Separating the Field Radiated by Two Rectilinear Sources

A. Natale, **M. A. Maisto**, **R. Solimene**, **G. Leone**, **and R. Pierri**
Dipartimento di Ingegneria Industriale e dell'Informazione, Seconda Universita' di Napoli
via Roma 29, Aversa 81031, Italy

Abstract— Consider two electric current sources radiating in free-space. They are assumed to be supported over two parallel strips whose positions are a priori known. Their radiated fields are collected over a single domain located in far-zone (with respect the whole radiating system). A two-dimensional scalar framework is set. The problem of separating the contributions of the two sources from their radiated field over a single domain is considered. A procedure based on two step is presented: in the first step the sources are reconstructed and in the second the field is computed by this currents. In order to reconstruct the sources, three different schemes of inversion are presented. Numerical examples are provided in order to compare the different strategies from the achievable performance and numerical complexity point of views.

1. INTRODUCTION

When dealing with electromagnetic field measurements for antenna diagnostics or even in inverse scattering problems one collects the radiated field (resp. the scattered field) over a domain and from such measurements the features of the antenna (resp. scatterer) under test have to be inferred. It often happens that beside the field due to the structure under test (SUT), the fields produced by further sources (generally unwanted) are collected as well. This give rises to clutter signals from which the useful field signals (i.e., those one due to the SUT) must be singled out. In this contribution, a related problem for a two-dimensional scalar configuration is tackled. More in detail, two strip electric currents are considered radiating in free-space. They are assumed to be supported over two parallel strips whose positions are a priori known. Their radiated fields are collected over a single domain located in far-zone (with respect the whole radiating system). The problem of concern herein is to distinguish between the contributions of two radiating sources. To this end, the problem is addressed by a two-step procedure. First, the currents are reconstructed by the radiated field measurements. This entails dealing with an inverse source problem that is tackled by comparing three different strategies of inversion. The first strategy consists in inverting the radiating operator, which rigorously describe the radiation by both the sources. In the second one, the radiated field data is inverted by considering each source as being radiating alone. Finally, the third method is a variant of the second one. Indeed, after having reconstructed one of the two sources the corresponding radiated field is synthetically computed and subtracted from the data. Then the second source is reconstructed. Same steps are applied to other source. The second step of the overall procedure then consists of computing the radiated field due to each source once they have been estimated according to the previous step. Numerical examples are provided in order to compare the different strategies from the achievable performance and numerical complexity point of views.

2. MATHEMATICAL FORMULATION

Consider two electric strip currents J_1 and J_2 directed along the $\hat{y}$-axis and supported over the segments of the $\hat{x}$-axis $\Sigma_1 = [-X_1, X_1]$ and $\Sigma_2 = [-X_2, X_2]$ located at $z_1 = 0$ and $z_2 = d$, respectively. Their radiated fields are collected over a curvilinear domain located in the far-zone of the whole radiating system. The observation is indicated by the observation angle θ which spans within the observation angular sector $\Theta = [\theta_{\min}, \theta_{\max}]$.

The field radiated (which has only the y component) by each strip source enjoys the well known far-zone integral form

$$E_i(u) = \exp\left(jwz_i\right) \int_{-X_i}^{X_i} \exp\left(jux\right) J_i(x) dx \tag{1}$$

where $i \in (1, 2)$, $u = k \sin\theta$, $w = k \cos\theta$, k being the free-space wavenumber. Now the field is regarded as a function of the spectral variable u which depends on Θ ranging within the interval $U = [u_{\min}, u_{\max}]$ for both the sources.

It is convenient to cast Eq. (1) in operator notation so to introduce the following radiation operators

$$\mathcal{A}_i : J_i \in L^2(\Sigma_i) \longrightarrow E_i \in L^2(U) \tag{2}$$

with $L^2(\Sigma_i)$ and $L^2(U)$ being the complex valued function which are square integrable in Σ_i and U.

As the sources radiate simultaneously, the field actually collected is the sum of both contributions

$$E(u) = E_1(u) + E_2(u) = \mathcal{A}_1 J_1 + \mathcal{A}_2 J_2 \tag{3}$$

Accordingly, we need to look for an operator which link the couple (J_1, J_2) to E. To this end, let us introduce the functional set

$$\Sigma = \{\underline{f} = (f_1, f_2) : f_1 \in L^2(\Sigma_1), f_2 \in L^2(\Sigma_2)\} \tag{4}$$

equipped with the scalar product

$$< \underline{f}, \underline{g} >_\Sigma = < f_1, g_1 >_{\Sigma_1} + < f_2, g_2 >_{\Sigma_2} \tag{5}$$

with $< \cdot, \cdot >_{\Sigma_1}$ and $< \cdot, \cdot >_{\Sigma_2}$ being the usual scalar products in $L^2(\Sigma_1)$ and $L^2(\Sigma_2)$, respectively. Now, the radiating operator can be compactly represented as

$$\underline{\mathcal{A}} : \underline{J} \in \Sigma \longrightarrow E \in U \tag{6}$$

with $\underline{\mathcal{A}}$ defined as

$$\underline{\mathcal{A}} = [\ \mathcal{A}_1 \quad \mathcal{A}_2\ ] \tag{7}$$

so that (3) can be rewritten as

$$E(u) = \underline{\mathcal{A}} \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} \tag{8}$$

3. SOURCES RECONSTRUCTION

In order to reconstruct the two sources by the radiated field measurements, the problem can be addressed following three different strategies of inversion. In the first one, the source are reconstructed by inverting the operator $\underline{\mathcal{A}}$. Let $\{v_n, \sigma_n, \underline{u}_n\}^\infty$ be the singular system of $\underline{\mathcal{A}}$. In particular, the v_ns and the $\underline{u}_n$s are the singular functions expanding the radiated field and the unknown, respectively, whereas the σ_ns are the singular values. Accordingly, the TSVD reconstruction is given by:

$$\underline{\tilde{J}} = \sum_{n=1}^{N_t} \frac{< E_s, v_n >_U}{\sigma_n} \underline{u}_n(x) \tag{9}$$

with $<,>$ denoting the scalar product in the data space and N_t is the truncation index. Let $\{v_{1n}, \sigma_{1n}, u_{1n}\}^\infty$ and $\{v_{2n}, \sigma_{2n}, u_{2n}\}^\infty$ be the singular systems of $\mathcal{A}_1$ and $\mathcal{A}_2$, respectively. Note that as $\mathcal{A}_1$ and $\mathcal{A}_2$ are related to compactly supporting Fourier Transforms, their SVD is known in closed form. Indeed, it is given in terms of the prolate spheroidal wave-functions (PSWF) [1]. This suggests the second inversion strategy, where the radiated field data is inverted by considering each source as being radiating alone. This way allows to obtain an advantage in terms of computational complexity respect to previous inversion scheme. By exploiting the apriori information about two source supports (that are two parallel strip spatially disjoint), the reconstructions can be expressed as:

$$\tilde{J}_1 = \sum_{n=1}^{N_{t1}} \frac{< E_s, v_{1n} >_U}{\sigma_{1n}} u_{1n}(x) \tag{10}$$

$$\tilde{J}_2 = \sum_{n=1}^{N_{t2}} \frac{< E_s, v_{2n} >_U}{\sigma_{2n}} u_{2n}(x) \tag{11}$$

The step-like behaviour of the singular values σ_{1n} and σ_{2n} suggests to choose the truncation indexes equal to $\frac{X_1[u_{\max} - u_{\min}]}{\pi}$ and $\frac{X_2[u_{\max} - u_{\min}]}{\pi}$ [1]. The third method is a variant of the second one. Indeed, after having reconstructed one of the two sources the corresponding radiated field is synthetically computed and subtracted from the data. Then the second source is reconstructed. Applying these steps to both the sources, their contributions on the field are compensated and this impacts positively the reconstructions.

The steps realizing the procedure are the following:

- $\tilde{J}_2$ is estimated by (9). From $\tilde{J}_2$, its radiated field is obtained by:
$$\tilde{E}_2 = \mathcal{A}_2 \tilde{J}_2$$
- $\tilde{E}_2$ is subtracted to the total field:
$$\tilde{E}_1 = \tilde{E}_s - \tilde{E}_2;$$
- Finally, the current defined on the other support is reconstructed by its approximated radiated field:
$$\tilde{J}_1 = \sum_{n=1}^{N_t} \frac{< \tilde{E}_1, v_{1n} >_U}{\sigma_{1n}} u_{1n}(x)$$

Same steps are applied for the other source.

4. NUMERICAL RESULTS

A numerical analysis is performed in order to compare the different procedure of inversion. Consider that the sources to be reconstructed are:
$$J_1 = \cos\left(\frac{\pi}{2X_1}x\right) \text{ with } x \in [-X_1, X_1],$$
$$J_2 = \cos\left(\frac{\pi}{2X_2}x\right) \text{ with } x \in [-X_2, X_2].$$
In Fig. 2, the reconstructions of the two sources obtained by the three inversion procedures are represented by varying of the distance d and of the truncation index N_t. In particular, the step-like behaviour of the singular values Fig. 1 suggests to choose the truncation index in correspondence of the knees: for the figures on left column the second knee is chosen and for the figures on right column the first. The singular values behaviour of $\underline{\mathcal{A}}$ exhibits a two step behaviour because of the overlapping of two sources contributions on the field in a spatial frequency band whose width depending by d ([2]). So each singular value in first step is associated both to one of $\mathcal{A}_1$ and $\mathcal{A}_2$, accordingly to the no-orthogonality of their singular functions. The number of the singular values in the first step depends on the distance between the two sources, indeed as it increases, the band on which v_{1n} and v_{2n} are no orthogonal decreases. As consequence, the singular values behaviour of $\underline{\mathcal{A}}$ approaches to single step.

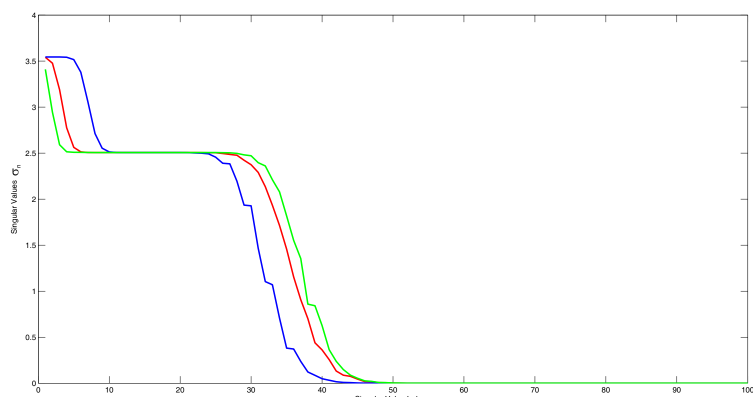

Figure 1: Singular values of the operator $\underline{\mathcal{A}}$ for $X_1 = 6\lambda$, $X_2 = 3\lambda$, $u_{\min} = -k$ and $u_{\max} = k$. The blue line refers to a distance $d = 10\lambda$, the red line to $d = 30\lambda$ and the green line to $d = 80\lambda$.

As can be seen, the performance of the third method allows to obtain better reconstruction starting from a distance $d = 80\lambda$, otherwise inverting $\underline{\mathcal{A}}$ by its eigen-spectrum gives better results only at larger d. The results obtained by the second method are the worst. Furthermore it is evident that the reconstruction of the wider source is better than the other because more singular functions associated only to wider source can be retained in 9. On the right column of Fig. 2, the role played by the truncation index is analysed. Truncating at the first knee heavily impacts the reconstruction of the wider source obtained by the first method because the singular functions associated only to this are filtered. In this case the results obtained by the third method are the best.

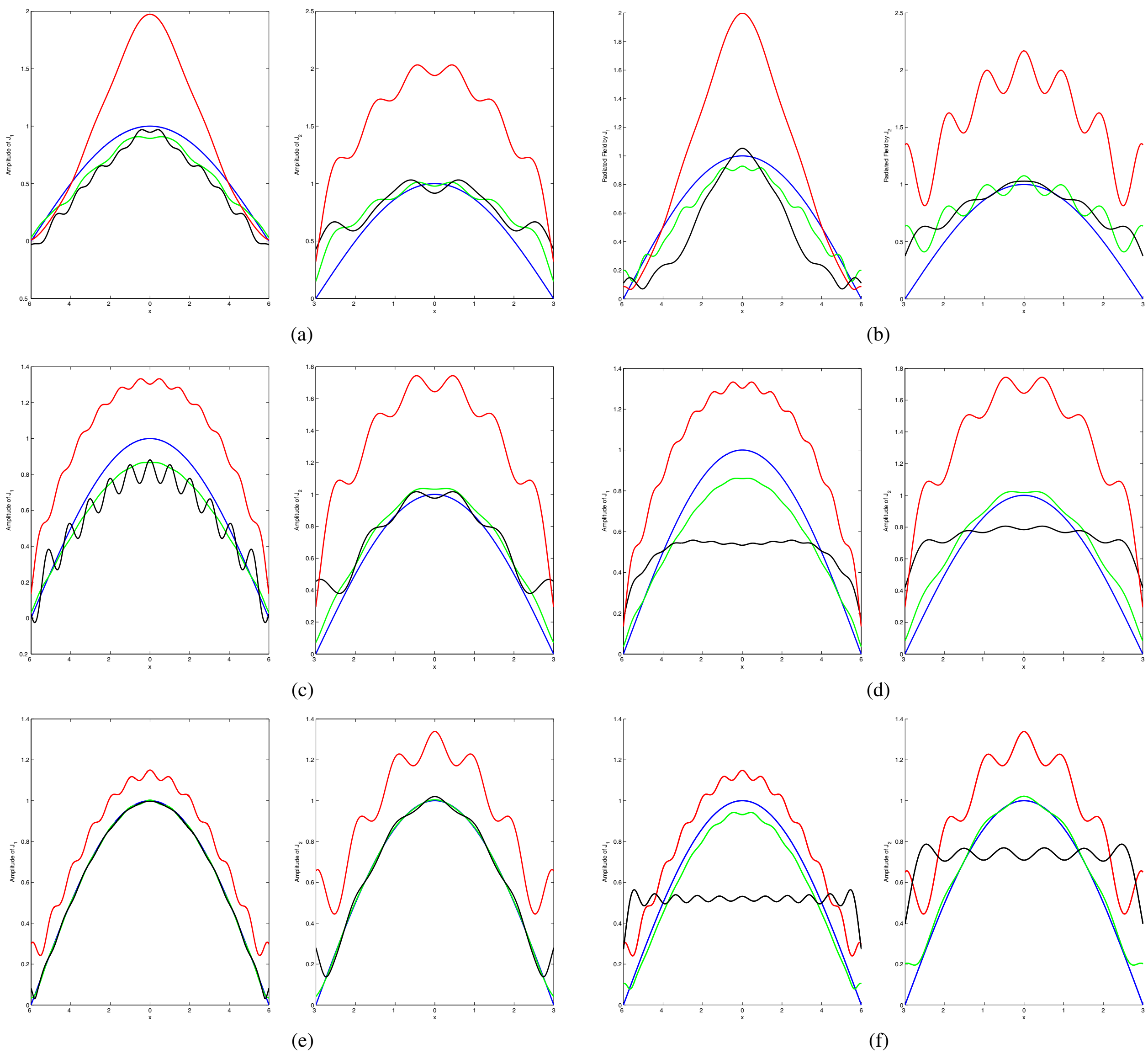

Figure 2: Reconstructions of two sources achieved by the three different methods for $X_1 = 6\lambda$, $X_2 = 3\lambda$, $u_{\min} = -k$ and $u_{\max} = k$. In particular the blue line refers to the sources to be determined, the black line to the reconstruction obtained by inverting $\underline{\mathcal{A}}$ by its eigen-spectrum, the red line to the reconstruction obtained by inverting the field as if each source radiates alone and the green line through the third method. On the left columns, the reconstructions are shown at varying the distance d between the source in $\{10\lambda, 80\lambda, 500\lambda\}$, respectively and the truncation index is chosen in correspondence of the second knee. On the right column same reconstructions are shown by choosing the truncation index in correspondence of the first knee.

5. FIELD SEPARATION

The second step of the overall procedure then consists of computing the radiated field due to each source once they have been estimated according to the previous section by the radiation operator.

$$E_1(u) = \mathcal{A}_1 J_{1r} \tag{12}$$

$$E_2(u) = \mathcal{A}_2 J_{2r} \tag{13}$$

where J_{1r} and J_{2r} are the sources reconstructed exploiting the seen method. In Fig. 3 the fields radiated by the sources of Fig. 2 are shown. As can be seen, the estimate of the field achieved by the second method does not approximate well the field at low frequency. The total field in 10 and 11 is spanned by a combination of two functions $\tilde{v}_{1n}$ and $\tilde{v}_{2n}$ linked to the singular functions of $\mathcal{A}_1$ and $\mathcal{A}_2$ that are not normalized for n belonging to the first step of σ_n. As consequence, the amplitude of the estimated field by the second method is doubled compare to the others for low

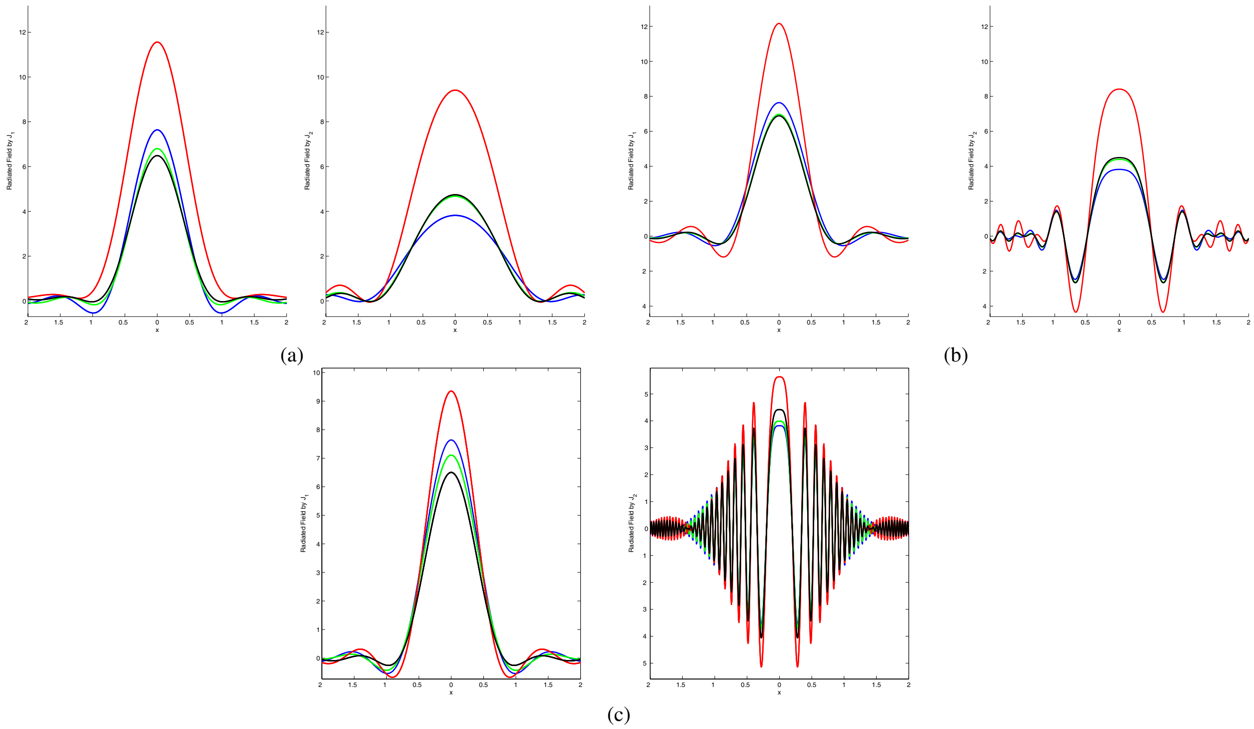

Figure 3: The radiated field due to sources shown on left column in Figure 2.

frequency because, being unitary the norm of v_{1n}, it is double compare to the norm of $\tilde{v}_{1n}$ and $\tilde{v}_{2n}$ for n belonging to the first step of σ_n.

6. CONCLUSION

The problem of separating the contributions of the two sources from their radiated field over a single domain has been tackled by introducing a based two step procedure. Three inversion strategies of the radiating operator realizing the first step are proposed. Their comparison has highlighted the better performance achievable by the third method by varying of the distance between the two sources and of the truncation index N_t.

ACKNOWLEDGMENT

This work was supported in part by POR Campania FSE 2007/2013-Embedded System.

REFERENCES

1. Slepian, D. and H. O. Pollak, "Prolate spheroidal wave function, Fourier analysis and uncertainty. I," *Bell Syst. Tech. J.*, Vol. 40, 43–63, 1961.
2. Solimene, R. and R. Pierri, "Number of degrees of freedom of the radiated field over multiple bounded domains," *Optics Letters*, Vol. 32, No. 21, 3113–3115, 2007.