Computation and Analysis of Terahertz Wire Grid Polarizer Self-resonance Using Transmission Line Theory

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Abstract— A terahertz metal wire-grid polarizer on a low-loss dielectric (crystalline quartz) substrate was simulated using High Frequency Structure Simulator (HFSS) and modeled with equivalent-circuit theory. The transmittance of the polarizer was calculated for electromagnetic radiation at normal incidence from 100 to 1000 GHz for both s-polarization (perpendicular to the grid) and p-polarization (parallel to the grid). The phase difference in S_{21} between the HFSS input and output ports was calculated and plotted versus frequency and versus fill-factors of 0.3, 0.5, 0.7, 0.9, and 0.95 for both polarizations. Analysis of the S-polarized S_{21} shows that the phase-angle differences are all negative. This is interpreted as the phase of the current in the wire grid leading the phase of the voltage across the gaps, and thus the electric field across the gaps. This behavior was seen for all fill-factors. Therefore, a capacitive effect is exhibited by the wire-grid polarizer for S-polarization. Analysis of the P-polarization results show the phase angle differences are positive for fill-factors less than or equal to 0.9 and negative at the fill-factor of 0.95. Thus, the wire-grid current lags the gap voltage for P-polarization at fill-factors ≤ 0.90 (an inductive effect) while wire current leads the gap voltage at fill-factor = 0.95 (a capacitive effect). This behavior is the spatial analog of a parallel LC circuit.

1. INTRODUCTION

Wire-grid polarizers have been fundamental passive optical devices for over one hundred years. They date back to 1888 when they were used by Heinrich Hertz to investigate radio waves [1]. Since then, wire-grid polarizers have been used throughout the electromagnetic spectrum from radio frequencies through the ultraviolet [2–5]. The wire-grid polarizer in this study was designed for the millimeter wave and terahertz (THz) regions. It consists of a regular periodic array of fine aluminum (Al) strips of width w_s and gaps w_g pattered on an insulating polycarbonate substrate. The objective was to study the characteristics of this basic design as the fill-factor, $FF = w_s/(w_s+w_g)$, was varied from low to high values. Full-wave numerical simulations were carried out using High Frequency Structure Simulator® (HFSS). The frequency response of the magnitude of the transmittance has previously been reported [6, 7]. Extraordinary optical transmission and exceptionally high extinction ratios that increased with increasing fill-factor were numerically predicted and experimentally observed. Here, the response of the phase angle of the transmittance will be presented and modeled.

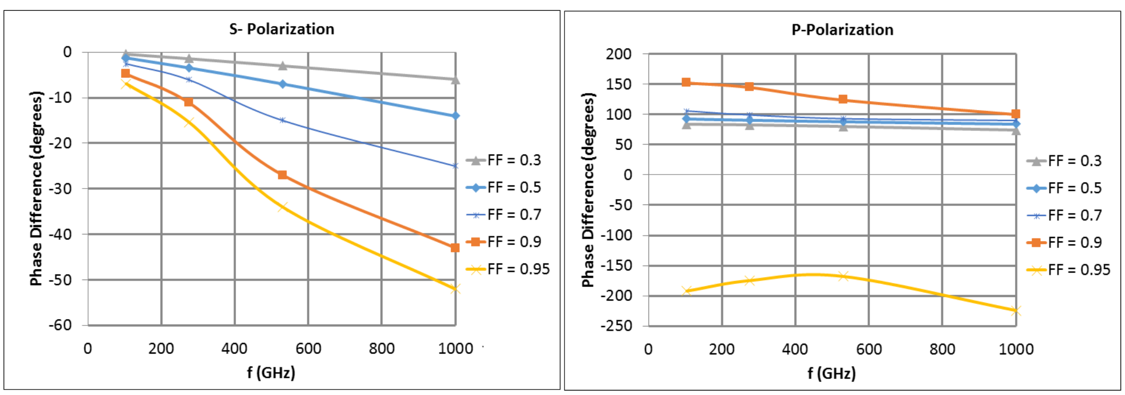
2. POLARIZER DESIGN AND SIMULATION

Based upon previous studies [6, 7], the period of the THz wire-grid polarizer was 40 μm, the thickness of the metal strips was 0.2 μm, and the thickness of the polycarbonate substrate was 1.5 mm. The numerical simulations were carried out for incident radiation polarized perpendicular (S) and parallel (P) to the strip axis at frequencies from 100 GHz to 1000 GHz in steps of 1 GHz. The fill-factor was varied by changing the strip width from 0 to 39.9 μm ($FF = 0$ to 0.9975). The simulated incident radiation was always at normal incidence to the substrate. To improve the accuracy of the simulations, the electrical properties of the Al strips where represented by the Drude model for AC conductivity [8]. The absorptive losses in the polycarbonate were modeled with a separate look-up table based upon a frequency-dependent dielectric function derived from transmission data taken with a THz photomixing spectrometer.

The phase angles calculated in this study are plotted in terms of the phase-angle difference between the input and output waveports with the wire strips and substrate in place, minus the phase difference between the input and output waveports with only the substrate in place (i.e., $FF = 0.0$). This subtracts out the uninteresting evolutionary phase effect of the substrate and allows us to focus on the effect of the wire grid only. As plotted in Figure 1, the phase angle difference is seen to decrease monotonically with increasing frequency for S polarization at all fill factors, and do the same for P polarization except at the highest FF (0.95), which displays a peaked behavior. For low fill-factors, the decrease is linear. For higher fill-factors, nonlinear dispersion emerges. The negative phase angle differences may be interpreted as the currents generated on the wire strips leading the voltages generated across the gaps (and hence the electric fields) in phase. This type of phase relationship is similar to that seen in capacitors.

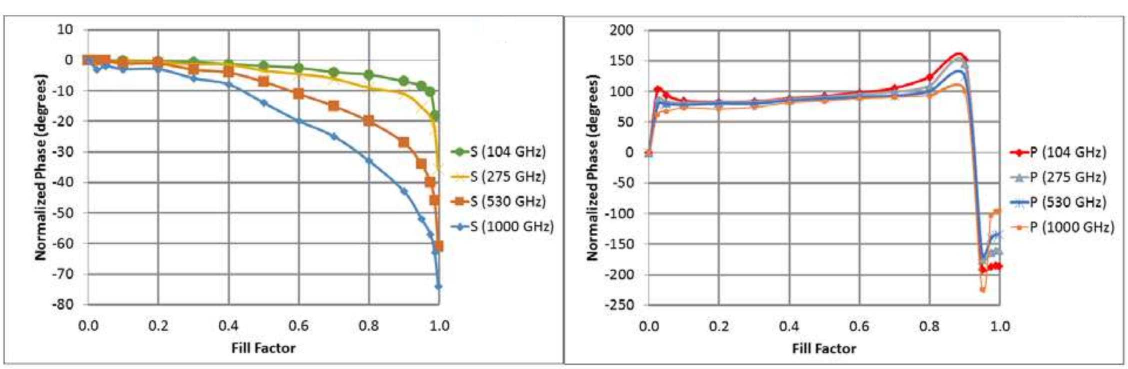
Physically, this makes sense for S-polarization as the surface currents flow across the wires and along the gap walls. For P-polarization, the phase differences are all positive for fill-factors less than 90%. The positive phase angle differences indicate that the currents on the wire strips lag the voltages (and the electric fields) across the gaps. This type of phase relationship is similar to that seen in inductors. At very high fill-factors, some threshold is crossed for P-polarization. A sign change occurs and the phase difference becomes negative (capacitive). This behavior is reminiscent of that seen in a parallel LC tank circuit with fill-factor taking the place of frequency. In a parallel LC circuit, the response is inductive below the resonant frequency and capacitive above it. In the wire-grid polarizer, the response to a P-polarized wave is inductive below some critical fill-factor and capacitive above it.

The behavior of the phase angle difference versus fill-factor, at four spot frequencies (104, 275, 530, and 1000 GHz) was calculated and plotted. For S-polarization, Figure 2(a) shows the phase angle difference increases in the negative sense with increasing fill-factor, a capacitive effect. This can be explained by the fact that increasing fill-factor means decreasing gap size and hence increasing capacitance. The plot also shows the phase angle difference increasing in the negative sense with increasing frequency in agreement with Figure 1(a). For P-polarization, Figure 2(b) shows the phase angle difference initially increasing in the positive sense with increasing fill-factor, an inductive effect. Then, between fill-factors of 0.9 and 0.95, a large negative phase shift occurs. This behavior is in agreement with Figure 1(b). Apparently, the critical fill-factor is somewhere between



(a) (b)

Figure 1: Calculated phase angle differences for various fill-factors versus frequency, (a) S-polarization, (b) P-polarization.



(a) (b)

Figure 2: Calculated phase angle differences versus fill-factor for 104 GHz, 275 GHz, 530 GHz, and 1000 GHz, (a) S-polarization, (b) P-polarization.

0.90 and 0.95. The impedance switches from inductive to capacitive within this range. Again, this is behavior analogous to that seen in an LC tank circuit.

3. ANALYSIS OF SIMULATION RESULTS

Based upon the behavior of the phase angle difference for P-polarization, the wire-grid self-resonance frequency was estimated for various fill-factors. The wire grid was modeled as an array of slotlines each having characteristic impedance Z_0 and effective dielectric constant $\varepsilon_{\mathit{eff}}$ on a 1.5-mm-thick polycarbonate substrate having $\varepsilon_r = 2.89$. These were calculated from analytic expressions [9] that were previously confirmed to be good approximations to more precise results from the method of moments [10]. From these, the specific capacitance C' and inductance L' were calculated from the transmission-line relations, $C' = (Z_0 v)^{-1} = (\varepsilon_{\mathit{eff}})^{1/2}/(Z_0 c)$, and $L' = Z_0(\varepsilon_{\mathit{eff}})^{1/2}/c$, where c is the speed of light in vacuum. The resulting curves for C' and L' are plotted in Figures 3(a) and (b) in the interesting range of fill factors between 0.85 and 1.0.

From C' and L', we can calculate corresponding sheet values knowing the grid period a, as $C_S = C' \cdot a$, and $L_S = L' \cdot a$. The resulting *distributed* resonant frequency is then given by

$$f_0 = \frac{1}{2\pi\sqrt{L_S C_S}} \tag{1}$$

For example, at the threshold fill factor of 0.90 in Figure 2(b), Figure 3(a) displays $C' \approx 5.0 \times 10^{-13}\,\mathrm{F/cm}$, and 3(b) displays $L' \approx 6 \times 10^{-9}\,\mathrm{H/cm}$. These become $C_S = 2.0\,\mathrm{fF/sq}$ and $L_S = 24\,\mathrm{pH/sq}$ upon substitution of the fixed grid period $a = 40\,\mu\mathrm{m}$ (0.004 cm), leading to a resonant frequency from (1) of $f_0 = 726\,\mathrm{GHz}$. This is well within the simulated frequency range and supports the existence of a distributed resonance effect.

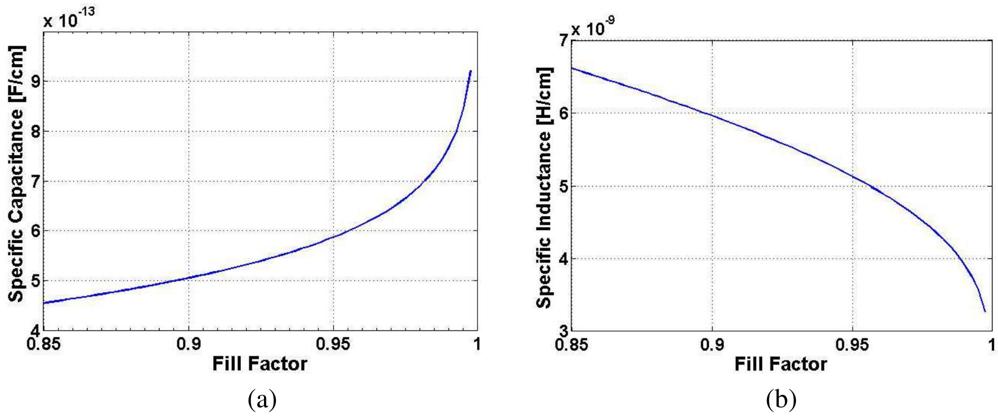


Figure 3: (a) Specific gap capacitance versus fill-factor, (b) specific sheet inductance for fill-factors, for fill-factors from 0.85 to 1.0.

4. CONCLUSION

Thus, full-wave numerical simulations of THz radiation incident on a wire-grid polarizer predict some interesting electromagnetic phenomena. For S-polarization, the wire-grid acts as an impedance with a capacitive reactance to the incident wave. For P-polarization, the wire-grid acts as an impedance with an inductive reactance at low fill-factors. At high fill-factors (somewhere between 0.90 and 0.95), the reactance changes abruptly to capacitive. This behavior is analogous to that seen in a lumped LC tank circuit and could possibly be exploited for use in plasmonic-like circuitry.

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