delta R < 2\pi V f_a/d\,|K_a|$. (b) $\Delta R \geq 2\pi V f_a/d\,|K_a|$.

3.2. GSF Comparison

To compare GSF between MF and CS, we test with 1-D range echoes from 20 scattering centers which has uniform amplitude and are regularly spaced within a microscopic resolution cell ($3000 \sim 3002.5$). The radar parameters are given in Table 2 and simulation result is given in Fig. 3. As shown, the results after MF and CS processing are almost the same.

Table 2: Simulation parameters.

Parameters	Band Width B (MHz)	Sampling Rate fs (MHz)	Pulse Width T (us)	Interval Δr (m)
Values	60	60	10	2.5

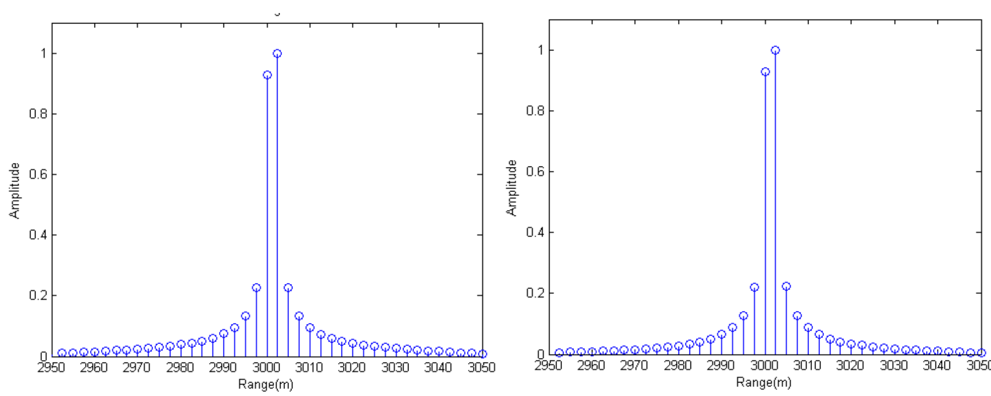

Figure 3: GSF comparison result.

3.3. SNR Comparison

To prove $SNR_{out} : SNR_{in} = M : N$, we changed input SNR and the ratio in sparse dictionary to achieve the statistical result on SNR. The results are as follows.

According to Table 4, Table 5 and Table 6, the ratio of output SNR to input SNR is around M/N, which can prove the validity of equation $SNR_{out} = \frac{P_{Sout}}{P_{Nout}} = \frac{P_{Sin}}{P_{Nin}} \cdot \frac{M}{N} = SNR_{in} \cdot \frac{M}{N}$. According to the comparison processing shown in Fig. 4, it is obvious that output SNR after MF is higher than that after CS processing when input SNR is given, which means MF has better anti-noise performance when compared to CS method.

Table 3: Simulation parameters.

Parameters	Values
Wavelength λ (m)	0.0566
Altitude H (km)	500
Slant Range R (km)	550
Antenna Length L (m)	10
Band Width B_r (MH_z)	20
Sampling Rate f_s (MH_z)	24
Pulse Width T_r (μs)	40
Satellite Velocity v (m/s)	5000
Pulse Repeat Frequency (PRF) f_a (Hz)	1000

Table 4: $M : N = 4$.

SNRin	SNRout	F
9.71E+02	3.86E+03	3.98
9.80E+01	3.88E+02	3.96
9.72E+00	3.89E+01	4.00
1.00E+00	3.79E+00	3.79
9.98E-02	4.40E-01	4.41
1.00E-02	4.04E-02	4.04
1.00E-03	4.00E-03	4.00

Table 5: $M : N = 10$.

SNRin	SNRout	F
1.02E+03	1.05E+04	10.28
9.90E+01	1.02E+03	10.32
9.95E+00	1.02E+02	10.21
1.02E+00	1.01E+01	9.90
1.00E-01	1.04E+00	10.40
1.00E-02	1.00E-01	10.00
9.74E-04	9.80E-03	10.06

Table 6: $M : N = 16$.

SNRin	SNRout	F
9.83E+02	1.58E+04	16.11
1.01E+02	1.49E+03	14.72
9.90E+00	1.58E+02	15.96
1.00E+00	1.60E+01	16.00
1.00E-01	1.56E+00	15.60
1.00E-02	1.58E-01	15.80
9.90E-04	1.65E-02	16.67

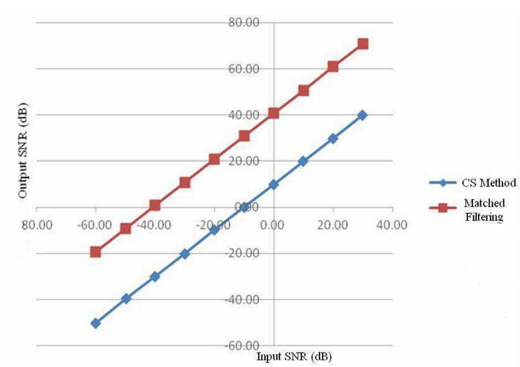

Figure 4: SNR comparison result.

4. CONCLUSIONS

According to theoretical analysis and simulation results, the resolution can be improved considerably by using CS when the sampling rate is greater than corresponding bandwidth, but ambiguity phenomenon will appear as the distance increases where does not occur when using MF. Besides macroscopic and microscopic resolution, GSF is also an important indicator to measure resolution, and it is proved that two methods' GSF are almost the same when sampling rate is close to bandwidth. In addition, output SNR for CS imaging varies with the number of rows and columns in sparse dictionary when input SNR is given, and its anti-noise ability is weaker than that when using MF which is designed to produce the maximum achievable SNR.

Each method has its own strength and weakness, and CS could not replace MF even if it has so many advantages because CS still has some characteristics not good as that of MF, such as SNR. So we must select appropriate imaging method based on specific situation.

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