Statistical Models of Noise Distribution in Broadband PLC Networks

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Abstract— The recent advancements in digital communication techniques have seen renewed interest in the possible utilization of the traditional power grid for the provision of broadband data and voice services. However, the power line communication (PLC) channel (the power network), presents a very challenging environment for communication purposes. This is due to the fact that severe noise and distortion effects are inherent in the medium. But, if these noise and distortion effects can be effectively analyzed and modeled, then the resulting models can be utilized in the optimal or sub-optimal design of a digital communication system for PLC applications. In this paper, frequency domain noise measurements done in broadband indoor PLC networks are presented. The measurement results obtained are then modeled using the alpha stable distribution, motivated by the flexibility of the distribution as a modeling tool and the fact that the measured noise distribution is heavily tailed and skewed.

1. INTRODUCTION

The traditional power grid, which was designed to deliver one way electric power, has seen tremendous evolution in the recent past. This has led to the emergence of an even more efficient, self-monitoring and controlled power system; the smart grid. The smart grid extends the traditional goal of using the power grid to deliver electric energy to include functionalities that enable real time load monitoring and control, automated meter reading, pre-paid billing as well as delivery of data services, among a plethora of other services. Traditionally, the target data rates in PLC technologies were low and the main application was in monitoring and data acquisition from different network elements. This constitutes what is referred to as narrowband power line communications (NB-PLC). However, recently, there has been a shift in PLC research, with most of the research activities focused on the use of the power grid to deliver higher data rate services. This has led to the development of new PLC technologies that seek to provide broadband services through the power grid; what is known as broadband power line communication (BB-PLC) [1–3].

Power line network systems are the most widely deployed in the world, and therefore the network provides a readily available infrastructure for the roll out of PLC technologies. PLC technologies are widely applied in smart grid applications, as mentioned earlier, networking of home equipment, monitoring and control of traffic lights in airport runways, monitoring and control of street lights, among a myriad of other applications. The fact that power network is ubiquitous renders it very attractive for exploitation as a communications medium. However, since the power grid was originally optimized for power delivery, it represents a very challenging environment for the delivery of communication services. This is due to the fact that almost all equipment connected to the power grid and even the power grid itself are possible sources of channel distortion and noise. These channel distortions manifest themselves in the form of frequency selective fading and attenuation. Moreover, the channel characteristics are highly dynamic in terms of frequency, time as well as location. Hence, different distortion effects will be experienced at different times of the day, varying mainly between peak and off peak hours, with more distortion expected during peak hours when there are a lot of activities going on in the power grid; that is, connection and disconnection of electrical equipment to/from the network. Also, the channel distortion varies across the frequency band of operation with more attenuation experienced at higher frequencies. Additionally, the attenuation tends to be highest in the low voltage (LV) networks and least in high voltage (HV) networks. However, the distortion effects will also highly depend on the weather conditions, networking loading as well as cable type and diameter [2–5].

Noise, the other biggest impairment in PLC networks, is responsible for degraded channel performance if not properly taken care of in design. The noise in PLC networks emanates from silicon controlled-rectifiers, dimmer switches, switching power supplies, brush motors and virtually all other equipment connected to the power network, for example, computers, printers, photocopiers, electric kettles, iron boxes, washing machines, et cetera. The noise in PLC network is categorized into generalized background noise, synchronous impulsive noise and sporadic (random) impulsive noise. Given the degradation that PLC noise causes in the channel performance, it is essential that the noise measurements be done and, characterized properly; parametrically or non-parametrically [6–8]. Different authors have treated the subject on PLC noise measurements, modeling and characterization in both time and frequency domains. Recently, Nyete et al. [5], proposed a multipath model of the channel frequency response based on a Rayleigh approximation. Also, Mulangu et al. [4] have also developed a framework for the estimation of the specific attenuation in broadband channels due to scattering points. On the other hand, Zimmerman and Dostert [7] have reported on the time domain and spectral analysis of impulsive noise, with particular focus on the pulse amplitude, interarrival time and impulse width distributions in typical power lines. Additionally, they have also examined the disturbance ratio and the impulsive rate in their work. In [9], Chan and Donaldson have also reported on the interarrival time, impulse width and amplitude probability distributions of noise impulses based on measurements carried out in both residential and industrial buildings. Their measurements are carried out with or without specific electrical loads on a 120 V network. Also, in [10], a similar approach to that adopted in [7, 9] above is also employed in the classification and characterization of impulsive noise in indoor power lines. In [11], Cortes et al. combine both time and frequency domain techniques to model the PLC noise measured in different buildings. Some other noise measurements, characterization and modelling efforts are those of Katayama et al. [12] and Vines et al. [13].

In this paper, we present results of an intensive PLC frequency domain noise measurement campaign carried out in different offices, laboratories and workshops in the Department of Electrical, Electronic and Computer Engineering building at the University of KwaZulu-Natal, Howard Campus, Durban, South Africa. The results of the measured noise power spectra probability distribution are then modelled using stable distribution. The choice of this statistical tool is informed by the fact that the noise spectrum probability distribution is seen to be heavily tailed and skewed, a property that the stable laws can perfectly model. Alpha stable distribution is a very attractive modeling tool especially in cases where impulsive characteristics are recorded; which means that the resulting distribution deviates from the traditional and widely used Gaussian distribution. Stable distributions have very attractive properties, some of which are highlighted below.

2. STABLE DISTRIBUTIONS

The stable family of distributions is a special class of distributions with heavier tails than the normal distribution; which means they give better and more robust estimates of the occurrence probability of extreme events. Noise occurrence in a PLC network is one such extreme event, considering its impulsive characteristics. Furthermore, the noise distribution in PLC networks is non-Gaussian and non-white. This distribution belongs to a class of distributions that are invariant under linear combinations; with three special cases, namely the Gaussian, Cauchy and Levy distributions. However the probability density function of this distribution does not exist in closed form. Only closed form expressions of the probability density function of the Gaussian, Cauchy and Levy distributions exist. Thus, the distribution is parameterized in terms of its characteristic function [14, 15]. The characteristic function is defined as [14, 15]:

$$\phi(t) = \begin{cases} \exp\left\{i\delta t - \gamma^\alpha |t|^\alpha \left[1 - i\beta \mathrm{sgn}(t) \tan\frac{\pi\alpha}{2}\right]\right\}, & \alpha \neq 1 \\ \exp\left\{i\delta t - \gamma|t| \left[1 + i\beta\frac{2}{\pi}\mathrm{sgn}(t)\ln|t|\right]\right\}, & \alpha = 1 \end{cases} \tag{1}$$

Where

$$\mathrm{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t = 0 \\ -1 & \text{for } t < 0 \end{cases}$$

And; $-\infty < \delta < \infty$, $\gamma > 0$, $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$.

From (1), we see that the stable distribution is defined by four parameters: the characteristic exponent, α, the skewness parameter β, the location parameter, δ, and the scale or dispersion parameter, γ. Additionally, we see that the stable distribution is a modelling tool that is very flexible, with the characteristic exponent controlling the heaviness of the distribution tails. A large value of α indicates less impulsiveness and vice versa. Again, as α approaches 2, the distribution tends to exhibit Gaussian tendencies. In fact, when $\alpha = 2$, the distribution is Gaussian. A theoretical justification of the stable distribution as a modelling tool comes from the Central Limit Theorem, which states that a physical phenomenon is Gaussian if there is an infinite number of independent and identically distributed (i.i.d) contributing sources (factors) with finite variance. Moreover, the Generalized Central Limit Theorem for an infinite number of i.i.d random variables states that if these variables converge with or without variance to a distribution by increasing their number, the limit distribution must be stable [14, 16].

3. NOISE MEASUREMENTS AND ANALYSIS

Indoor PLC noise measurements were carried out in different rooms at the Electrical, Electronic and Computer Engineering Department building at Howard College, University of KwaZulu-Natal, Durban, South Africa. The noise measurements were done in the frequency domain using a Rhode and Schwarz FS300 spectrum analyzer and a simple coupler designed for the task. The noise was measured in the frequency range 0–30 MHz. The indoor supply voltage in South Africa is 230 V 50 Hz. Some of the results obtained are shown in Figures 1 and 2 below. We observe from Figure 1 that the noise captured in room 501 is less impulsive than that captured in the other two rooms (Figure 2). Actually, much of the noise in this room can be simply classified as background noise, whose average level is approximately -49 dBm. Room 501 is laboratory that was not in use at the time of the measurements. It only had one active PC and several florescent bulbs. All other equipment housed in the room were dormant during the measurement period. On the contrary, we see that the noise captured in the electronic workshop is the most impulsive. This is due to the fact that this room has a lot of activities going on including drilling operations and fabrication of PCBs. The room also houses several computers and soldering equipment. In all the noise measured, we see, for example in Figures 1 and 2 that the noise level between 0–10 MHz is the highest and this can be attributed to the fact that most of radio broadcast interference is prevalent in this band.

Figure 3 shows the probability density function (pdf) of the measured noise spectra. From this figure, we see that the much of the noise captured has a low power spectral density (PSD), and therefore can be classified as background noise; noise that is always present in the system either due to narrowband disturbances or colored background noise (thermal noise), which emanates from sources with low power but cannot be easily traced to their specific origin.

Additionally, we see that strong impulses are also captured, and despite the fact that their probability of occurrence is low, they are very intense when they occur. These impulses are sources

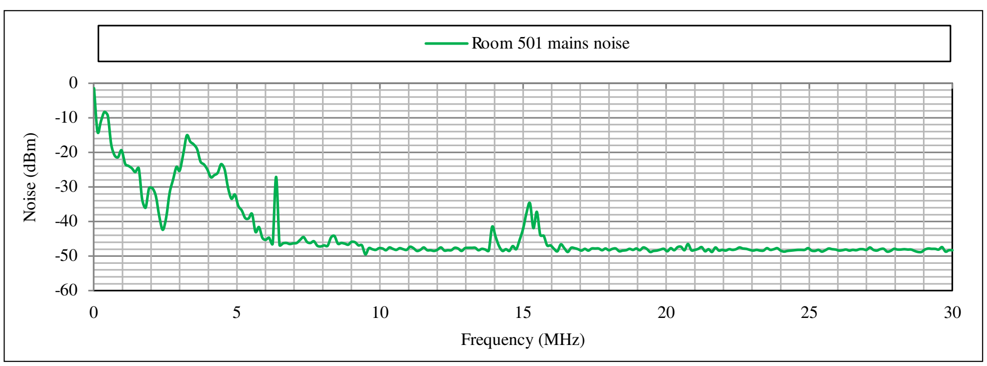

Figure 1: Noise spectrum captured in a room 501.

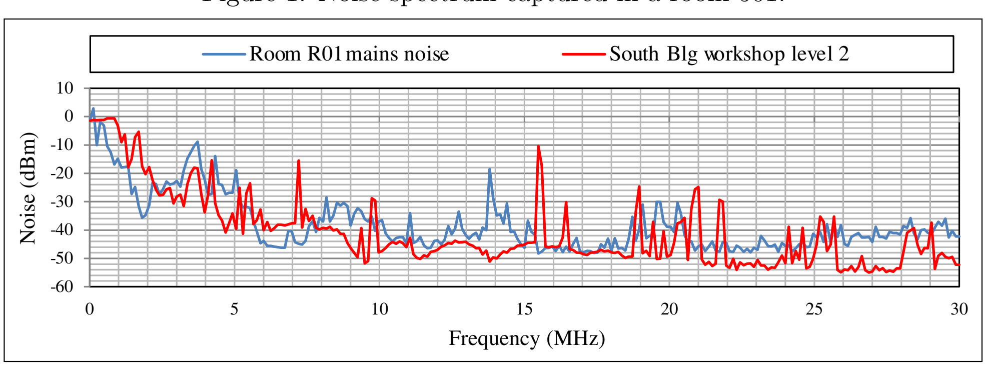

Figure 2: Noise captured in an electronic workshop and a microwave lab (R01).

of high noise power, and when they occur, they can wipe out data streams resulting in deteriorated system performance. The difference between the lowest and highest noise values recorded is about 60 dB. This result closely ties with observations in [7, 8], where we see that the impulsive noise can be as high as 50 dB above the background noise. A simple classification of the strength of the noise captured has been derived and is shown in Table 1.

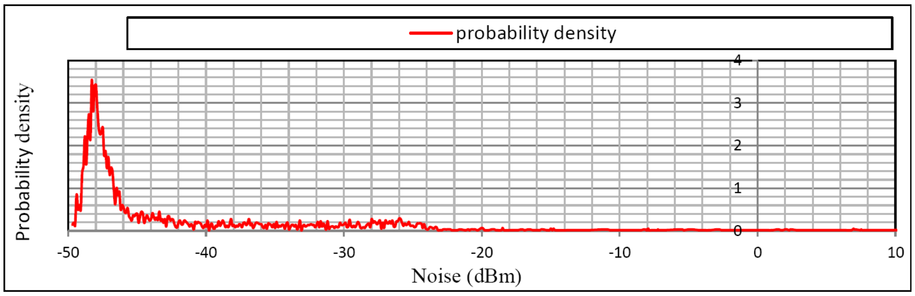

Figure 3: Probability density function of the measured noise characteristics.

Table 1: Measured noise classification.

Strength, n, dBm	Noise category
$-50 \leq n \leq -45$	Background noise
$-45 < n \leq -25$	Weak impulses
$n < -25$	Strong impulses

4. NOISE MODELING

From Figures 1, 2, and 3, we observe that the measured noise distribution is non-white, non-Gaussian, and more importantly, it is heavily tailed and skewed. The stable family of distributions is best suited to model the measured pdf shown in Figure 3 since they possess a heavy tailed characteristic (controlled by the characteristic exponent) and are also able to model any case of symmetry or asymmetry, which is controlled by the symmetry parameter. However, since the pdf of the stable distribution does not exist in closed form, we have to estimate the four parameters that define its characteristic function. Several methods exist in literature that can be used to estimate the parameters; for example, the Koutrouvelis' method [17], Dumouchel method [18] and Paulson et al. technique [19]. However, only Koutrouvelis' method is adopted in this paper due to its superior performance compared to earlier techniques, like those in [18, 19].

Koutrouvelis' method is a Fourier based method that estimates the stable distribution parameters by regressing the empirical characteristic function (ECF) onto the parameterized characteristic function. The ECF estimation interval is stored in a look-up table, and is chosen based on the initial parameter estimates and the size of the sample. Then, the regression procedure is repeated with an interval on the ECF based on the new estimates of the parameters. The entire process is repeatedly carried out until a predefined criterion of convergence is satisfied, typically a set number of iterations. This procedure is quite tasking in terms of the computational intensity but with the current supercomputing capabilities, it is not much of a problem. The method proposed by Koutrouvelis is easier to implement and faster than earlier methods; for example those proposed in [18, 19]. This regression type estimation method is based on an observation concerning the characteristic function. First, the following is implied from (1), [17]:

$$\log\left(-\log|\phi(t)|^2\right) = \log\left(c^\alpha\right) + \alpha\log|t| \tag{2}$$

where $c^\alpha = \gamma$, the scale parameter, and the real and imaginary parts of $\phi(t)$, $\mathrm{Re}\phi(t)$ and $\mathrm{Im}\phi(t)$, respectively, for $\alpha \neq 1$, are:

$$\mathrm{Re}\phi(t) = \exp\left(-|ct|^\alpha\right)\cdot\cos\left[\delta t - |ct|^\alpha\beta\mathrm{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right] \tag{3}$$

$$\mathrm{Im}\phi(t) = \exp\left(-|ct|^\alpha\right)\cdot\sin\left[\delta t - |ct|^\alpha\beta\mathrm{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right] \tag{4}$$

Apart from principal values, from Equations (3) and (4), we deduce the following:

$$\arctan\left(\mathrm{Im}\phi(t)/\mathrm{Re}\phi(t)\right) = \delta t - \beta c^{\alpha}\tan\left(\frac{\pi\alpha}{2}\right)\mathrm{sgn}(t)|t|^{\alpha} \tag{5}$$

Parameters c and α, which control Equation (2), are obtained by regressing $y = \log(-\log|\phi_n(t)|^2)$ on $\omega = \log|t|$ in the model given below:

$$y_k = \mu + \alpha\omega_k + \varepsilon_k, \quad k = 1, 2\ldots, K, \tag{6}$$

where $(t_k;\ k = 1, 2\ldots, K)$ is an appropriate set of real numbers, $\mu = \log(2c^{\alpha})$, and ε_k denotes an error term. After determining the fixed values of c and α, β and δ are obtained from (5), by regressing $z = g_n(u) + \pi k_n(u)$ on u and $\mathrm{sgn}(u)|u|^{\alpha}$ in the model:

$$z_l = \delta u_1 - \beta c^{\alpha}\tan\left(\frac{\pi\alpha}{2}\right)\mathrm{sgn}(u_l)|u_l|^{\alpha} + \eta_l, \quad l = 1, 2, \ldots, L \tag{7}$$

where η_l denotes the error term and $(\mu_l;\ l = 1, 2\ldots, L)$ is an appropriate set of real numbers, integer $k_n(u)$ is introduced to take care of any possible nonprincipal branches of the arctan function, and $g_n(u) = \arctan(\mathrm{Im}\phi(t)/\mathrm{Re}\phi(t))$. The reader is referred to [17] for more details on this method. The estimates obtained for the four parameters of the measured noise spectrum distribution are shown in Table 2.

Table 2: Noise stable distribution parameters.

	Characteristic exponent (α)	Symmetry Index (β)	Scale parameter (γ)	Location parameter (δ)
Parameter values	1.72	1	4.29	−45.69

From the estimated parameters, a stable distribution noise model was fitted. This model is shown in Figure 4 below. From this figure, we see that the fitted Alpha stable model approximates the measured statistics satisfactorily. The model approximates the location of the measured maximum density ($\delta = 45.69$ dBm) very well as well as the left skewness of the measured statistics (which is maximum as per the value of the symmetry parameter obtained, that is, $\beta = 1$).

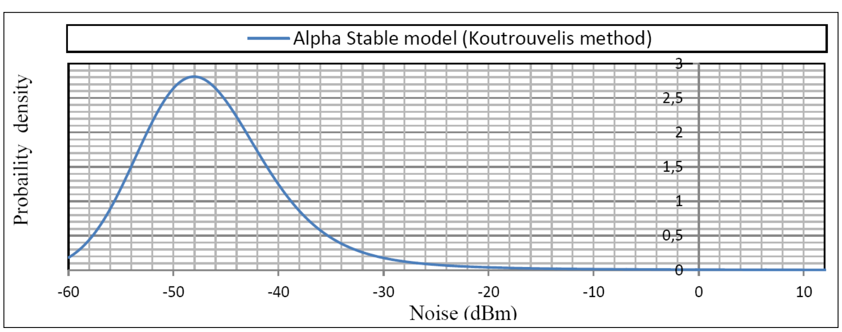

Figure 4: Alpha stable PLC noise model.

Also, we see that the heavy tailed (long tailed) characteristic of the measured data is very well captured by the fitted model. Thus the model is a good approximation of the measured noise data. However, the sharp peaked/sloped nature of the measured noise statistics, as seen in Figure 3, makes it very difficult to fit the data parametrically. This is an aspect of the noise data distribution that needs further investigation. More noise measurements are required to validate the results obtained and the approach used.

5. CONCLUSION AND FUTURE WORK

Noise in communication networks is undoubtedly the most severe source of performance degradation in virtually all wired and wireless systems. In this paper, a framework for modeling the noise characteristics in indoor power lines using alpha stable models has been introduced. The measured statistics are seen to conform to most of the observations in other studies carried out by previous authors in the same area. We see that strong impulsive noise has a much lower probability of occurrence but is characterized by high power as compared to the background noise, and vice versa. The impulsive noise can be as high as one million times above the average background noise as seen from our results, although other studies have found it to go as high as one hundred thousand times. The results obtained here form a basis for an alternative approach in the modeling and characterization of noise in indoor broadband PLC networks. Possible avenues for future research would be to carry out more noise measurements to ascertain the validity of the results obtained here, and even employ alternative methods to model similar noise in PLC networks.

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