Enhanced Group Velocity Characteristics of a ENG Cladded Metamaterial Loaded Helical Guide

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Abstract— In this paper, we propose an ENG Cladded Metamaterial Loaded Helical (CMLHG) waveguide structure, to achieve enhance slow wave in THz frequency spectrum. Both analytical as well as numerical computation has been done to explore the dispersion behavior of the structure. It is observed that the waveguide supports propagation of forward wave (FW) mode having parallel phase and group velocities over a wide bandwidth with an enhanced slow wave characteristic. A comparative study of slow wave behavior has been done between the ENG CMLHG waveguide and two other cases (i) when helix is in free space and (ii) when helix is cladded with DPS material and loaded with metamaterial. It is found that present structure supports slower wave as compared to those structures.

1. INTRODUCTION

There is a tremendous interest in slow down the velocity of wave. Dynamically controlled wave flow is vital for developing photonics devices. Specifically, the feature of slowing light is useful in order to regulate the data traffic at network nodes. Device like optical buffer is based on the mechanism of slow wave.

For achieving slow wave different methods are proposed such as coherent population oscillations, stimulated Brillouin scattering, quantum dot semiconductor optical amplifier etc.. Those are found unsuitable in incorporating on an optical integrated chip (IC). For rectifying this disability other methods are reported which uses dispersive behaviour of metamaterials and finds easy to accommodate on optical ICs.

Metamaterials are newly discovered composite material having negative permittivity and permeability value. It exhibits unusual characteristics such as negative refraction, backward wave, negative Goos-Hansen effect etc.. This artificial material was first investigated by Russian physicist Veslogas [1] and named this bizarre material to left-handed material. Due to presence of left-handed traied in between electric field (E), magnetic field (H) and propagation vector (k). These striking predictions were not widely accepted until their experimental verification 30 years later by Smith et al. [2]. They designed metamaterial through split ring resonators (SRR) and thin wire array.

Different research groups studied various waveguide structures which were loaded with metamaterial or itself made up of it. Metamaterial slab waveguide were studied by Bait et al. [3] and they observed that TM and TE mode propagate with slower phase velocity. Gennaro et al. [4] studied metamaterial experimentally and they found that group velocity is reduced in metamaterial medium. Savo et al. [5] studied slow wave experimentally at microwave frequency range through a planar waveguide consisting of a dielectric core cladded by single-negative metamaterial. Erfaninia et al. [6] studied multilayered metamaterial waveguide and reported reduction in group velocity as compared to conventional structures. In our recent work [7], we showed that wave velocity is greatly reduced in metamaterial loaded helical guide (MLHG) as compared to conventional cases (helix loaded with free-space and dielectric or DPS material). Helical guide is a well known slow wave structure which is widely used in travelling wave tube and travelling wave antenna etc..

In this work, we propose ENG CMLHG structure which consists of a helical guide having metamaterial core and ENG (epsilon material) cladding. For that structure, dispersion relation is derived analytically and computed numerically.

2. DISPERSION EQUATION

The structure of interest is shown in Fig. 1, here region-I (core) consists of metamaterial (permittivity and permeability ϵ_1 and μ_1 respectively) and region-II (cladding) is of ENG material (permittivity and permeability ϵ_2 and μ_2 respectively). Region outside to the cladding layer is region-III, free-space medium (permittivity ϵ_3 and μ_3 respectively).

The electric field components in different regions are:

$$E_{z,1} = -k_1^2 A I_n(k_1\rho)\, e^{jn\phi} e^{-j\beta z} \tag{1}$$

$$E_{\phi,1} = \left[\frac{n\beta}{\rho} A I_n(k_1\rho) + i\omega\mu_1 B I_n'(k_1\rho)\right] e^{jn\phi} e^{-j\beta z} \tag{2}$$

$$E_{z,2} = -k_2^2 \left[C I_n(k_2\rho) + D K_n(k_2\rho)\right] e^{jn\phi} \tag{3}$$

$$E_{\phi,2} = \left[\frac{n\beta}{\rho}\left[C I_n(k_2\rho) + D K_n(k_2\rho)\right] + i\omega\mu_2 \left[E I_n'(k_2\rho) + F K_n'(k_2\rho)\right]\right] e^{jn\phi} e^{-j\beta z} \tag{4}$$

$$E_{z,3} = -k_3^2 G I_n(k_3\rho)\, e^{jn\phi} e^{-j\beta z} \tag{5}$$

$$E_{\phi,1} = \left[\frac{n\beta}{\rho} G I_n(k_3\rho) + i\omega\mu_3 H I_n'(k_3\rho)\right] e^{jn\phi} e^{-j\beta z} \tag{6}$$

Here k_1, k_2 and k_3 are transverse wave vectors of different regions I, II and III respectively. For different regions I, II and III the i respectively is 1, 2 and 3. β is longitudinal propagation coefficient and related with propagation vector (k_{oi}) by wave equation $k_i = \sqrt{\beta^2 - k_{oi}^2}$. $I_n(k_i\rho)$ and $K_n(k_i\rho)$ are modified Bessel functions of first and second kind. Its derivative represent by (‘). ρ is radial measure which defines core radial length by $\rho < a$, cladding by $a < \rho < b$ and region-III by $\rho > b$.

Obtained dispersion relation for HE-mode after applying boundary conditions on Equations (1) to (6) on different region interfaces:

$$\begin{aligned} &V\alpha\gamma P(R-Q)(I-W) + V^2P^2(\alpha-\gamma)^2 + V^2(\alpha T - \gamma S)(\gamma R - Q\alpha) + P^2(\gamma Y_c - j_c\alpha) \\ &(\gamma I - \alpha W) + (\gamma Y_c R - Q j_c \alpha)(T\alpha W - s\gamma I) + VP\alpha\gamma(Y_c - j_c)(T - S) = 0 \end{aligned} \tag{7}$$

Used coefficients in above equation are:

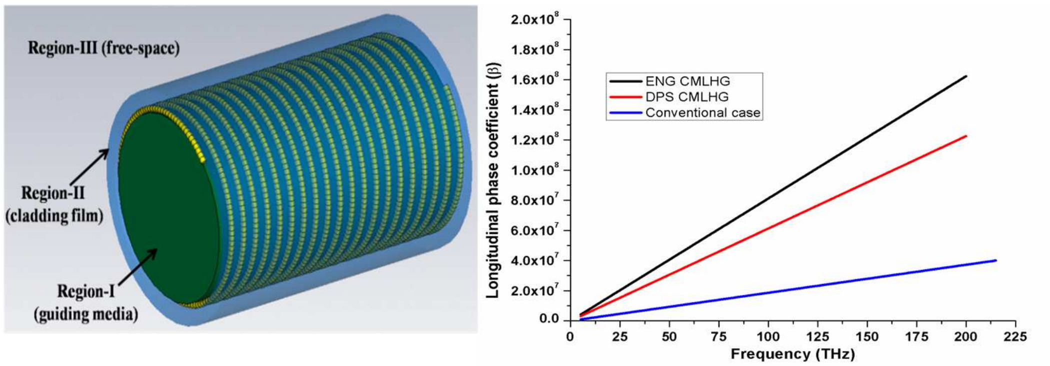
$$\begin{aligned}
&V = k_2^2 - \frac{n\beta}{a}\cot(\psi), \quad U = k_1^2 - \frac{n\beta}{a}\cot(\psi), \quad x = \omega\mu_1 k_1, \quad x_1 = \omega\epsilon_1 k_1, \\
&y = \omega\mu_2 k_2, \quad y_1 = \omega\epsilon_2 k_2, \quad z = \omega\mu_3 k_3, \quad z_1 = \omega\epsilon_3 k_3, \quad r_1 = \frac{k_1}{k_2}, \quad r_2 = \frac{k_2}{k_3}, \\
&M = \frac{U^2 I_n(k_2a)}{x r_1^2 \cot(\psi) I_n'(k_2a)}, \quad N = \frac{x_1 \cot(\psi) I_n'(k_2a)}{r_1^2 I_n(k_2a)}, \quad j_c = \frac{y\cot(\psi) I_n'(k_2a)}{I_n(k_2a)}, \\
&Y_c = \frac{y\cot(\psi)K_n'(k_2a)}{K_n(k_2a)}, \quad W = M + N + \frac{y_1\cot(\psi)I_n'(k_2a)}{I_n(k_2a)}, \quad I = M + N + \frac{y_1\cot(\psi)K_n'(k_2a)}{K_n(k_2a)}, \\
&P = \frac{n\beta}{b(1-r_2^2)}, \quad R = \frac{yI_n'(k_2b)}{I_n(k_2b)} - \frac{zr_2^2K_n'(k_2b)}{K_n(k_2b)}, \quad Q = \frac{yK_n'(k_2b)}{K_n(k_2b)} - \frac{zr_2^2K_n'(k_3b)}{K_n(k_3b)}, \\
&S = \frac{y_1 I_n'(k_2b)}{I_n(k_2b)} - \frac{z_1 r_2^2 K_n'(k_3b)}{K_n(k_3b)}, \quad T = \frac{y_1 K_n'(k_2b)}{K_n(k_2b)} - \frac{z_1 r_2^2 K_n'(k_3b)}{K_n(k_3b)}, \\
&\alpha = I_n(k_2a)k_n(k_2b), \quad \gamma = I_n(k_2b)k_n(k_2a).
\end{aligned} \tag{8}$$

3. RESULT AND ANALYSIS

The Dispersion behaviour of structure is obtained by plotting longitudinal propagation coefficient (β) as function of frequency. For root finding we used Findroot method of Mathematica software package.

A comparative dispersion graph is presented in Fig. 1(b), for ENG CMLHG waveguide and two other cases: (i) when helix is in free space and (ii) when helix is cladded with DPS material and loaded with metamaterial. It is found that present structure provides almost 4 times lesser phase velocity in comparison to the helix in free space. It supports propagation of fundamental mode. Higher order modes are not observed in the guide so that most of the power is carried out by fundamental mode. Obtained slope of dispersion graph is positive that corresponds to forward wave (FW) mode which having parallel phase and group velocities.

A very large value of longitudinal propagation coefficient is observed in ENG clad MLHG structure that enhances slow wave behaviour of guide and achieved minimum normalized phase velocity



(a) (b)

Figure 1: Cladded metamaterial loaded helical guide (CMLHG) structure. Figure 1(b) Dispersion characteristic (vs frequency) is plotted for different cases (i) Conventional case, helical guide in free-space (ii) helix is cladded with DPS material and loaded with metamaterial and (iii) ENG CMLHG waveguide represents in blue, red and black colour respectively.

(v_p/c) is 0.0258. As compared to DPS clad MLHG structure reduction in slow wave behaviour is enhanced by 26.4%. In different simulations it observed that in ENG Clad MLHG structure the value of v_p/c is reached at 0.0011 by adjusting physical parameter of the guide such as radius and pitch angle and further it could reduced more.

Other advantages of ENG cladding is that it works as an insulator in order to avoid possible interaction between helix copper winding and other on-chip elements. It enhances slow wave characteristic and provides wider bandwidth as well.

4. CONCLUSION

In this work we propose cladded MLHG structure. From analytical computation it is observed that wave velocity is greatly reduced in ENG clad MLGH as compared to conventional cases. Other unique advantage of this structure is a wider bandwidth.

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