On the Limits of Numerical Modelling of Electromagnetic Field Coupling through Small Apertures

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Abstract— While the modelling of electromagnetic field coupling through apertures constitutes a well-known problem class in electromagnetic theory, it still is a nontrivial problem to determine accurate electromagnetic field solutions for specific cases of aperture coupling. In this contribution this circumstance is illustrated by a number of explicit examples which are, in increasing order of complexity, given by single circular apertures in planar shields, single circular apertures in rectangular cavities, and by an aperture array within a rectangular cavity. It is seen that both analytic solutions and numerical standard methods exhibit restrictions due to their approximative character in terms of validity and discretization, respectively. As a result it is concluded that, depending on the type of problem, standard solution techniques still are limited for accurately predicting aperture coupling such that refined hybrid numerical methods become required.

1. INTRODUCTION

The modelling of electromagnetic field coupling through apertures is a canonical problem that has been studied by many researchers, see, e.g., [1] and references therein. Within the electromagnetic compatibility (EMC) community the related concepts and methods are an important basis for the design of efficient metallic shielding enclosures [2]. Such metallic enclosures are commonly used to shield against radiated EM fields or to prevent leakage effects from interior components. The shielding effectiveness (SE) then typically is reduced by apertures and slots which may be necessary for ventilation or as lead throughs.

Depending on the electrical size and thickness of relevant apertures a variety of analytic models exist to solve the electromagnetic boundary value problems that are related to compute the aperture coupling and SE [1, 2]. Most of them are approximations and their limitations need to be kept in mind, even though they can provide benchmark solutions in certain frequency regimes or lead to a useful understanding of parameter dependencies. Additionally, for practical EMC analyses of realistic systems it is tempting to use modern numerical standard techniques in order to determine aperture couplings and shielding efficiencies. A straightforward use of such methods, however, requires some caution and care since apertures often constitute geometries that are characterized by sharp edges and small geometries such that careful discretizations are required to properly model diffraction phenomena or complicated near-field contributions, for example.

In the following the still prevailing difficulties that are connected to an accurate determination of aperture coupling are illustrated by a variety of analytical and numerical methods that are applied to three canonical and one realistic geometry. The canonical geometries comprise the infinite and finite planar shield with a small circular aperture and a rectangular cavity with a small circular aperture. Both analytical and numerical results can be applied in these case. The realistic geometry involves a rectangular cavity with finite aperture array where no closed form solutions are available. As numerical standard tools the method of moments (MoM), as implemented in the CONCEPT-II software package [3], and the Finite Integration Technique (FIT), as implemented in the CST Microwave Studio [4], are used. These methods are applied with the best of our knowledge and from the perspective of an EMC analyst who is interested in obtaining accurate solutions with meaningful computational effort. It is not our interest to show which methods are "better", but it is our aim to make clear that the field coupling through small apertures still constitutes a demanding computational problem, even with fine and careful discretisations.

2. ANALYTICAL AND NUMERICAL COMPUTATION OF APERTURE COUPLING: CASE STUDIES

2.1. Infinite and Finite Planar Shield with Small Circular Aperture

To begin with, the problem of finding the transmitted EM field through a planar, infinitesimally thin, and perfectly conducting screen of infinite extent perforated with a finite aperture is considered, compare Fig. 1. As an analytic approach, the Bethe theory for small apertures [5] can be used to represent the field scattered by the aperture as superposition of an electric-dipole field and

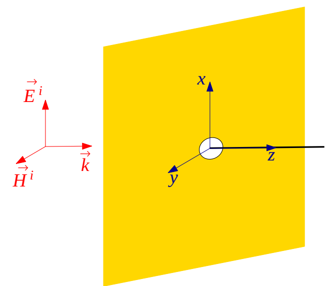

Figure 1: An electromagnetic plane-wave propagates towards a perfectly conducting infinitesimally thin planar screen with a small circular aperture, the propagation direction is normal to the shield in z-direction.

a magnetic-dipole field due to electric and magnetic dipole sources, respectively, both being located at the center of the aperture.

The aperture equivalent electric and magnetic dipole moments $\vec{p}_e$ and $\vec{p}_m$ are related to the electric and magnetic (imaged) polarizability tensors $\overleftrightarrow{\alpha}_e$ and $\overleftrightarrow{\alpha}_m$ of an aperture in an infinite, perfectly conducting plane by

$$\vec{p}_e = 2\varepsilon_0 \overleftrightarrow{\alpha}_e \cdot \vec{E}_{sc} \quad \text{and} \quad \vec{p}_m = -2 \overleftrightarrow{\alpha}_m \cdot \vec{H}_{sc}, \tag{1}$$

where $\vec{E_{sc}}$, $\vec{H_{sc}}$ are the short-circuit fields at the aperture [6]. On this basis the electric dipole field at an observation point $\vec{r}$ radiated by $\vec{p}_e$ and $\vec{p}_m$ from the origin $\vec{r'} = \vec{0}$ can be derived from [7]

$$\vec{E}\,(\vec{r}) = -\frac{1}{\epsilon}\vec{\nabla} \times \left[\vec{p}_e \times \vec{\nabla}' \frac{e^{-jk|\vec{r}-\vec{r'}|}}{4\pi\left|\vec{r}-\vec{r'}\right|}\right] + j\omega\mu\left[\vec{p}_m \times \vec{\nabla}' \frac{e^{-jk|\vec{r}-\vec{r'}|}}{4\pi\left|\vec{r}-\vec{r'}\right|}\right]. \tag{2}$$

In the setup of Fig. 1 the z-axis of the coordinate system is perpendicular to the aperture such that the polarizabilities for a circular aperture with radius r_0 are given by

$$\alpha_{e,z} = \frac{2}{3} \cdot r_0^3 \quad \text{and} \quad \alpha_{m,x} = \alpha_{m,y} = \frac{4}{3} \cdot r_0^3. \tag{3}$$

It follows

$$E_{sc,\perp} = E_{sc,x} = 0, \quad H_{sc,y} = 2\hat{H}^i = 2\hat{E}^i/\eta_0, \quad H_{sc,z} = 0, \tag{4}$$

where $\hat{E}^i$, $\hat{H}^i$ denote the amplitudes of the incoming field and $\eta_0 = 377\,\Omega$ is the intrinsic impedance of free space. With these values the dipole moments become

$$\vec{p}_e = 0, \quad p_{m,x} = p_{m,z} = 0, \quad p_{m,y} = -2 \cdot \frac{4}{3} r_0^3 \cdot \hat{H}^i. \tag{5}$$

It follows that the electric field component E_x, for example, transmitted through the small circular aperture and evaluated along the z-axis is given by

$$|E_x(0,0,z)| = \left| j\omega\mu \cdot p_{m,y} \cdot \frac{e^{-jkz}}{4\pi z} \cdot \frac{1}{z + jk} \right|, \quad z > 0. \tag{6}$$

As a result, Fig. 2 shows the penetrating electric field along the z-axis through an infinite, perfectly conducting screen with circular apertures of radii $r_{0,1} = \lambda/10$ and $r_{0,2} = \lambda/100$, respectively, where $\hat{E}^i = 1\,\text{V/m}$ and $f = 600\,\text{MHz}$. To account for the infinite extent of the screen, the Babinet principle [7] was used within the MoM, while within the FIT it was possible to model the infinite extent by a suitable setting of boundary conditions. It is known that the analytic result obtained by the Bethe theory exhibits an unphysical singularity at $\vec{r'}$ where the aperture and equivalent dipoles are located. Distant from the aperture the agreement with the numerical solutions improves. It also is seen that for decreasing aperture radius the numerical solutions tend to yield increasingly different results.

Additionally, a perfectly conducting, finite planar screen of dimension $2\lambda \times 2\lambda$ has been considered with the setup of Fig. 1, the corresponding results are shown in Fig. 3. For this finite geometry the application of Bethe's theory is no longer meaningful. The numerical solutions qualitatively agree well but show considerably differences close to the aperture where a minimum of the field values occurs.

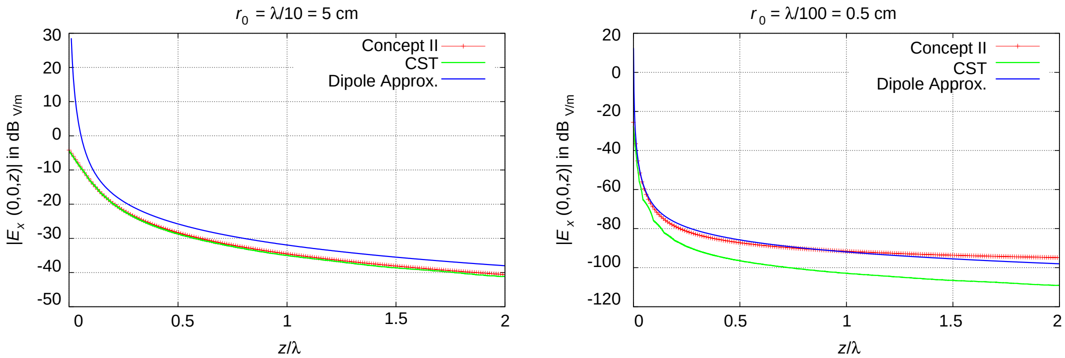

Figure 2: Penetrating electric field component E_x for an infinite planar screen with a circular aperture of different radii $r_{0,1} = \lambda/10$ and $r_{0,2} = \lambda/100$.

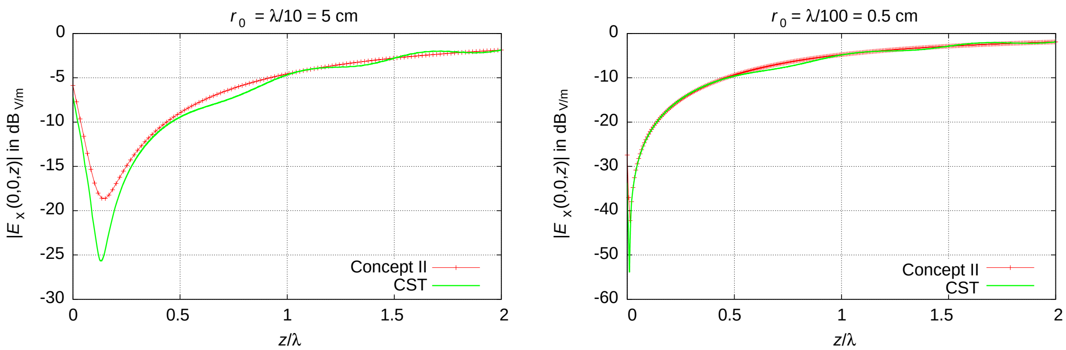

Figure 3: Penetrating electric field component E_x for a finite planar screen with a circular aperture of different radii $r_{0,1} = \lambda/10$ and $r_{0,2} = \lambda/100$.

2.2. Rectangular Cavity with Small Circular Aperture

To tend to a more practical geometry, the electromagnetic coupling through a small circular aperture of a rectangular cavity with dimensions a, b, and c is considered next. The circular aperture is located with its center at coordinates (x_A, y_A, z_A) and illuminated by an exterior plane wave, compare Fig. 4. An analytic description of the nonvanishing electric field components within the rectangular cavity is given in [8] and yields, e.g., the expression for the z-component of the electric field in the form

$$E_z(\vec{r}, j\omega) = \frac{4}{3}\frac{d^3}{V}\sum_{n_x=1}^{\infty}\sum_{n_y=1}^{\infty}\sum_{n_z=0}^{\infty}\epsilon_{0,n_z} k_x \sin(k_x x)\cos(k_x x_A)\sin(k_y y)\sin(k_y y_A)$$
$$\times\cos(k_z z)\cos(k_z z_A)\frac{jk\hat{E}^i(j\omega)}{k_x^2 + k_y^2 + k_z^2 - k^2 + j\delta}, \tag{7}$$

with $k_x = n_x\pi/a$, $k_y = n_y\pi/b$, $k_z = n_z\pi/c$. The Neumann symbol $\epsilon_{0,i}$ assumes the values 1 for $i = 1$ and 2 for $i \neq 1$. The parameter δ characterizes losses within the cavity. These losses can be included in the wave number according to

$$k \to k\left(1 - \frac{j}{2Q}\right), \quad \delta = \frac{k^2}{Q}, \tag{8}$$

where $Q(k)$ denotes the quality factor of the resonator [9].

The electric field component E_z inside a perfectly conducting rectangular cavity with dimensions $a = 0.20\,\mathrm{m}$, $b = 0.509\,\mathrm{m}$, $c = 0.40\,\mathrm{m}$ and a single circular aperture of radius $r_0 = 2.5\,\mathrm{cm}$ has been evaluated for frequencies 400 MHz to 1200 MHz at the interior position $(0.13, 0.15, 0.15)\,\mathrm{m}$. The center of the aperture is located at $(0.20, 0.15, 0.15)\,\mathrm{m}$ and it is illuminated by a normally incident plane wave with $\hat{E}^i = 1\,\mathrm{V/m}$, as also indicated in Fig. 4. The corresponding analytic and numerical results are shown in Fig. 5.

For a proper discussion it is noted that the analytic solution has been obtained on the basis of a quality factor $Q = 0.05\sqrt{f/\mathrm{Hz}}$ to account for resonator losses that are due to the presence

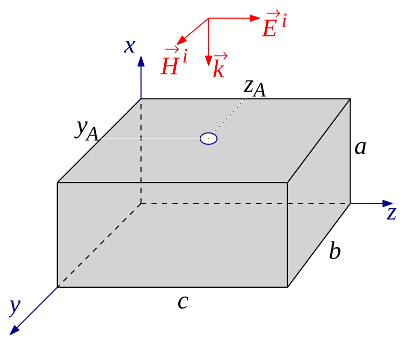

Figure 4: Geometry of rectangular cavity with circular aperture, excited by an electromagnetic plane wave.

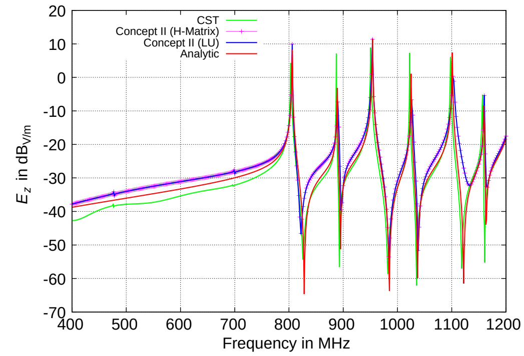

Figure 5: Electric field component E_z within the rectangular cavity with circular aperture of radius $r_0 = 2.5\,\mathrm{cm}$, evaluated at position $(0.13,\,0.15,\,0.15)\,\mathrm{m}$.

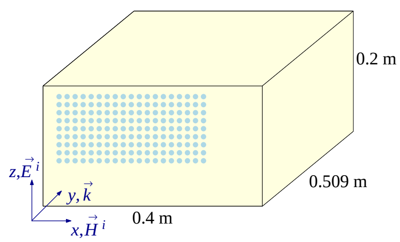

Figure 6: Geometry of rectangular cavity with circular aperture, excited by an electromagnetic plane wave.

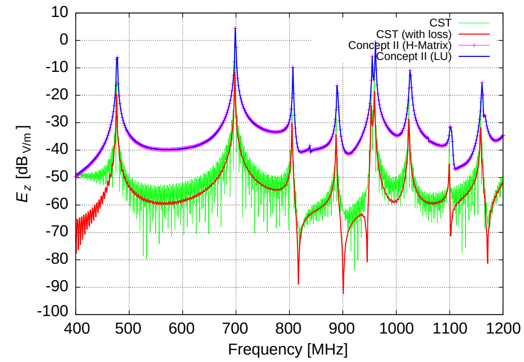

Figure 7: Electric field component E_z within the rectangular cavity with aperture array, evaluated at position $(0.150,\,0.359,\,0.130)\,\mathrm{m}$.

of the aperture. Two versions of MoM calculations have been used, one based on a traditional LU-decomposition and one based on an advanced solution algorithm which takes advantages of the formalism of $\mathcal{H}$-matrices and already proved useful for the modelling of electromagnetic coupling within resonators [10, 11]. The calculation in CST required an exceedingly long computation time to achieve the necessary bound for sufficient energy decay within the resonator. The agreement between the results in Fig. 5 is quite satisfying but already required a rather careful discretization of the geometry, in particular in the vicinity of the aperture.

2.3. Rectangular Cavity with Aperture Array

A more realistic aperture array of a rectangular cavity is shown in Fig. 6. The array consists of 19×9 apertures of radius $r_0 = 0.4\,\mathrm{cm}$; the upper left hole has coordinates $(0.04,\,0,\,0.16)\,\mathrm{m}$ and the centers of adjacent apertures are separated by $1.2\,\mathrm{cm}$ from each other. Again, a plane wave with amplitude $\hat{E}^i = 1\,\mathrm{V/m}$ is taken to illuminate the structure and the E_z-component is evaluated at position $(0.150,\,0.359,\,0.130)\,\mathrm{m}$. For this configuration, a closed form solution does not exist, such that results have been obtained on a numerical basis only.

For the method of moments solution both the traditional LU-decomposition and $\mathcal{H}$-matrix algorithm have been used. The numerical solution based on FIT, again, requires a rather long computation time and it turns out to be difficult to reach the necessary bound for sufficient energy decay, as also can be seen from spurious oscillations in the frequency domain result. Therefore an additional calculation has been performed with slight losses of $\tan(\delta) = 0.001$ added to the inside volume of the resonator.

The numerical results are displayed in Fig. 7. All curves capture the main resonance peaks correctly. The two MoM curves show a very good agreement and also the two FIT curves agree, if averaged, fairly well with each other, where it is clearly understood that the introduction of artificial losses leads to lower resonance peaks. But the conceptually different numerical methods

MoM and FIT lead, in this example, to different field levels by about 15 dB for most of the frequency range. While the discretization of the apertures has been peformed with similar care and effort if compared to the simpler examples, it is now no longer obvious from Fig. 7 which result is the more accurate one. Therefore it appears that a limit is reached where conventional numerical mehods may no longer accurately predict the solution of a nontrivial electromagnetic aperture problem. Similar observations obtained by a larger variety of different numerical field solvers already have been observed in the context of electromagnetic coupling into an aircraft geometry with electrically larger apertures involved [12].

3. CONCLUSION

It has been demonstrated by examples of increasing complexity that the classic problem of aperture coupling still is a challenging task if quantitatively accurate results are required. For canonical problems it is possible to have analytical results available that may serve as a reference for numerical results. For realistic cases of aperture coupling, which are of great interest to the EMC community, analytical results typically are no longer available. In this case, the application of different standard numerical tools can lead to different results such that for a definite answer it appears as necessary to provide either additional measurement results or to possibly develop refined numerical methods of higher accuracy that are adapted to particular aperture configurations.

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