On the Connection between Jones Matrix and Sinclair Matrix

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Abstract— The Sinclair matrix and the Jones matrix describe the scattering behavior of radar targets using different coordinate systems. To analyze the physical scattering mechanisms occurring at a target, the Sinclair matrix can be decomposed with a coneigenvalue decomposition. This decomposition is mathematically challenging and still not fully understood. In contrast the Jones matrix can be decomposed with an eigenvalue decomposition, which can be easily implemented. In the literature a simple connection between Jones and Sinclair matrices was derived which gives rise to the question if the coneigenvalue decomposition of the Sinclair matrix can be replaced by an eigenvalue decomposition of a suitable Jones matrix by exploiting this connection. Within this paper it will be proven that, depending on the type of connection, this is either not possible or requires high mathematical effort. Therefore the coneigenvalue decomposition remains the method of choice for the aforementioned analysis of scattering mechanisms.

1. INTRODUCTION

The polarimetric scattering behavior of radar targets can be described with two different 2×2 matrices: The Sinclair matrix $\underline{\mathbf{S}}$ which specifies the backscattering of a target and which is commonly used by polarimetric monostatic radar systems, and the Jones matrix $\underline{\mathbf{J}}$ which characterizes the forward scattering of a target and originates from optics [1, 2]. Both matrices are suitable for the description of radar targets in the monostatic case, but rely on different coordinate systems. The Sinclair matrix is based on the backward scattering alignment (BSA) convention, where the wave vector of the incident wave is parallel to the wave vector of the scattered wave. In this convention the polarization of incident and scattered wave are described by the antenna polarization, which is defined as the polarization of a wave radiated by an antenna [3]. Therefore the BSA convention is also termed to be *antenna-oriented*. The Jones matrix is based on the forward scattering alignment (FSA) convention, where the wave vector of the incident wave is antiparallel to the wave vector of the scattered wave. Since the wave vectors are oriented along the propagation direction of the waves, the FSA convention is termed to be *wave-oriented*.

The decomposition of the Sinclair matrix into so-called Huynen-Euler parameters [4] has gained some attention in recent publications [5, 6]. This decomposition is based on a diagonalization of the Sinclair matrix [7, 8]

$$\underline{\mathbf{\Sigma}} = \underline{\mathbf{U}}^T \, \underline{\mathbf{S}} \, \underline{\mathbf{U}} \tag{1}$$

where $\underline{\mathbf{U}}$ is a unitary matrix describing a change of the polarization basis of the incident and scattered electromagnetic wave. This allows to characterize the target by fundamental scattering mechanisms independent from the polarization of the incident and scattered wave. To these mechanisms belong single and double reflections as well as circularly polarizing and linearly polarizing structures. Since all scatterers influence in which directions and at which frequencies the scattering occurs they give a physical interpretation to the radar-cross section signatures of radar targets. This interpretation is made possible by the Huynen-Euler parameters, which are used to differentiate between the mechanisms. As can be seen from (**??**) the approach requires a coneigenvalue decomposition [9]. Up to now, little research has been conducted in the field of consimilarity transformations and coneigenvalues. Some of the implications of these decompositions are therefore still unknown [7, 8]. Moreover, there exist only few decompositions of $\underline{\mathbf{S}}$ into coneigenvalues which are either mathematically cumbersome [5] or miss out some of the Huynen-Euler parameters [9, 10].

In contrast the Jones matrix can be decomposed into [7, 8]

$$\underline{\mathbf{\Lambda}} = \underline{\mathbf{U}}^H \, \underline{\mathbf{J}} \, \underline{\mathbf{U}}. \tag{2}$$

Since $\underline{\mathbf{U}}$ is a unitary matrix this equation describes an eigenvalue decomposition. The mathematics of eigenvalues and similarity transformations are known and well understood [9]. Moreover, there are various methods available for eigenvalue decomposition which are both numerically stable and computationally efficient [11].

In [12] a misleading, but simple connection between the Jones matrix $\underline{\mathbf{J}}$ and the Sinclair matrix $\underline{\mathbf{S}}$ was established:

$$\underline{\mathbf{J}} = \underline{\mathbf{T}}\,\underline{\mathbf{S}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underline{\mathbf{S}} \tag{3}$$

This equation was derived for linear polarizations only. Unfortunately in most publications it is not emphasized that this connection is limited to linear polarizations [1, 2], leading the reader to the wrong assumption that (3) is generally valid. If this would be the case, the decomposition of Sinclair matrices could be extremely simplified by transforming them to Jones matrices and performing an eigenvalue decomposition afterwards:

$$\underline{\mathbf{\Lambda}} = \underline{\mathbf{U}}^H\,\underline{\mathbf{J}}\,\underline{\mathbf{U}} = \underline{\mathbf{U}}^H\,\underline{\mathbf{T}}\,\underline{\mathbf{S}}\,\underline{\mathbf{U}}. \tag{4}$$

For clarity, this decomposition will be called *simplified decomposition* in the following. Within this paper it will be shown that the simplified decomposition will not deliver valid results if the transformation matrix $\underline{\mathbf{T}}$ in (3) is used. Moreover, it will be shown that this is also the case for any other transformation matrix $\underline{\mathbf{T}}$ which does not change with $\underline{\mathbf{S}}$. Due to this it will become clear that coneigenvalue decompositions are still the only possibility to extract Huynen-Euler parameters from a Sinclair matrix and that additional research in the field of consimilarity transformations and coneigenvalues is indespensable.

2. TARGET-INDEPENDENT TRANSFORMATION MATRICES

As described by J. R. Huynen in [4], a coneigenvalue decomposition of the Sinclair matrix $\underline{\mathbf{S}}$ allows to characterize the scattering mechanism occuring at the target independent from the polarization of the incoming and scattered wave. Thus the coneigenvalues of two targets $\underline{\mathbf{S}}_1$, $\underline{\mathbf{S}}_2$ will be identical if the response of both targets is caused by the same scattering mechanism. It should be noted that despite his groundbreaking work in the field of radar polarimetry Huynen did not realize the fundamental difference between coneigenvalues and eigenvalues — a pitfall which was discussed in detail by Lüneburg in [7, 8].

If the simplified decomposition described in Section 1would be valid, then the eigenvalues of two targets $\underline{\mathbf{J}}_1$, $\underline{\mathbf{J}}_2$ must be identical if the response of both targets is caused by the same scattering mechanism. Since two matrices with the same (con)eigenvalues are (con)similar to each other [9], it follows that the Jones matrices $\underline{\mathbf{J}}_1$, $\underline{\mathbf{J}}_2$ of two targets must be similar if the Sinclair matrices $\underline{\mathbf{S}}_1$, $\underline{\mathbf{S}}_2$ of two targets are consimilar. If the simplified decomposition can be applied depends strongly on the transformation matrix $\underline{\mathbf{T}}$: If $\underline{\mathbf{T}}$ is target-independent, which means that $\underline{\mathbf{T}}$ does not change with $\underline{\mathbf{S}}$ and therefore always has the same entries $\underline{t}_{mn}$, then the simplified decomposition is not applicable. This will be shown with the following proof:

Theorem 1. *Let M_2 be the set of all 2×2-matrices. It does not exist a single full-rank matrix $\underline{\mathbf{T}}$ for arbitrary $\underline{\mathbf{S}}_1, \underline{\mathbf{S}}_2 \in M_2$ for which the following property holds: If $\underline{\mathbf{S}}_1$ is consimilar to $\underline{\mathbf{S}}_2$, then $\underline{\mathbf{J}}_1 = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_1$ is similar to $\underline{\mathbf{J}}_2 = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_2$.*

Proof. Assume that there exists a full-rank $\underline{\mathbf{T}}$ which fulfills the stated property. Then one should be able to derive the entries $\underline{t}_{mn}$ of $\underline{\mathbf{T}}$. Therefore the following matrices are used:

$$\underline{\mathbf{S}}_{1,A} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix} \qquad \underline{\mathbf{S}}_{2,A} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \underline{\mathbf{U}}_{A} = \begin{bmatrix} +j & 0 \\ 0 & -j \end{bmatrix}$$

$$\underline{\mathbf{S}}_{1,B} = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \underline{\mathbf{S}}_{2,B} = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix} \qquad \underline{\mathbf{U}}_{B} = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$$

$$\underline{\mathbf{S}}_{1,C} = \begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix} \qquad \underline{\mathbf{S}}_{2,C} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad \underline{\mathbf{U}}_{C} = \begin{bmatrix} 0 & +j \\ +j & 0 \end{bmatrix}$$

$$\underline{\mathbf{S}}_{1,D} = \begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix} \qquad \underline{\mathbf{S}}_{2,D} = \begin{bmatrix} 0 & +j \\ +j & 0 \end{bmatrix} \qquad \underline{\mathbf{U}}_{D} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix} e^{+j\frac{\pi}{4}}$$

$\underline{\mathbf{S}}_{1,i}$ is consimilar to $\underline{\mathbf{S}}_{2,i}$ over the unitary matrix $\underline{\mathbf{U}}_i$ for all $i \in \{A, B, C, D\}$.

- $\underline{\mathbf{J}}_{1,A} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{1,A}$ and $\underline{\mathbf{J}}_{2,A} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{2,A}$ yields

$$\underline{\mathbf{J}}_{1,A} = \begin{bmatrix} \underline{t}_{11} & \underline{t}_{12} \\ \underline{t}_{21} & \underline{t}_{22} \end{bmatrix} \qquad \underline{\mathbf{J}}_{2,A} = \begin{bmatrix} -\underline{t}_{11} & -\underline{t}_{12} \\ -\underline{t}_{21} & -\underline{t}_{22} \end{bmatrix}$$

  If $\underline{\mathbf{S}}_{1,A}$ is consimilar to $\underline{\mathbf{S}}_{2,A}$ then $\underline{\mathbf{J}}_{1,A}$ must be similar to $\underline{\mathbf{J}}_{2,A}$. Thus $\mathrm{tr}(\underline{\mathbf{J}}_{1,A}) = \mathrm{tr}(\underline{\mathbf{J}}_{2,A})$. From this follows $\underline{t}_{11} = -\underline{t}_{22}$.

- $\underline{\mathbf{J}}_{1,B} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{1,B}$ and $\underline{\mathbf{J}}_{2,B} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{2,B}$ yields

$$\underline{\mathbf{J}}_{1,B} = \begin{bmatrix} \underline{t}_{11} & -\underline{t}_{12} \\ \underline{t}_{21} & -\underline{t}_{22} \end{bmatrix} \qquad \underline{\mathbf{J}}_{2,B} = \begin{bmatrix} -\underline{t}_{11} & \underline{t}_{12} \\ -\underline{t}_{21} & \underline{t}_{22} \end{bmatrix}$$

  If $\underline{\mathbf{S}}_{1,B}$ is consimilar to $\underline{\mathbf{S}}_{2,B}$ then $\underline{\mathbf{J}}_{1,B}$ must be similar to $\underline{\mathbf{J}}_{2,B}$. Thus $\mathrm{tr}(\underline{\mathbf{J}}_{1,B}) = \mathrm{tr}(\underline{\mathbf{J}}_{2,B})$. From this follows $\underline{t}_{11} = \underline{t}_{22}$. Together with $\underline{t}_{11} = -\underline{t}_{22}$ this results in $\underline{t}_{11} = \underline{t}_{22} = 0$.

- $\underline{\mathbf{J}}_{1,C} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{1,C}$ and $\underline{\mathbf{J}}_{2,C} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{2,C}$ yields

$$\underline{\mathbf{J}}_{1,C} = \begin{bmatrix} \underline{t}_{12} & 0 \\ 0 & \underline{t}_{21} \end{bmatrix} \qquad \underline{\mathbf{J}}_{2,C} = \begin{bmatrix} -\underline{t}_{12} & 0 \\ 0 & -\underline{t}_{21} \end{bmatrix}$$

  If $\underline{\mathbf{S}}_{1,C}$ is consimilar to $\underline{\mathbf{S}}_{2,C}$ then $\underline{\mathbf{J}}_{1,C}$ must be similar to $\underline{\mathbf{J}}_{2,C}$. Since $\underline{\mathbf{J}}_{1,C}$, $\underline{\mathbf{J}}_{2,C}$ are diagonal matrices, the diagonal entries are eigenvectors. Since both matrices must have the same eigenvectors, it holds either $\underline{t}_{12} = \underline{t}_{21} = 0$ or $\underline{t}_{12} = -\underline{t}_{21}$, depending on the order of the eigenvectors.

- If $\underline{t}_{12} = -\underline{t}_{21}$, then $\underline{\mathbf{J}}_{1,D} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{1,D}$ and $\underline{\mathbf{J}}_{2,D} = \underline{\mathbf{T}}\,\underline{\mathbf{S}}_{2,D}$ yields

$$\underline{\mathbf{J}}_{1,D} = \begin{bmatrix} \underline{t}_{12} & 0 \\ 0 & -\underline{t}_{12} \end{bmatrix} \qquad \underline{\mathbf{J}}_{2,D} = \begin{bmatrix} j\underline{t}_{12} & 0 \\ 0 & -j\underline{t}_{21} \end{bmatrix}$$

  If $\underline{\mathbf{S}}_{1,D}$ is consimilar to $\underline{\mathbf{S}}_{2,D}$ then $\underline{\mathbf{J}}_{1,D}$ must be similar to $\underline{\mathbf{J}}_{2,D}$. The diagonal entries of $\underline{\mathbf{J}}_{1,D}$ and $\underline{\mathbf{J}}_{2,D}$ must be the same eigenvectors again. Therefore $\underline{t}_{12} = \underline{t}_{21} = 0$.

From $\underline{t}_{11} = \underline{t}_{12} = \underline{t}_{21} = \underline{t}_{22} = 0$ follows $\mathrm{rank}(\underline{\mathbf{T}}) = 0$, which contradicts the initial assumption. □

It was shown that the simplified decomposition will not succeed if $\underline{\mathbf{T}}$ is full-rank and target-independent. For completeness it should be mentioned that this is also true if $\mathrm{rank}(\underline{\mathbf{T}}) = 1$. This can be shown by describing $\underline{\mathbf{T}}$ with two linearly dependent row vectors $\underline{\mathbf{t}}_1 = \underline{c}_1[\underline{t}_1 \;\; \underline{t}_2]$ and $\underline{\mathbf{t}}_2 = \underline{c}_2[\underline{t}_1 \;\; \underline{t}_2]$ so that $\underline{\mathbf{T}} = [\underline{\mathbf{t}}_1 \;\; \underline{\mathbf{t}}_2]^T$. The proof follows from the matrices $\underline{\mathbf{S}}_{1,A}$, $\underline{\mathbf{S}}_{2,A}$, $\underline{\mathbf{S}}_{1,B}$ and $\underline{\mathbf{S}}_{2,B}$ by reasoning similar to Theorem 1. For the trivial case $\mathrm{rank}(\underline{\mathbf{T}}) = 0$ the eigenvalues of $\underline{\mathbf{J}}$ are always zero and different targets are therefore indistinguishable. Thus the simplified decomposition will not succeed for any target-independent $\underline{\mathbf{T}}$. This also includes the transformation matrix in (2) which was derived in [12].

3. TARGET-DEPENDENT TRANSFORMATION MATRICES

It is possible to give a physical explanation of the results of Section 2. The coneigenvalue decomposition in (1) changes the polarization basis of $\underline{\mathbf{S}}$ in a way which leads to a diagonalization of $\underline{\mathbf{\Sigma}}$ [7, 8]. Since the transformation matrix in (2) was derived for a linear polarization base only [12], this matrix is susceptible to fail for nonlinear polarization bases.

However, if $\underline{\mathbf{T}}$ changes with $\underline{\mathbf{S}}$ and thus depends on the target's properties, the simplified decomposition can be applied. This is shown in the following proof:

Theorem 2. *Let M_2 be the set of all 2×2-matrices. For arbitrary $\underline{\mathbf{S}}_1, \underline{\mathbf{S}}_2 \in M_2$ distinct full-rank matrices $\underline{\mathbf{T}}_1$, $\underline{\mathbf{T}}_2$ can be constructed for which the following property holds: $\underline{\mathbf{J}}_1 = \underline{\mathbf{T}}_1\,\underline{\mathbf{S}}_1$ is similar to $\underline{\mathbf{J}}_2 = \underline{\mathbf{T}}_2\,\underline{\mathbf{S}}_2$ iff $\underline{\mathbf{S}}_1$ is consimilar to $\underline{\mathbf{S}}_2$.*

Proof. In the following all matrices will be denoted using the index $i \in \{1, 2\}$. This results in the six matrices $\underline{\mathbf{S}}_i$, $\underline{\mathbf{T}}_i$ and $\underline{\mathbf{J}}_i$.

- Let there be $\underline{\mathbf{J}}_0 \in M_2$ which is similar to both $\underline{\mathbf{J}}_i$. In this case $\underline{\mathbf{J}}_0 = \underline{\mathbf{U}}_i^H \, \underline{\mathbf{J}}_i \, \underline{\mathbf{U}}_i$ for unitary $\underline{\mathbf{U}}_i$. Due to the transitivity of matrix similarity also both $\underline{\mathbf{J}}_i$ are similar to each other. The matrices $\underline{\mathbf{T}}_i$ can now be defined to $\underline{\mathbf{T}}_i = \underline{\mathbf{U}}_i \, \underline{\mathbf{U}}_i^T$. This yields

$$\begin{aligned} \underline{\mathbf{J}}_0 &= \underline{\mathbf{U}}_i^H \, \underline{\mathbf{J}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^H \, \underline{\mathbf{T}}_i \underline{\mathbf{S}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^H \, \underline{\mathbf{U}}_i \, \underline{\mathbf{U}}_i^T \, \underline{\mathbf{S}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^T \, \underline{\mathbf{S}}_i \, \underline{\mathbf{U}}_i \end{aligned}$$

  Since this is a consimilarity transformation, both $\underline{\mathbf{S}}_i$ are similar to $\underline{\mathbf{J}}_0$. Therefore also both $\underline{\mathbf{S}}_i$ are consimilar to each other.

- Conversely, let there be $\underline{\mathbf{S}}_0 \in M_2$ which is consimilar to both $\underline{\mathbf{S}}_i$. In this case $\underline{\mathbf{S}}_0 = \underline{\mathbf{U}}_i^T \, \underline{\mathbf{S}}_i \, \underline{\mathbf{U}}_i$ for unitary $\underline{\mathbf{U}}_i$. Due to the transitivity of matrix consimilarity also both $\underline{\mathbf{S}}_i$ are consimilar to each other. Moreover, $\underline{\mathbf{S}}_i = \underline{\mathbf{T}}_i^{-1} \underline{\mathbf{J}}_i$ since both $\underline{\mathbf{T}}_i$ have full rank. The matrices $\underline{\mathbf{T}}_i$ can now be defined to $\underline{\mathbf{T}}_i^{-1} = \underline{\mathbf{U}}_i^* \, \underline{\mathbf{U}}_i^H$. This yields

$$\begin{aligned} \underline{\mathbf{S}}_0 &= \underline{\mathbf{U}}_i^T \, \underline{\mathbf{S}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^T \, \underline{\mathbf{T}}_i^{-1} \underline{\mathbf{J}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^T \, \underline{\mathbf{U}}_i^* \, \underline{\mathbf{U}}_i^H \, \underline{\mathbf{J}}_i \, \underline{\mathbf{U}}_i \\ &= \underline{\mathbf{U}}_i^H \, \underline{\mathbf{J}}_i \, \underline{\mathbf{U}}_i \end{aligned}$$

  Since this is a similarity transformation, both $\underline{\mathbf{J}}_i$ are similar to $\underline{\mathbf{S}}_0$. Therefore also both $\underline{\mathbf{J}}_i$ are similar to each other.

Therefore $\underline{\mathbf{J}}_1$ is similar to $\underline{\mathbf{J}}_2$ iff $\underline{\mathbf{S}}_1$ is consimilar to $\underline{\mathbf{S}}_2$. □

Although this approach seems to enable the simplified decomposition in an easy way, the transformation matrices $\underline{\mathbf{T}}_i$ in the proof of Theorem 2 require the knowledge of the matrices $\underline{\mathbf{U}}_i$. These matrices are connected to $\underline{\mathbf{S}}_i$ via a consimilarity transformation to the unknown matrix $\underline{\mathbf{S}}_0$. Therefore the construction of the transformation matrices $\underline{\mathbf{T}}_i$ is again based on consimilarity. Since the simplified decomposition was analyzed to prevent the application of methods from this field, this decomposition does not allow to retrieve the Huynen-Euler parameters in the desired manner.

4. CONCLUSION

Within this paper it was shown that the matrix $\underline{\mathbf{T}}$ constructed in [12] cannot be used for a simplification of the coneigenvalue decomposition of $\underline{\mathbf{S}}$ by eigenvalue decomposition of $\underline{\mathbf{J}} = \underline{\mathbf{T}} \, \underline{\mathbf{S}}$. This is also true for any other matrix $\underline{\mathbf{T}}$ which does not depend on $\underline{\mathbf{S}}$. If $\underline{\mathbf{T}}$ depends on $\underline{\mathbf{S}}$, then the eigenvalue decomposition of $\underline{\mathbf{T}} \, \underline{\mathbf{S}}$ will deliver valid results. However, the proof of Theorem 2 implies that $\underline{\mathbf{T}}$ can only be acquired using consimilarity transformations. Therefore a simplification of the coneigenvalue decomposition is not possible without shifting the underlying mathematical problem to other parts of the decomposition process. Due to this coneigenvalue decompositions still require extensive research to solve the problems related to the decomposition of the Sinclair matrix into Huynen-Euler parameters.

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