Two-dimensional Spatial Frequency Technique for Calculating Electromagnetic Scattering from Large Objects

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Abstract— Spatial frequency analysis has been proposed as a way to speed up the solution of the integral equation (IE) method for calculating electromagnetic scattering from large objects. In this paper, a new, faster technique based on spatial frequency analysis is developed for describing scattering from two-dimensional conducting objects. The technique utilizes the spatial frequency representation of the integral equation and generates a closed form analytical expression for estimating the induced currents without the need for matrix inversion. The technique is applied to the scattering from two-dimensional objects such as infinite conducting cylinders and parallel plate structures. Simulation results are compared with the standard Method of Moments solution and its faster solutions such as the conjugate gradient method. The results indicate that the proposed technique performs faster than the standard MoM solution with results which are comparable to the latter technique.

1. INTRODUCTION

In computational electromagnetics, there is a need for fast and accurate methods for calculating electromagnetic scattering from large objects and surfaces. Typically, in scattering problems an integral equation (IE) is derived and solved for calculating the fields or the currents on a scattering structure [1]. Numerical solution methods such as the method of moments (MoM) discretize the IE into a matrix form and solve the problem by inverting this matrix [2]. However, since these matrices are generally dense for large scattering objects, matrix inversion is not computationally efficient. More efficient iterative methods such as the conjugate gradient (CG) method have since been developed to handle the numerical complexity of solving large matrix systems [3]. These iterative methods have significantly reduced the computational demands of solving the IE for scattering and radiation problems.

Another important technique which has shown great potential for reducing the computational demands of large matrix inversion problems is the use of the spatial frequency technique. First proposed by Bojarski [4], the spatial frequency technique transforms the matrix equation into the spatial frequency domain by the use of the computationally efficient FFT algorithm [5]. Generally, the IE represents a set of boundary conditions which are the result of a convolution of a Green's function and surface currents or fields. Using the property of the Fourier transform, a convolution in space is converted to a product in the spatial frequency domain. The dense matrix multiplication in the spatial domain representation of the IE is thereby transformed into a multiplication by a diagonal matrix.

In addition, another significant advantage of the spatial frequency method is that it converts derivatives of fields or currents into simple multiplication by the appropriate spatial frequency component [6]. In many scattering or radiation problems the effective Green's function is derived by taking the derivatives of an original Green's function. In all of these cases the modified Green's functions for different polarizations can be derived easily from one original Green's function by utilizing this property of the spatial frequency method.

In previous work, the spatial frequency approach was used for analyzing scattering from infinite, one-dimensional conducting surfaces [7]. In this study, the spatial frequency method is extended to scattering from two-dimensional, conducting objects, such as metallic cylinders of arbitrary cross-sections. In discrete processing, only Green's functions represented by circulant Toeplitz matrices can be converted to diagonal matrices in the spatial frequency domain [6]. However, in practice, the Green's function matrices are generally Toeplitz matrices and not circulant Toeplitz matrices. A method for converting the Toeplitz matrix to a circulant Toeplitz matrix without altering the problem and increasing the computer storage requirements is developed and used in this technique. An analytical expression for the induced currents are obtained in terms of a diagonal Green's function matrix in the spatial domain and the spatial frequency transform of the incident fields on the conducting surface.

A Taylor series expansion is used to avoid the need for matrix inversion. By truncating the Taylor series, one is able to generate results with different degrees of accuracy. Simulation results for scattering from cylinders of different sizes and cross-sections are generated and compared with the method of moments for accuracy. In addition to the analysis of electromagnetic scattering, this technique also lends itself well to the synthesis of scattering patterns. Several examples of how the spatial frequency technique may be applied to the synthesis of two-dimensional targets with prescribed scattering cross-sections is also presented.

2. THEORETICAL FORMULATION

The spatial frequency technique is studied in two-dimensions by applying the technique to scattering from infinite conducting cylinders with rectangular and triangular cross-sections and parallel plate structures. If a unit amplitude plane wave, with TM polarization, is incident on the structure at an angle θ_0, the boundary condition for the tangential electric field component on the conducting surface can be written in terms of the surface current as [3]

$$\frac{k_0\eta}{4}\int_{l'} J_z(r')H_0^{(2)}(k_0\left|r-r'\right|)dr' = -e^{j(k_0x(r)\sin\theta_0+k_0y(r)\cos\theta_0)}, \tag{1}$$

where k_0, and η are the plane wave's propagation constant and wave impedance, respectively. $J_z(\cdot)$ and $H_0^{(2)}(\cdot)$ are the z-directed current density on the conducting surface and the Hankel function of the second type, which represents the two-dimensional Green's function. The right hand side of Equation (1) represents the tangential component of the scattered electric field on the conducting surface, which is equal and opposite to the tangential component of the incident electric field on the surface. The integral is performed along a contour defined by the cross-section of the conducting structure.

The discretized form of the IE given in Equation (1) may be written as a matrix equation

$$Gx = e, \tag{2}$$

where x is the column vector representing the sampled current densities on the conducting surface and e is the column vector representing the sampled scattered electric field components on the conducting cylinder. The elements of the matrix G are generated from the two-dimensional Green's function. Note in a regular rectangular grid, the matrix G, which describes the convolution operation, is both symmetric and Toeplitz.

In the MoM method, since e is known, x is obtained by inverting the matrix G [2]. Fast algorithms such as the CG algorithm use an iterative approach to avoid the direct inversion of the matrix [3]. In the proposed spatial frequency method, the $N \times N$ two-dimensional FFT is converted to a vector of length $N \times N$. Each $N \times N$ vector represents the electric fields or the currents in the spatial frequency domain. A matrix F is defined as the two-dimensional FFT operator. The columns of the matrix, F, represent each discretized spatial point from the region of interest, in the spatial frequency domain. Since each spatial point is orthogonal to all the others, the F matrix is unitary.

The Toeplitz matrix G is transformed into a larger circulant Toeplitz matrix by padding additional rows and columns [7]. It has been shown that the minimum number of padding elements required in each dimension to convert a $N \times N$ Toeplitz matrix to a circulant Toeplitz matrix is $N-1$ [6]. In this study, the additional padding is equal to the number of observation points on the conductor, which results in an extended region with the same dimension as the discretized dimension of the conducting region and thus, a doubling of the size of the matrix. In this paper, the extended region is referred to as the complementary space as it is orthogonal to the conducting surface. The new matrix equation representing the IE in the spatial frequency domain is written as

$$G_C\hat{x} = e + \Delta e, \tag{3}$$

where G_C is the larger circulant Toeplitz matrix, $\hat{x}$ is the original current vector padded with zeros, and Δe is some unknown electric field component in the complementary space. When the two-dimensional, FFT operator F is applied to both sides of (3), it yields

$$FG_C\hat{x} = FG_cF^HF\hat{x} = Fe + F\Delta e. \tag{4}$$

Since G_C is circulant Toeplitz, Equation (4) can be re-written as

$$G_DF_1x = F_1e + F_2\Delta e, \tag{5}$$

where G_D is the diagonal matrix, whose diagonal elements are equal to the FFT of the primary row of matrix G_C. F_1 and F_2 are columns of F which correspond to the two orthogonal subspaces consisting of points on the conducting surface and points in the complementary space, respectively. Since these subspaces are orthogonal, projecting Equation (5) onto the F_1 subspace yields,

$$F_1^H G_D F_1 x = e. \tag{6}$$

Similarly, projecting Equation (5) onto the F_2 subspace yields,

$$F_2^H G_D F_1 x = \Delta e. \tag{7}$$

Using the diagonal property of G_D, inverting Equation (5) yields,

$$x = F_1^H H_D F_1 e + F_1^H H_D F_2 \Delta e, \tag{8}$$

where H_D is the diagonal matrix which is the inverse of the matrix, G_D.

Using Equation (7) and substituting for Δe, the current and field vectors are related by

$$F_1^H H_D F_1 e = (I - A) x, \tag{9}$$

where the matrix A is given by $A = F_1^H H_D F_2 F_2^H G_D F_1$. Solving for x and then using the Taylor series expansion yields

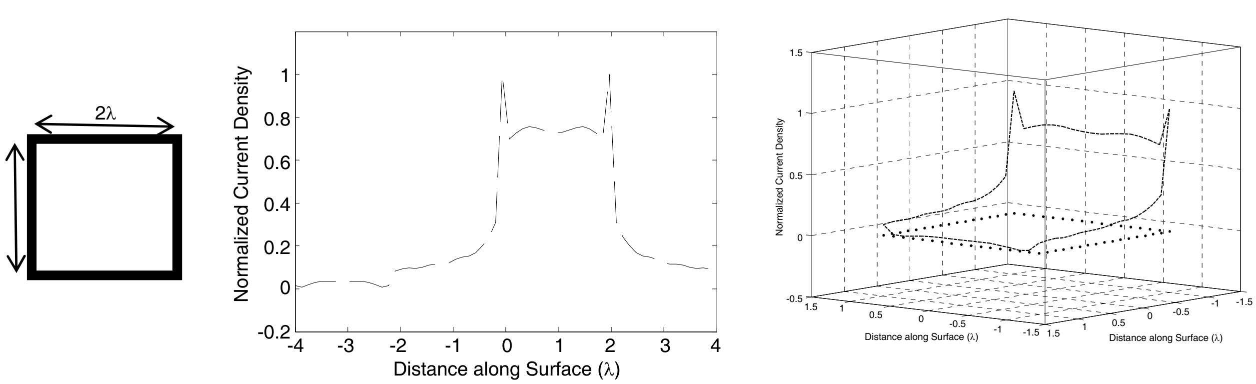
$$x = \left(I + A + A^2 + \ldots\right) F_1^H H_D F_1 e. \tag{10}$$

In Equation (10), the matrix inversion is replaced by the Taylor series expansion of $(I - A)^{-1}$. A is low rank matrix which may be inverted using the Miller method for inverting a sum of two matrices, where one matrix is a low rank matrix [8]. The Miller method for inverting matrix sums is based on the broader Sherman-Morrison formula [9]. In this study, the Taylor series expansion is used instead. The Taylor series form in Equation (10) allows one to choose the level of accuracy needed by truncating the series.

3. SIMULATION RESULTS

The spatial frequency method described in the previous section is verified by generating scattering simulations using MATLAB. The method is initially tested with the direct MoM solution which involves a direct matrix inversion. Figure 1 shows the currents induced on a conducting square cylinder whose side dimension is 2λ, when a TM wave is incident normally on one side of the cylinder. Figure 1(a) shows the cross-section of the cylinder. The cylinder is considered to be of infinite extent in the z-direction. Figure 1(b) shows the induced current density along the surface of the conducting surface. Figure 1(c) shows a three-dimensional perspective of this current density.

The spatial frequency method is verified and validated by using this method for generating the induced currents for the same square cylinder shown in Figure 1. Figure 2 shows the simulated currents using the spatial frequency method for a TM wave incident on one side of the cylinder. The

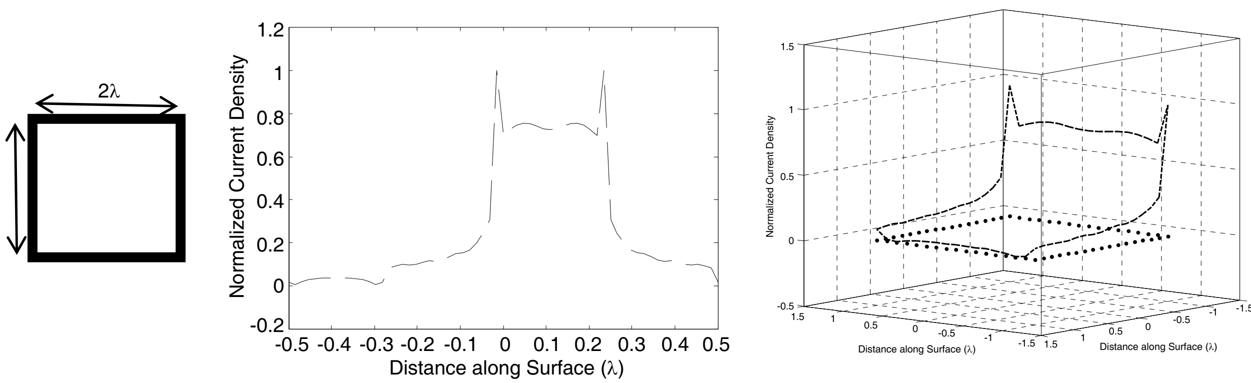


(a) (b) (c)

Figure 1: The induced currents calculated using the method of moments on a square box of dimension 2λ whose cross-section is shown in (a). The currents along the length of the surface are shown in (b). (c) shows the three-dimensional view of the current distribution.

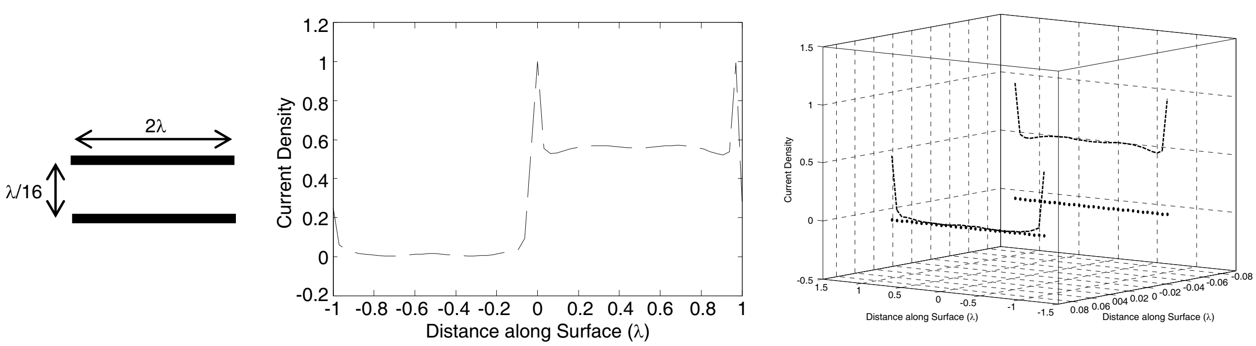
current densities shown in Figure 2 are identical to the current densities generated by the standard MoM as shown in Figure 1. Clearly, the results generated by the spatial frequency method are consistent with the well-established MoM approach.

The spatial frequency method is also used to calculate the scattering from two infinite parallel conducting plates. The width of the plates is 2λ and they are separated by a distance of $\lambda/16$. Figure 3 shows the induced current densities on the two plates when a TM wave is incident normally



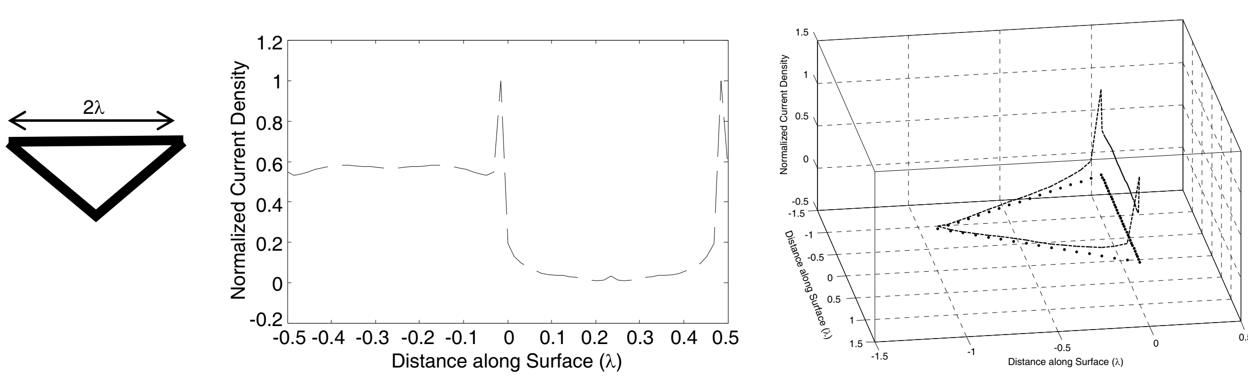
(a) (b) (c)

Figure 2: The induced currents calculated using the spatial frequency method on a square box of dimension 2λ whose cross-section is shown in (a). The currents along the length of the surface are shown in (b). (c) shows the three-dimensional view of the current distribution.



(a) (b) (c)

Figure 3: The induced currents calculated using the spatial frequency method on two parallel plates of dimension 2λ and separation $\lambda/16$ as shown in (a). The currents along the length of the two surfaces are shown in (b). (c) shows the three-dimensional view of the current distribution.



(a) (b) (c)

Figure 4: The induced currents calculated using the spatial frequency method on a right angled triangular cylinder with a hypotenuse of dimension of 2λ as shown in (a). The currents along the length of the surface are shown in (b). (c) shows the three-dimensional view of the current distribution.

on one plate. As expected, the currents induced on the top plate is higher than the currents induced on the bottom plate.

Figure 4 shows the currents induced in a cylinder with a right-angled triangular cross-section. The cross-section is shown in Figure 4(a). The cross-section consists of two equal perpendicular sides of size 1.414λ and a hypotenuse of 2λ. A TM wave is assumed to be incident on the largest side.

4. CONCLUSIONS

A fast and accurate technique for calculating the scattering from two-dimensional conducting surfaces is developed. The technique utilizes the FFT to transform the integral equation from the spatial domain to the spatial frequency domain. The two-dimensional FFT is used in this paper to represent the scattered electric fields as a product of the spatial frequency representation of the induced currents and the spatial frequency representation of the two-dimensional Green's function. Simulations are performed for infinite cylinders of different cross-sections and parallel plate structures. The results from spatial frequency method are compared with the results from the standard MoM solution and they appear to produce identical results. The spatial frequency method is shown to be significantly faster than the standard MoM method. In the future, a three-dimensional version of the model will be presented to describe scattering from three-dimensional conducting objects.

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