# Moving Radar Target Detection Using an Improved OFDM Chirp Waveform Scheme

**Jiahua Zhu**, **Pengzheng Lei**, **Chongyi Fan**, **Xiaotao Huang**, **and Zhimin Zhou**
College of Electronic Science and Engineering
National University of Defense Technology, Changsha, Hunan 410073, China

**Abstract**— Orthogonal frequency division multiplexing (OFDM) chirp waveform has almost constant modulus, large time-bandwidth and good performance on correlation. It is a novel waveform scheme based on the OFDM principle and the linear frequency modulation (LFM) waveform. In this paper, the OFDM chirp waveform generation model is improved to the condition with arbitrary numbers of orthogonal basic signals, and the detection performance of this waveform is also analyzed. It is proved that the OFDM chirp waveform performs better than the traditional OFDM and LFM waveforms on moving target detection using the energy detector (ED) and the matched filter detector (MFD).

## 1. INTRODUCTION

Waveform diversity for moving target detection is always a challenging problem in radar field. Linear frequency modulation (LFM) waveform is one of the most usually used schemes for target detection, since it has the following qualities [1–3]. Firstly, it has constant envelope in time domain, which leads to a high efficiency of the transmitter modules of a phased-array antenna. Secondly, its spectrum approximates to rectangular shape with the increase of time-bandwidth product, which could maximize the spectral efficiency and signal-to-noise ratio (SNR). Thirdly, it can further exploit the linear frequency-time characteristics for easier signal processing. On the other hand, the orthogonal frequency division multiplexing (OFDM) waveform is recently introduced to radar field since it can solve the range-Doppler coupling problem caused by LFM waveform. Due to its orthogonal character in frequency domain [4, 5], the OFDM waveform could accomplish the transmission and reception in multi-channels by just one antenna and obtain much more information from targets, which means this scheme has the potential to save the cost as well as enhance the target detection performance. But unfortunately, OFDM waveform faces a drawback that its time and frequency synchronization is crucial to ensure subcarrier orthogonality [6, 7].

In order to solve the aforementioned problems, a novel waveform design scheme — the OFDM chirp waveform, which combines the OFDM principle with the LFM waveform, was proposed by Dr. Wen-Qin Wang [8, 9] and Dr. Jung-Hyo Kim [1, 10] respectively at the year 2012 and they all illustrated it has almost constant modulus, large time-bandwidth and good performance on correlation. Noticed that Dr. Kim's method only suitable for designing two orthogonal signals, Dr. Ming-Yang Zhou [7] improved the method to three orthogonal signals, and proved its better performance on the ambiguity function than the method with only two orthogonal signals.

However, the previous researches mainly discuss the processing techniques of multiple-input multiple-output (MIMO) synthetic aperture radar (SAR) systems. Few achievement about target detection based on OFDM chirp waveform was published before. Since the OFDM chirp waveform combined the orthogonality of subcarriers and intrinsic characteristics of traditional LFM waveform [1], it could be promising to have better performance than both the LFM and the OFDM waveform on moving target detection.

In this paper, the OFDM chirp waveform generation model demonstrated in [1] and [7] is extended to arbitrary numbers of orthogonal signals in theory. Moreover, the OFDM chirp waveform is proved to have good target detection performance, which is better than the traditional OFDM and chirp waveforms. The remainder sections of this paper are organized as follows. Section 2 makes a brief introduction of the OFDM chirp waveform modulation and extends its waveform generation model to arbitrary numbers of orthogonal basic signals. Based on the energy detector (ED) and the matched filter detector (MFD), Section 3 discusses the target detection performance of OFDM chirp waveform in detail. Then, some simulations and comparison analyses are given in Section 4, proving that the OFDM chirp waveform performs better than LFM and OFDM waveform on moving target detection. Finally, the paper is concluded in Section 5.

## 2. SIGNAL DESCRIPTION AND MODELING

According to the researches made by Dr. Jung-Hyo Kim and Dr. Ming-Yang Zhou, the orthogonality of the waveforms is independent on the types of input sequences [1], since they used the orthogonality of discrete frequency, like orthogonal subcarriers. On the basis of this quality, they built a three orthogonal basic signals model based on OFDM chirp waveform. But actually, this model could be further improved to the condition with arbitrary numbers of orthogonal basic signals.

Assume that the input sequence is $S[\bar{p}]$ with $N$ discrete spectral components in spectrum, which are separated by $M\Delta f$ as shown in Fig. 1. Interleave $(M-1)N$ zeros in the input sequence $S[\bar{p}]$ and we can get $\mathbf{S}_1[p]$ in Fig. 1. Then, we could shift $\mathbf{S}_1[p]$ by $\Delta f, 2\Delta f, \ldots, (M-1)\Delta f$ to obtain $\mathbf{S}_2[p], \mathbf{S}_3[p], \ldots, \mathbf{S}_M[p]$ respectively. The OFDM modulation could be obtained by transforming those data sequences into the time domain through the $MN$ point inverse discrete Fourier transform (IDFT). Therefore, the $M$ OFDM modulators have the same number of orthogonal subcarrier, and we could get $M$ OFDM modulators with each orthogonal subcarrier that are mutually shifted by $\Delta f$. Notice that although both modulators contain $MN$ subcarriers, there are only $N$ useful subcarriers that carry the input data.

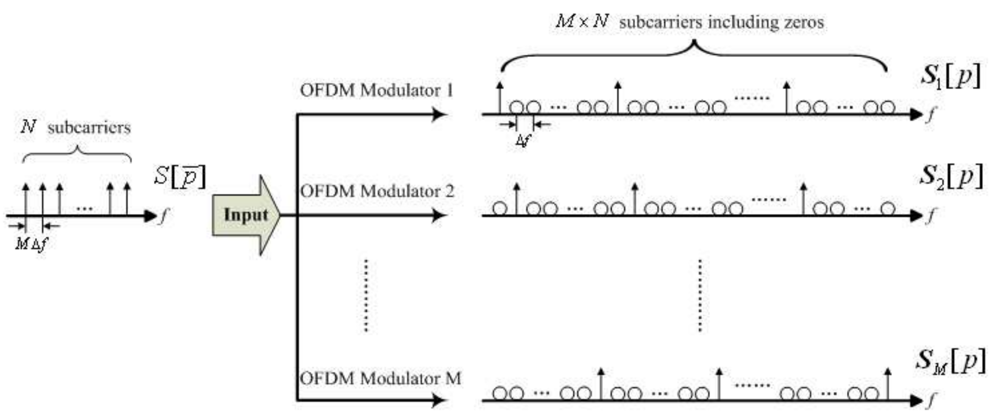


Figure 1: OFDM chirp waveform generation scheme.

In Fig. 1, the arrows represent subcarriers and the circles represent insert zeros. We first use an LFM signal spectrum as the input of each OFDM modulator, which could be given as

$$S[\bar{p}] = \mathcal{F}\{s[n]\} = \mathcal{F}\{\exp[j\pi k(nT_s)^2]\} \tag{1}$$

where $s[n]$ is the $N$-point discrete time samples of a complex LFM signal, $k$ denotes the chirp rate, $T_s$ means the sampling interval and $\mathcal{F}\{\cdot\}$ is the Fourier transform operator. Then, we can generate $M$ input data sequence with zeros interleaved as follows using $S[\bar{p}]$.

$$\mathbf{S}_1[p] = [S[0], 0, 0, \ldots, 0, 0, S[1], 0, 0, \ldots, 0, 0, \ldots, S[N-1], 0, 0, \ldots, 0, 0] \tag{2}$$

$$\mathbf{S}_2[p] = [0, S[0], 0, 0, \ldots, 0, 0, S[1], 0, 0, \ldots, 0, \ldots, 0, S[N-1], 0, 0, \ldots, 0] \tag{3}$$

$$\mathbf{S}_M[p] = [0, 0, \ldots, 0, 0, S[0], 0, 0, \ldots, 0, 0, S[1], \ldots, 0, 0, \ldots, 0, 0, S[N-1]] \tag{4}$$

where $p = 0, 1, \ldots, MN-1$. Then using the method aforementioned, we can express $s_1[n]$ as a repetition of $s[n]$ over the length of $MN$.

$$s_1[n] = s[n]\mathrm{rect}\left[\frac{n}{N}\right] + s[n-N]\mathrm{rect}\left[\frac{n-N}{N}\right] + \ldots + s[n-(M-1)N]\mathrm{rect}\left[\frac{n-(M-1)N}{N}\right] \tag{5}$$

In the same way, the other $M-1$ signals could be described through a phase shift

$$s_2[n] = s_1[n]\exp\left(jn2\pi\frac{1}{MN}\right) \tag{6}$$

$$s_3[n] = s_1[n]\exp\left(jn2\pi\frac{2}{MN}\right) \tag{7}$$

$$s_M[n] = s_1[n]\exp\left(jn2\pi\frac{M-1}{MN}\right) \tag{8}$$

By introducing $t = nT_s$ and $\Delta f = 1/(MNT_s)$, they can also be given in continuous time domain

$$s_1(t) = s(t)\mathrm{rect}\left(\frac{t}{T_P}\right) + s(t-T_p)\mathrm{rect}\left(\frac{t-T_P}{T_P}\right) + \ldots + s(t-(M-1)T_p)\mathrm{rect}\left(\frac{t-(M-1)T_P}{T_P}\right) \quad (9)$$

$$s_2(t) = s_1(t)\exp(j2\pi\Delta f t) \quad (10)$$

$$s_3(t) = s_1(t)\exp(2\cdot j2\pi\Delta f t) \quad (11)$$

$$s_M(t) = s_1(t)\exp((M-1)\cdot j2\pi\Delta f t) \quad (12)$$

where $T_P$ is the chirp duration and

$$s(t) = \exp\left(j\pi k t^2\right) \quad (13)$$

Thus, the model of OFDM chirp waveform with arbitrary numbers of orthogonal basic signals is deduced in this section. However, with the increase of $M$, the subcarrier numbers of each OFDM modulator would correspondingly increase $M$ times. Although it could bring a higher processing precision, it also aggravates the burden of computing devices and needs higher cost. Without loss of generality, we then use $M = 3$ as an example in next sections to discuss its target detection performance considering the computation burden and processing precision.

## 3. TARGET DETECTION

There are two versions of the target detection problem. The first one we refer to as the ideal scenario, when the target channel response $\mathbf{h}_t$ is assumed known. In the other one called realistic scenario, the target channel response $\mathbf{h}_t$ is unknown. Although unrealistic, the ideal scenario provides straightforward bounds on the detection performance achievable by the realistic scenario [11].

In this paper, two detectors are used to analyze the detection performance under the ideal scenario:

(1) MFD, which is the optimal detector when no clutter is presented in the scene;
(2) ED, which is the generalized likelihood ratio test (GLRT) for conventional detection.

Given a simple radar detection scene with one moving point target and background white Gaussian noise, so the detection problem is equivalent to the binary hypothesis problem as follows:

$$\begin{cases} \mathcal{H}_0: & \mathbf{y} = \mathbf{w} \\ \mathcal{H}_1: & \mathbf{y} = \mathbf{s}\otimes\mathbf{h}_t + \mathbf{w} \end{cases} \quad (14)$$

where $\mathbf{y}$ and $\mathbf{s}$ is the receiving echo wave and the transmitting signal of each OFDM modulator respectively, "$\otimes$" is the convolution operator, $\mathbf{w} \sim C\mathcal{N}(0, \sigma^2)$ is the complex white Gaussian noise and $\sigma^2$ is its variance. Besides, all parameters in (14) are expressed in vector forms.

### 3.1. MFD

According to the Neyman-Pearson (NP) criterion [12], the data probability density functions (PDFs) $p(\mathbf{y}|\mathcal{H}_0)$ and $p(\mathbf{y}|\mathcal{H}_1)$ under the conditions $\mathcal{H}_0$ and $\mathcal{H}_1$ can be given as

$$p(\mathbf{y}|\mathcal{H}_0) = \frac{1}{(\pi\sigma^2)^{MN}}\exp\left[-\frac{1}{\sigma^2}\mathbf{y}^H\mathbf{y}\right] \quad (15)$$

$$p(\mathbf{y}|\mathcal{H}_1) = \frac{1}{(\pi\sigma^2)^{MN}}\exp\left[-\frac{1}{\sigma^2}(\mathbf{y}-\mathbf{s})^H(\mathbf{y}-\mathbf{s})\right] \quad (16)$$

After some algebraic manipulations we have

$$L(\mathbf{y}) = \frac{p(\mathbf{y}|H_1)}{p(\mathbf{y}|H_0)} > \gamma \Rightarrow \mathrm{Re}\left(\mathbf{s}^H\mathbf{y}\right) > \gamma' \quad (17)$$

where $\gamma' = \sqrt{\sigma^2\varepsilon/2}Q^{-1}(P_{\mathrm{FA,MF}})$, $P_{D,\mathrm{MF}} = Q(Q^{-1}(P_{\mathrm{FA,MF}}) - \sqrt{2\varepsilon/\sigma^2})$, $\varepsilon = \sum_{n=0}^{MN-1}|\mathbf{s}[n]|^2$, $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}t^2)dt$, $P_{\mathrm{FA}}$ and $P_D$ represent the false-alarm and detection probabilities, respectively. Besides, $\varepsilon$ and $Q(x)$ denote the energy of $\mathbf{s}$ and the right tail probability function, respectively.

### 3.2. ED

The likelihood ratio of ED should compute the total received signal energy across the operating frequency range [13], thus the likelihood ratio function is shown as

$$L(\mathbf{y}) = \frac{p(\mathbf{y}|H_1)}{p(\mathbf{y}|H_0)} = \frac{1/\sigma^2 \sum_{n=0}^{MN-1} |\mathbf{y}[n]|^2}{1/\sigma^2 \sum_{n=0}^{MN-1} |\mathbf{w}[n]|^2} > \gamma \tag{18}$$

To calculate the $P_{\mathrm{FA,ED}}$ and $P_{D,\mathrm{ED}}$, the Monte Carlo method [12] is introduced since the analytical expression of test statistics $L(\mathbf{y})$ cannot be obtained.

## 4. SIMULATIONS AND PERFORMANCE ANALYSIS

In this section, some simulation results are put forward to illustrate the target detection performance of the OFDM chirp waveform, which are compared with the counterparts of conventional OFDM and LFM waveforms.

As it is mentioned before, one moving point target was set in the radar detection scene with background white Gaussian noise. The original distance between the target and radar is 1000 m and the target has a constant velocity $v_t = 10\,\mathrm{m/s}$ towards the radar. In order to compare the performance of those three kinds of waveforms, we first set some global parameters for each waveform: The operating frequency $f_c = 1\,\mathrm{GHz}$; signal bandwidth $B = 200\,\mathrm{MHz}$; pulse number is 1000; pulse repetition frequency is 500 Hz; discrete time samples of each subcarriers or the whole signal is $N = f_s \cdot T_P$. To satisfy the orthogonal quality of the OFDM, $T_P$ should be no less than $M/B$ and here we set $T_P = 8.5\,\mu\mathrm{s}$, $M = 3$; $\Delta f$ should be no less than $1/T_P$ and here we set $\Delta f = 66.7\,\mathrm{MHz}$. For LFM signal $k = B/T_P$. Besides, the energy of each waveform is normalized to 1 and we control the noise add to them is at the same energy level. Then, the receiver operating characteristic (ROC) curves of three waveforms using two kinds of detectors when the SNR is $-30\,\mathrm{dB}$ are shown in Fig. 2.

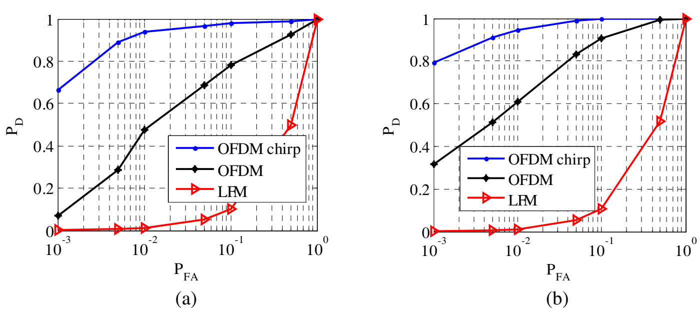


Figure 2: ROC curves of the OFDM chirp, OFDM and LFM waveforms using (a) ED; (b) MFD.

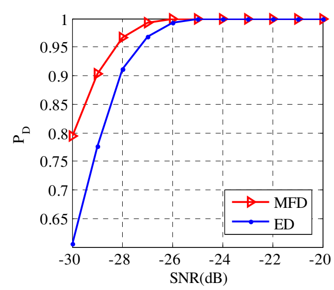


Figure 3: Detection probability versus SNR for the OFDM chirp waveform using two detectors.

From Fig. 2 it is obvious to see that the OFDM chirp waveform performs better than the other two waveforms. Exactly, the $P_D$ of the OFDM chirp waveform using ED is about 0.63 when $P_{\mathrm{FA}} = 10^{-3}$, while the counterparts of OFDM and LFM waveforms are only 0.07 and 0.01, respectively. Additionally, the $P_D$ of the OFDM chirp waveform using MFD is about 0.79, while the counterparts of OFDM and LFM waveforms are only 0.31 and 0.02, respectively. As we know, it is apparent that the OFDM chirp waveform has much more orthogonal subcarriers than LFM waveform, thus more target information could be exploited for target detection algorithms and higher detection probability could be obtained. Besides, using chirp signals as the OFDM symbols can suppress the cross-correlation components better than the conventional OFDM waveform, since LFM signal is less sensitive to the time and frequency synchronization. Therefore, the OFDM chirp waveform provides higher detection probability in comparison with that of OFDM waveform.

The performance of two detectors in different SNRs when $P_{\mathrm{FA}} = 10^{-3}$, using the OFDM chirp waveform as an example, is further compared as Fig. 3 illustrated. It proves that as the optimal detector in our detection scene, the MFD has better performance than the ED. This is because the MFD uses phase information of signals and maximizes the receiving SNR through convolution. The results also show that without clutter is presented, the MFD has about 1.5 dB gain over the ED at $P_D = 0.8$. However, it is stressed that the MFD has a lower degree of stability and robustness than the ED [13], which means its performance may drop off significantly in dense clutter environment. This work will be discussed in our future researches.

## 5. CONCLUSION

In this paper, we improve a novel OFDM chirp waveform generation model to the condition with arbitrary numbers of orthogonal basic signals and introduce this waveform scheme to moving target detection. Then we use the MFD and ED respectively to evaluate the target performance of the waveforms. The simulation results prove that the OFDM chirp waveform performs better than the traditional OFDM and LFM waveforms on moving target detection, and the MFD shows a significant performance gain over the ED when clutter is not exist.

In subsequent work, the detection scene will be extended to more complex conditions such as dense clutter and multipath to further research the performance of OFDM chirp waveform.

## ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 60972121 and in part by the University Innovation Foundation for Postgraduate of National University of Defense Technology (NUDT) under Grant S140402.

## REFERENCES

1. Kim, J. H., M. Younis, A. Moreira, and W. Wiesbeck, "A novel OFDM chirp waveform scheme for use of multiple transmitters in SAR," *IEEE Geoscience and Remote Sensing Letters*, Vol. 10, No. 3, 568–572, May 2013.
2. Curlander, J. C. and R. N. McDonough, *Synthetic Aperture Radar Systems and Signal Processing*, Wiley, U.S., 1991.
3. Richards, M. A., *Fundamentals of Radar Signal Processing*, McGraw-Hill Companies, U.S., 2005.
4. Le, N. P., L. C. Tran, and F. Safaei, "Energy-efficiency analysis of per-subcarrier antenna selection with peak-power reduction in MIMO-OFDM wireless systems," *International Journal of Antennas and Propagation*, Vol. 2014, 1–13, 2014.
5. Zhu, J. H., C. Y. Fan, X. T. Huang, and Z. M. Zhou, "Ground moving target detection using airborne radar based on OFDM waveform," *14th International Radar Symposium*, 929–934, Dresden, Germany, Jun. 19–21, 2013.
6. Morelli, M., C. J. Kuo, and M. Pun, "Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review," *Proceedings of the IEEE*, Vol. 95, No. 7, 1394–1427, Jul. 2007.
7. Zhou, M. Y., H. Y. Liu, W. Z. Liu, and Y. M. Gu, "An improved OFDM chirp waveform design," *2013 IEEE 5th International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications (MAPE)*, 478–482, Chengdu, China, Oct. 29–31, 2013.
8. Wang, W. Q., "OFDM waveform diversity design for MIMO SAR imaging," *2012 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, 2090–2093, Munich, Germany, Jul. 22–27, 2012.
9. Wang, W. Q., "Mitigating range ambiguities in high PRF SAR with OFDM waveform diversity," *IEEE Geoscience and Remote Sensing Letters*, Vol. 10, No. 1, 101–105, Jan. 2013.
10. Kim, J. H., "Multiple-input multiple-output synthetic aperture radar for multimidal operation," Ph. D. Dissertation, Karlsruhe Institute of Technology, Germany, Dec. 2011.
11. Moura, J. M. F. and Y. W. Jin, "Detection by time reversal: Single antenna," *IEEE Transactions on Signal Processing*, Vol. 55, No. 1, 187–201, Jan. 2007.
12. Kay, M. S., *Fundamentals of Statistical Signal Processing, Volume II: Detection Theory*, Prentice Hall PTR, U.S., 1998.
13. Jin, Y. W., J. M. F. Moura, Y. Jiang, D. D. Stancil, and A. G. Cepni, "Time reversal detection in clutter: Additional experimental results," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 47, No. 1, 140–154, Jan. 2011.