# Power Line Noise Measurements and Statistical Modelling in the Time Domain

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**Abstract**— The power grid provides a readily available medium for communication purposes. This means that the power grid is the cheapest communication medium since no new cabling is required. However, the power grid was designed for the delivery of power and not for communication applications. As such no considerations have been made in the design of power networks to accommodate communication services. However, with the application of current digital signal processing techniques, the power network can be adapted to support communication services. This adaptation requires a full understanding of the power network characteristics, which are noise, distortion and channel effects. These power network characteristics are studied through measurements, modelling and characterization. The results obtained through such studies are then utilized in the redesign of conventional receiver structures and modulation schemes so as to adequately suit them for powerline communication applications. In this paper, we present time domain power line noise measurement results, and then apply a statistical technique for the characterization and modelling of the time domain noise amplitudes in typical indoor power networks. The results obtained are validated through error analysis. The models developed are vital in the optimization of communication applications through the power network.

## 1. INTRODUCTION

The power network is the most expansive network in the world. Hence, given its ubiquitous nature, the power network has been attractive to communication engineers who have been busy optimizing this network for use as a communication channel. The optimization of this network for use as a communication channel is not a trivial task due to the fact that power lines were designed for the delivery of power and not communication services. The problems associated with the power network as a communication channel have mainly to do with the fact that the channel characteristics are very dynamic due to the many branches of the network, electrical equipment connected to the network and the varying cable diameters that are found in different sections of the grid. The various electrical equipment connected to the power network are a source of noise especially during switch ON/OFF. And again the different branching nodes in the network form reflection points for the signal which results in signal fading. The power network channel therefore suffers from two main impairments; reflections and temporal variations, and noise. Given the structure of a typical power network, communication signals suffer from many reflections resulting in a complex echo scenario and in turn frequency-dependent attenuation occurs [1–5].

Powerline communications (PLC) technology enables the double utilization of the power grid to deliver both communication and power services. Smart grid technologies that PLC supports include remote meter reading, load management and control, and power equipment monitoring, among a plethora of other applications. Most of the smart grid services utilize narrowband PLC technologies which operate in low frequencies and the data rates are low. Narrowband PLC technologies operate between the frequencies 3 kHz–500 kHz, with provision of data rates of up to a few kbps for single carrier cases and up to 800 kbps in multicarrier cases. However, in the recent times, PLC research has been mainly concentrated on broadband PLC technologies. This kind of shift in focus has been motivated by the possible utilization of the PLC channel (power grid) to deliver higher data rate broadband content like internet and video services. Broadband PLC or high frequency PLC operates between 1.8 MHz–250 MHz and provides data rates of up to 200 Mbps for home area networks. This PLC technology is also widely used in the networking of in-home equipment and delivery of high definition television services [1, 2, 5].

Noise in power line networks is completely different from the classical additive white Gaussian noise (AWGN) found in many other communication systems. The noise in power line networks is additive, non-white and non-Gaussian. There are three different classes of noise in power line networks, namely: colored background noise, narrowband interference and appliance (impulsive) noise. The colored background noise together with the narrowband interference constitutes what is generally referred to as background noise. On the other hand, appliance noise manifests itself in three forms: impulsive noise that is periodic and synchronous with the mains cycle, periodic noise that is asynchronous with the mains cycle and finally non-periodic noise which is sporadic (random). Impulsive noise is the most severe type of noise and is the cause of channel degradation in many PLC systems. Impulsive noise levels can be up to 50 dB above the average background noise. PLC noise has also been studied by various authors both in frequency and time domain. Some of the time domain noise measurements, analysis and modelling have been carried out by several authors among them Chan and Donaldson [6], Degardin et al. [7], and Zimmermann and Dostert [8]. These authors have emphasized on noise properties like pulse width, arrival time and inter-arrival times, and therefore not much work has been done to characterize the noise amplitudes. In this paper, we introduce an alternative modelling approach for the measured instantaneous noise amplitudes in the time domain for indoor PLC networks. We statistically fit the instantaneous noise amplitudes using the kernel density method and then develop a stable distribution model using the kernel model as the reference model. The measurements are carried out in different rooms in the Department of Electrical, Electronic and Computer Engineering building, University of KwaZulu-Natal, Howard College, Durban, South Africa.

## 2. THE KERNEL DENSITY METHOD AND STABLE DISTRIBUTIONS

The kernel density method belongs to a class of density estimation tools that are non-parametric. That is to say that the estimates obtained using the kernel density methods are purely data-based with no particular form of the probability density function (pdf) being assumed. Thus, the kernel method produces the least error when compared to parametric methods which model data sets via a fixed set of parameters whose final outlook cannot be altered. The kernel density is defined by the following equation [9]:

$$f\left(x\right)=\frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x-X_i}{h}\right) \tag{1}$$

where $n$ is the number of data points, $h$ is the smoothing parameter, $X_i$ is the $i$th data point and $K(\cdot)$ is the kernel function. On the other hand, the stable distribution is parametric and is defined by its characteristic function, $\emptyset(t)$, given as [10–13]:

$$\emptyset\left(t\right)=\begin{cases}\exp\left\{i\delta t-\gamma^{\alpha}\left|t\right|^{\alpha}\left[1-i\beta\text{sgn}\left(t\right)\tan\frac{\pi\alpha}{2}\right]\right\}, & \alpha\neq1\\ \exp\left\{i\delta t-\gamma\left|t\right|\left[1+i\beta\frac{2}{\pi}\text{sgn}(t)\ln\left|t\right|\right]\right\}, & \alpha=1\end{cases} \tag{2}$$

where, $\text{sgn}(t)$ is the signum function and; $-\infty<\delta<\infty$, $\gamma>0$, $0<\alpha\leq2$, $-1\leq\beta\leq1$. $\delta$ is the location parameter, $\gamma$ is the dispersion parameter, $\alpha$ is the characteristic exponent and $\beta$ is the symmetry parameter. Thus, the stable distribution is very flexible modelling tool that is controlled by the four parameters defined in Equation (2). Also, this distribution models the tail probabilities very well, an aspect that is taken care of by the characteristic exponent. Additionally, this distribution can model any degree of skewness that any data distribution may possess, an aspect that is taken care of by the symmetry parameter. The dispersion parameter models the data spread around the mean or median value. Finally, the location parameter determines the shift in the data peak. However, the closed form expression for the pdf of this distribution does not exist, and therefore it is entirely defined by the characteristic function. Hence, the four parameters that define the characteristic function have to be determined for any model. Different techniques have been proposed for the estimation of these four parameters, among them, the techniques based on extreme value theory, fractional lower moments and order statistics by Tsihrintzis and Nikias [10], the Fourier based techniques of Kogon and Williams [11] and Koutrouvelis [12], and the percentile method by McCulloch [13]. Above all, the alpha stable distribution, being a Gaussian distribution general case, is a very attractive tool for modelling phenomena that is impulsive, because it satisfies the stability property and the generalized central limit theorem.

## 3. MEASUREMENT METHODOLOGY, RESULTS AND DISCUSSION

Comprehensive time domain noise measurements were carried out in the Department of Electrical, Electronic and Computer Engineering building in different offices and laboratory rooms. The noise was measured using a Tektronix TDS 2024B four channel digital storage oscilloscope. A locally designed coupler was used to isolate the measurement equipment from the ac mains and filter the noise from the power lines. The noise measurement campaign was carried out for a period of eight

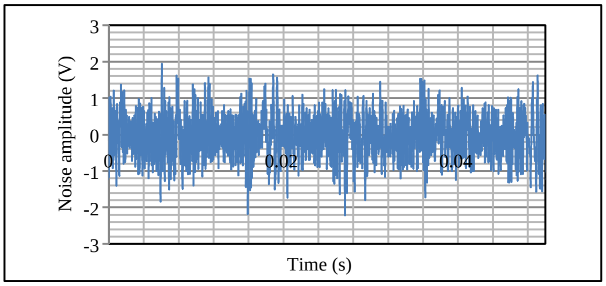


Figure 1: Noise waveform captured in an office.

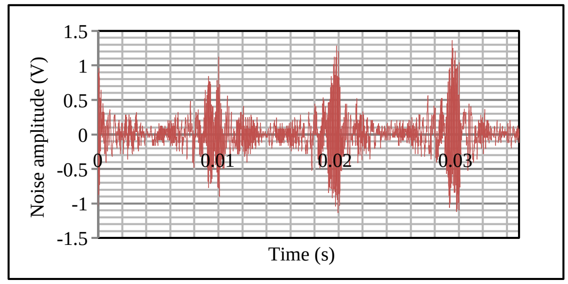


Figure 2: Noise waveform captured in a laboratory.

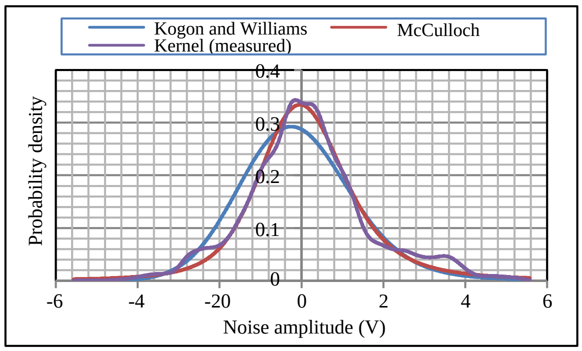


Figure 3: Alpha stable noise models.

months. Additional noise measurements were carried out in an indoor test bed environment with a 230 V 50 Hz uninterruptible power supply (UPS) used to isolate the test bed from the ac mains. Some of the noise waveform measured in different rooms are shown in Figures 1 and 2 above. Figure 1 shows the noise waveform obtained in an office room that has a printer, several desktop computers and air conditioners. Figure 2, on the other hand, shows the noise waveform measured in a laboratory which houses an anechoic chamber, and several desktop computers, lighting tubes. From Figure 1, we see that the noise measured at the office is very sporadic and does not display any periodic or synchronism features with respect to the ac mains. However, the noise measured in Figure 2 displays periodic features such that its period is half that of the ac mains, that is $T = 0.01$ sec, or $F = 100$ Hz.

From the numerous instantaneous noise amplitudes obtained, the measured pdf was developed using the kernel density technique. The pdf is shown in Figure 3, together with other parametric models developed. From this figure, we see that the pdf tails are heavier than those of the Gaussian distribution. These heavy tails are a confirmation that the measured noise contains both background and impulsive components. The heavy tails are a representative of the impulsive component of the measured noise. Such heavy tails can be modeled using the alpha stable distribution, since the heaviness of the tails can be sufficiently modeled with the right value of the characteristic exponent.

## 4. NOISE CHARACTERIZATION AND MODELING

From the heavy-tailed characteristic of the measured noise distribution, and for the reasons explained in Section 2 above, we have chosen the alpha stable distribution to develop a PLC noise model for indoor power networks. Since the closed form of the alpha stable pdf does not exist, but the closed form of the characteristic function exists, we have to determine the four characteristic parameters that define the PLC noise model. Two methods are used in this paper to estimate the four parameters of the noise model. These are: the Fourier based method of Kogon and Williams [11] and the quantile (percentile) method of McCulloch [13]. McCulloch's method has less computational intensity when compared with empirical characteristic function (ECF) based Fourier methods. For this method, five pre-determined sample percentiles, that are accompanied by tables, for $\beta$ $(-1, 1)$ and $\alpha$ $(0.6, 2.0)$, are used to consistently estimate the four parameters of the stable distribution. First we estimate the characteristic exponent $\alpha$ and the symmetry parameter

$\beta$ using the percentiles of the empirical distribution. We define:

$$\upsilon_\alpha = \frac{x_{0.95} - x_{0.05}}{x_{0.75} - x_{0.25}}, \quad \upsilon_\beta = \frac{x_{0.95} + x_{0.05} - 2x_{0.25}}{x_{0.95} - x_{0.05}} \tag{3}$$

where $x_p$ is the empirical distribution $p$th percentile, having $\upsilon_\alpha = \phi_1(\alpha, \beta)$ and $\upsilon_\beta = \phi_1(\alpha, \beta)$, or by way of inversion, $\alpha = \psi_1(\upsilon_\alpha, \upsilon_\beta)$ and $\beta = \psi_2(\upsilon_\alpha, \upsilon_\beta)$. Also,

$$\psi_1(\upsilon_\alpha, \upsilon_\beta) = \psi_1(\upsilon_\alpha, -\upsilon_\beta) \tag{4}$$

Bi-linear interpolation to estimate $\alpha$ and $\beta$ is then carried out using tables developed for the functions $\psi_1(\cdot)$ and $\psi_2(\cdot)$ for different values of $\upsilon_\alpha$ and $\upsilon_\beta$. Similarly, the location parameter $\delta$ and the dispersion parameter $\gamma$ are estimated using the corresponding tabulated functions and the previous estimates for $\alpha$ and $\beta$.

Kogon and Williams method is a modified Fourier based technique that eliminates the need for the cumbersome iterations required in Koutrouvelis technique. This is achieved by making a choice of the interval on the ECF which is robust over a wide range sample sizes on a given parameter space. The method's performance is at par with Koutrouvelis method and at the same time reduces the amount of computation intensity and the need for an interval look-up table. Here the parameter estimator is derived by considering the log-characteristic function, given by:

$$\Psi(\omega) = -|c\omega|^\alpha + j\left(\delta\omega + |c\omega|^\alpha \beta \frac{\omega}{|\omega|} \tan\frac{\pi\alpha}{2}\right) \tag{5}$$

where $c$ is the dispersion (spread) parameter. In this case, since $\alpha$ is never unity in practice, the real and imaginary components of the log-characteristic function are given by:

$$\mathrm{Re}[\Psi(\omega)] = -|c\omega|^\alpha = -c^\alpha |\omega|^\alpha \tag{6}$$

$$\mathrm{Im}[\Psi(\omega)] = \left(\delta\omega + |c\omega|^\alpha \beta \frac{\omega}{|\omega|} \tan\frac{\pi\alpha}{2}\right) \tag{7}$$

Again, a further manipulation of the real portion of the log-characteristic can be done by taking another logarithm as follows:

$$\mathrm{In}\{-\mathrm{Re}[\Psi(\omega)]\} = \alpha \mathrm{In}|\omega| + \alpha \mathrm{In} c \tag{8}$$

The estimation of the parameters of a stable distribution of any signal that is random, $x(n)$ is done by computing the ECF and then fitting it to the parameterized characteristic function in a least squares (LS) sense. In this case, we define the ECF as follows:

$$\Phi'(\omega) = \frac{1}{N}\sum_{n=1}^{N} e^{j\omega x(n)} = \frac{1}{N}\sum_{n=1}^{N} \cos[\omega x(n)] + j\sin[\omega x(n)] \tag{9}$$

where $N$ is the noise $x(n)$ sample size. Since, $\Psi'(\omega) = \mathrm{In}\Phi'(\omega)$, both the log-ECF real and imaginary parts are the obtained from the ECF as follows:

$$\mathrm{Re}\left[\Psi'(\omega)\right] = \mathrm{In}\left|\Phi'(\omega)\right| \tag{10}$$

$$\mathrm{Im}\left[\Psi'(\omega)\right] = \mathrm{Arctan}\left(\frac{\mathrm{Im}[\Phi'(\omega)]}{\mathrm{Re}[\Phi'(\omega)]}\right) \tag{11}$$

where Arctan$(\cdot)$ is the principal value of the arctangent function. Making use of Equation (8), we then obtain the estimates of the characteristic component $\alpha'$ and the spread parameter $c'$ with a LS fit on the following equation:

$$\mathrm{In}\left\{-\mathrm{Re}\left[\Psi'(\omega)\right]\right\} = \alpha' \mathrm{In}|\omega| + \alpha' \mathrm{In} c' \tag{12}$$

$\alpha'$ is the slope of $\mathrm{In}\{-\mathrm{Re}[\Psi'(\omega)]\}$ versus $\mathrm{In}|\omega|$ plot and $\alpha'\mathrm{In}c'$ is the $y$ intercept of the same plot. Once $\alpha'$ and $c'$ have been determined, we then estimate $\beta'$, the index of symmetry and $\delta'$, the location parameter by computing the LS fit to Equation (7), for the log-ECF imaginary part as:

$$\mathrm{Im}\left[\Psi'(\omega)\right] = \left(\delta'\omega + \beta' \left|c'\omega\right|^{\alpha'} \frac{\omega}{|\omega|} \tan\frac{\pi\alpha'}{2}\right) \tag{13}$$

Table 1: Alpha stable model parameters.

| Method | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| McCulloch | 1.469 | 0.238 | 0.844 | 0.231 |
| Kogon and Williams | 1.859 | 1 | 0.964 | −0.002 |

Table 2: Model ISE errors.

| Mode | ISE |
|---|---|
| McCulloch | 0.172 |
| Kogon and Williams | 0.831 |

The alpha stable model parameters obtained using the two methods are shown in Table 1, and the models are shown in Figure 3.

From Table 1, we see that the values of the symmetry parameter for the two methods are very different from each other. The value of this parameter for Kogon and Williams's method implies that the model is fully skewed to the left. However, from Figure 3, we see that the model matches very well with both the measured model and that obtained using McCulloch's method. This is to say that as $\alpha \to 2$, the value of $\beta$ doesn't matter. In this case we see that $\alpha = 1.86$, which is close to 2. The other parameters are not very different from each other for the two models. However, to determine which of the two models is optimum, we validate the models through error analysis. The error criterion employed is the integral of square error (ISE). The errors obtained are shown in Table 2. From Table 2, we see that the errors obtained are small, which means that the models obtained are a good fit to the measured data. Also, the least error is obtained for the McCulloch method, from which we conclude that this model is the best fit for the measured noise characteristics.

## 5. CONCLUSION AND FUTURE WORK

Communication networks suffer from degraded channel performance due to noise. This noise manifests itself in different forms. In this paper, we have developed a time domain noise model for the PLC network based on channel measurements. This model is necessary for the optimal design of receiver structures and modulation schemes for power line communications. Further measurements and alternative modeling approaches should be explored to ascertain the validity of the outcomes in the current research work.

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