Local Field Effects for Left-handed Planar Metamaterials

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Abstract— We consider dielectric and magnetic properties of left-handed planar metamaterials with negative permittivity and permeability. In our work we investigate planar metamaterials consist of anisotropic particles. We obtain dependence of permittivity on dielectric polarizability and permeability on magnetic polarizability of the particles. Local field effects are taken into account in calculations. These effects are caused by interaction between single particles. We also obtain the generalized Clausius-Mossotti relations for planar metamaterials made of anisotropic particles in the long wave approximation. We calculate a response function to an external electromagnetic field in case of planar metamaterial. This function describes connection between external electromagnetic field and the local field. On the one hand, such connection allows us to analyze structure of electromagnetic field in the near-field regime. On the other hand, the relationship between the field interacting with particles in the planar metamaterials and the external field is of key importance for calculations of characteristics of the electromagnetic radiation generated by charged particles or particle beams passing through or near the planar structures.

1. INTRODUCTION

Metamaterials are artificially designed materials that have effective properties, such as anomalous refraction or reflection. Particularly, planar metamaterials have unusual properties such as manipulation of the polarization states of light diffracted from the surface. These structures also demonstrate extremely high confinement of electromagnetic fields and thus strong variations of field intensities, which can significantly enhance near-field optical forces [1–3]. In work [4–6] it has demonstrated that planar metamaterials with desired values of permittivity and permeability can be obtained by using different methods, for example, hybridization. Also such effective properties are achieved by the optimization of the parameters of constituent molecules of metamaterials. Therefore planar metamaterials are in the center of many modern researches. In addition the planar nature of these structures means they could be incorporated into thin-film coatings for numerous optical components.

2. PERMITTIVITY AND PERMEABILITY OF 2D METAMATERIALS WITH LOCAL FIELD EFFECTS

Let us consider planar metamaterial located at the plane $z = 0$. This material has the thickness b that is smaller than $n^{-1/2}$ — the average intermolecular distance ($n = N/S$ is the surface concentration of molecules of metamaterial). We investigate planar metamaterial which consists of anisotropic molecules with dielectric polarizability tensor and magnetic polarizability tensor:

$$\alpha^e_{ij}(\omega) = \alpha^e_{\perp}(\omega)\left(\delta_{ij} - e_i e_j\right) + \alpha^e_{\parallel}(\omega)\, e_i e_j, \tag{1}$$

$$\alpha^m_{ij}(\omega) = \alpha^m_{\perp}(\omega)\left(\delta_{ij} - e_i e_j\right) + \alpha^m_{\parallel}(\omega)\, e_i e_j. \tag{2}$$

In this medium Fourier-transform for density of micro-currents has the form:

$$\begin{aligned}\left\langle j^{mic}_i\left(\mathbf{r}',\omega\right)\right\rangle = & -i\omega\alpha^e_{ij}(\omega)\left\langle\sum_n E^{mic}_j(\mathbf{R}_b,\omega)\delta\left(\mathbf{r}'-\mathbf{R}_b\right)\right\rangle \\ & +c\alpha^m_{ij}(\omega)\left\langle\sum_n\left(rot\mathbf{H}^{mic}(\mathbf{R}_b,\omega)\right)_j\delta\left(\mathbf{r}'-\mathbf{R}_b\right)\right\rangle.\end{aligned} \tag{3}$$

In our previous work [7] we obtain relation between local (electric and magnetic) fields and macroscopic fields for 3d metamaterials takin into account distributional function. In case of planar metamaterials we have:

$$w\left(X_{ba}, Y_{ba}, Z_a + Z_{ba}\right) = \frac{1}{S}\eta\left(Z_a + Z_{ba}\right)\left[1 - f\left(X_{ba}, Y_{ba}\right)\right], \tag{4}$$

where $\eta(Z_b)$ is the distribution of metamaterial molecules along the z axis, $f(X_{ba}, Y_{ba})$ can be found from equation for the radial distributional function. From the Maxwell's equations takin into account Eq. (4) we obtain Fourier-transform of the resulting relations between local fields and macroscopic fields:

$$E_i(z,\omega) = T_{ik}(z,\omega) E_k^{loc}(z,\omega), \tag{5}$$

$$B_i(z,\omega) = Q_{ik}(z,\omega) B_k^{loc}(z,\omega), \tag{6}$$

where

$$T_{ik}(z,\omega) = \delta_{ik} - 4\pi n \alpha_{ij}^e(\omega) \int d^3 l e^{il_z z} \frac{l_j l_k}{l^2} \eta(l_z) f\left(\mathbf{l}_\parallel\right), \tag{7}$$

$$Q_{ik}(z,\omega) = \delta_{ik} - 4\pi n \alpha_{ij}^m(\omega) \int d^3 l e^{il_z z} \frac{l_j l_k}{l^2} \eta(l_z) f\left(\mathbf{l}_\parallel\right). \tag{8}$$

We do our calculations in a long-wave limit. According to this concept, if the wavelength of the field is large in comparison with the lattice constant d, the effective field acting on a molecule is formed by adding the fields of many molecules lying in some volume of the crystal with linear dimensions of L [8]:

$$c/\omega \gg L \gg d. \tag{9}$$

This limit allows us to neglect the spatial dispersion in Eqs. (5), (6). The integrals in Eqs. (7), (8) are symmetric tensors. These tensors depend on the unit normal vector to the surface of planar metamaterial. For this reason such integral can be represented as the sum of two terms which are proportional to tensors δ_{jk} and $e_j e_k$. So, the integral from Eqs. (7), (8) can be written in the form:

$$\int d^3 l e^{il_z z} \frac{l_j l_k}{l^2} \eta(l_z) f\left(\mathbf{l}_\parallel\right) = c\delta_{jk} + b e_j e_k. \tag{10}$$

We also know that the probability of unlimited approaching of molecules is negligibly small. Using this fact one can obtain convolution of Eq. (10) with δ_{jk}:

$$3c + b = \int d^3 l e^{il_z z} \eta(l_z) f\left(\mathbf{l}_\parallel\right) = \int d^2 l_\parallel f\left(\mathbf{l}_\parallel\right) \int dl_z e^{il_z z} \eta(l_z) = \eta(z). \tag{11}$$

On the other hand, convolution of (10) with $e_j e_k$ gives:

$$c + b = \int d^3 l e^{il_z z} \frac{l_z^2}{l^2} \eta(l_z) f\left(\mathbf{l}_\parallel\right). \tag{12}$$

In this problem the thickness b is smaller than all the characteristic lengths. For this reason the distribution function $\eta(z)$ is more rapid than other function in Eq. (12). So, integral in the right side of Eq. (12) can be represented in the form:

$$\int d^3 l e^{il_z z} \frac{l_z^2}{l^2} f\left(\mathbf{l}_\parallel\right) = \int d^2 l_\parallel f\left(\mathbf{l}_\parallel\right) \int dl_z \left(1 - \frac{l_z^2}{l_z^2 + l_\parallel^2}\right) e^{il_z z} \eta(l_z) = \eta(z) - \frac{1}{2} \int d^2 l_\parallel f\left(\mathbf{l}_\parallel\right) l_\parallel e^{-|z| l_\parallel}. \tag{13}$$

Thus we can write Eq. (12) as follows:

$$c + b = \eta(z) - 1/2\zeta(z), \tag{14}$$

where

$$\zeta(z) = \int d^2 l_\parallel f\left(\mathbf{l}_\parallel\right) l_\parallel e^{-|z| l_\parallel}. \tag{15}$$

Combining Eqs. (11), (14), we can derive coefficients c and b:

$$c = 1/4\zeta(z), \tag{16}$$

$$b = \eta(z) - 3/4\zeta(z). \tag{17}$$

Taking into account these coefficients one can rewrite Eqs. (7), (8):

$$T_{ik}(z,\omega) = \left(1 - 4\pi n \alpha_\parallel^e \zeta(z)\right)(\delta_{ik} - e_i e_k) + \left(1 + 2\pi n \alpha_\perp^e (\zeta(z) - 2\eta(z))\right) e_i e_k, \tag{18}$$

$$Q_{ik}(z,\omega) = \left(1 - 4\pi n\alpha_{\parallel}^{m}\zeta(z)\right)(\delta_{ik} - e_i e_k) + \left(1 + 2\pi n\alpha_{\perp}^{m}(\zeta(z) - 2\eta(z))\right) e_i e_k. \tag{19}$$

The tensors $Q_{ik}^{-1}(z,\omega)$, $T_{ik}^{-1}(z,\omega)$ describe relations between local magnetic and macroscopic magnetic fields, local electric and macroscopic electric fields, correspondingly. Also they define magnetic and dielectric properties of planar metamaterial. Using this fact Eqs. for permittivity and permeability can be determined as:

$$\varepsilon_{ij}(z,\omega) = \delta_{ij} + 4\pi n\alpha_{ik}^{e}(\omega)\eta(z)T_{kj}^{-1}(z,\omega), \tag{20}$$

$$\mu_{ij}(z,\omega) = \delta_{ij} + 4\pi n\alpha_{ik}^{m}(\omega)\eta(z)Q_{kj}^{-1}(z,\omega). \tag{21}$$

On the other hand, the permittivity and permeability are tensors and can be submitted in the form:

$$\varepsilon_{ij}(z,\omega) = \varepsilon_{\parallel}(z,\omega)(\delta_{ij} - e_i e_j) + \varepsilon_{\perp}(z,\omega)e_i e_j, \tag{22}$$

$$\mu_{ij}(z,\omega) = \mu_{\parallel}(z,\omega)(\delta_{ij} - e_i e_j) + \mu_{\perp}(z,\omega)e_i e_j, \tag{23}$$

where

$$\mu_{\parallel}(z,\omega) = 1 + \frac{4\pi n\alpha_{\parallel}^{m}(\omega)\eta(z)}{1 - \pi n\alpha_{\parallel}^{m}(\omega)\eta(z)}, \tag{24}$$

$$\mu_{\perp}(z,\omega) = \frac{1 + 2\pi n\alpha_{\perp}^{m}(\omega)\zeta(z)}{1 + 2\pi n\alpha_{\perp}^{m}(\zeta(z) - 2\eta(z))}. \tag{25}$$

Equations for components of permittivity are similar to Eqs. (24), (25) and depend on the components of dielectric polarizability of constituent molecules. In these equations denominators do not vanish because the dielectric polarizability and magnetic polarizability are also complex values.

3. DISCUSSION

In this work planar metamaterials are discussed. These materials draw more and more attention in the modern scientific investigations because of possibility of obtaining exotic properties via constructing their microstructure [9]. We obtain equations for permittivity (22) and permeability (23) of planar metamaterials. These equations are analogues the well-known Clausius-Mossotti relations, but for planar magnetic medium. Permittivity and permeability describe relation between macroscopic and microscopic properties of planar metamaterials. We also calculate response functions $T_{ik}^{-1}(z,\omega)$ and $Q_{ik}^{-1}(z,\omega)$ which define dielectric and magnetic properties of magnetic medium and describe relations between local and macroscopic fields (electric and magnetic). In our approach we use local field effects. This fact allows us to analyze optical properties of planar metamaterials which are very interesting for new investigations. Particularly, for design and engineering optical devices based on planar metamaterials which have extremely interesting properties (high specific rotation of polarization supported by plasmonic or waveguide resonances) [10]. Local field affects is also attractive for research nonlinear optical processes, which scale with a high power of the field. In case of nonmagnetic medium our results coincide with work [11].

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