Cerenkov Radiation in Presence of Squeezed Electromagnetic Vacuum

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Abstract— In this work we investigate the Cerenkov radiation in the case of motion of a charged particle on a background of the squeezed electromagnetic vacuum. Using the quantum approach (time dependent perturbation theory) we obtain the explicit expression for the Cerenkov radiated power. The analysis of the obtained expression shows that in contrast to the "classical" case (where the Cerenkov radiation is a spontaneous process) the Cerenkov radiation in the squeezed vacuum has an induced character. The expression for radiated power contains two terms, namely the first term corresponds to radiation power, while the second one corresponds to absorption power.

1. INTRODUCTION

Cerenkov effect [1] have a long history but it remains relevant in relation to the different areas of application of such a phenomena (the wide list of publication on this subject takes place, see, e.g., [2–4] and related references). The expression for the radiated power was obtained by solving the equations of classical electrodynamics in the medium. A large number of studies have been devoted to other aspects of the Cerenkov effect. They concern the extension to the case of magnetic media, a detailed analysis of radiation in crystals, the role of boundaries, etc. The classical theory of spontaneous Cerenkov effect is fairly accurate, however the quantum analysis gives a more complete understanding of the phenomenon. Moreover, the quantum approach is most useful when considering the motion of a charged particle in a dielectric medium in the external electromagnetic wave. The radiation in this case has an induced nature and quantum approach gives a better understanding of the physical nature of phenomena.

Creation of powerful sources of electromagnetic radiation initiated extensive research in the area of particle-field interaction. The squeezed states of light [5] play an important role in such researches. These states generalize the well-known class of coherent states and are characterized by the fact that the uncertainty of one of the canonical variables of the electromagnetic field is less than in a conventional coherent state. The squeezed states are important in such areas as optical communication, interferometry, nonlinear optics, the detection of gravitational waves, etc. It should also be noted that the squeezed states are objects of purely quantum nature. So the description of the motion of a charged particle on a background of the squeezed vacuum can be performed using the quantum field theory only. This problem is of great practical interest (new sources of electromagnetic radiation such as free-electron lasers and Cerenkov lasers).

In this work we investigate the Cerenkov radiation in the case of motion of a charged particle on a background of the squeezed electromagnetic vacuum. Due to "purely" quantum nature of the squeezed vacuum we use the quantum approach, namely the time dependent perturbation theory.

2. CERENKOV RADIATION: QUANTUM DESCRIPTION

As is well known the Cerenkov radiation is electromagnetic radiation emitted when a charged particle (such as an electron) passes through a dielectric medium at a speed greater than the phase velocity of light in that medium. There are several approaches that describe this phenomenon, namely, the classical Tamm-Frank approach based on the solution of equations of classical electrodynamics, the Fermi approach based on the effect of radiative attenuation of an electron when it moves in the medium, as well as the of quantum-mechanical approach.

Here we would like to present the quantum-mechanical consideration of the Cerenkov radiation (we use the time-dependent perturbation theory). As is known, the hamiltonian of a charged particle in the electromagnetic field in non-relativistic approach has the form

$$\hat{H} = \frac{1}{2m_e}\left(\hat{\vec{p}} - \frac{e}{c}\vec{A}\right)^2 - e\varphi. \tag{1}$$

Taking the gauge $(\nabla \cdot \vec{A}) = 0$, $\varphi = 0$ we can write

$$\hat{H} = \hat{H}_0 + \hat{V}_{int}, \hat{H}_0 = \frac{\hat{\vec{p}}^2}{2m_e}, \hat{V}_{int} = -\frac{e}{m_e c}\hat{\vec{p}}\hat{\vec{A}}, \tag{2}$$

where $\hat{\vec{A}}$ should be written in the second quantized form (also we neglect the quadratic term)

$$\hat{\vec{A}} = \sum_{\vec{k}\sigma}\sqrt{\frac{2\pi\hbar v_c^2}{\omega_{\vec{k}} V\Sigma(\omega_{\vec{k}})}}\left(\vec{e}_{\vec{k}\sigma}\hat{a}_{\vec{k}\sigma}e^{i\vec{k}\hat{\vec{r}} - i\omega_{\vec{k}}t} + \vec{e}^{\,*}_{\vec{k}\sigma}\hat{a}^{+}_{\vec{k}\sigma}e^{-i\vec{k}\hat{\vec{r}} + i\omega_{\vec{k}}t}\right), \tag{3}$$

where $v_c = c/n(\omega_{\vec{k}})$ is a phase velocity of the light in the medium with refractive index $n(\omega_{\vec{k}})$.

Using (2) we can write the expression for transition amplitude in the form[1]:

$$a_{fi}(t) = -\frac{i}{\hbar}\int_0^t \langle f|\langle 1_{\vec{k}\sigma}|\hat{V}_{int}(t')|0_{\vec{k}\sigma}\rangle|i\rangle \mathrm{d}t' = -\frac{i}{\hbar}\int_0^t \langle f|\hat{V}_{\vec{k}\sigma}(t')|i\rangle \mathrm{d}t',\ \hat{V}_{\vec{k}\sigma}(t') = \langle 1_{\vec{k}\sigma}|\hat{V}_{int}(t')|0_{\vec{k}\sigma}\rangle. \tag{4}$$

The probability of single-photon process is determined by the expressions:

$$w(t) = \sum_{f,\vec{k},\sigma}|a_{fi}(t)|^2 = \frac{1}{\hbar^2}\sum_{\vec{k},\sigma}\int_0^t\int_0^t \langle i|\hat{V}^{+}_{\vec{k}\sigma}(t_1)\hat{V}_{\vec{k}\sigma}(t_2)|i\rangle \mathrm{d}t_1\mathrm{d}t_2, \tag{5}$$

and

$$\hat{V}_{\vec{k}\sigma}(t') = -\frac{e}{m_e c}\sqrt{\frac{2\pi\hbar v_c^2}{\omega_{\vec{k}} V\Sigma(\omega_{\vec{k}})}}e^{-i\vec{k}\hat{\vec{r}} + i\omega_{\vec{k}}t}(\vec{e}^{\,*}_{\vec{k}\sigma}\hat{\vec{p}}). \tag{6}$$

Using the explicit expression for $\hat{V}_{\vec{k}\sigma}(t)$ (6) we can write:

$$w(t) = \left(\frac{e}{\hbar m_e c}\right)^2\sum_{\vec{k},\sigma}\frac{2\pi\hbar v_c^2}{\omega_{\vec{k}} V\Sigma(\omega_{\vec{k}})}\int_0^t\int_0^t \langle i|(\vec{e}_{\vec{k}\sigma}\hat{\vec{p}})e^{i\vec{k}\hat{\vec{r}} - i\omega_{\vec{k}}t_1}e^{-i\vec{k}\hat{\vec{r}} + i\omega_{\vec{k}}t_2}(\vec{e}^{\,*}_{\vec{k}\sigma}\hat{\vec{p}})|i\rangle \mathrm{d}t_1\mathrm{d}t_2. \tag{7}$$

Taking into account the explicit form of $\hat{\vec{r}}$ operator in Heisenberg representation[2] ((8) and making some transformations we obtain the following result

$$w(t) = \left(\frac{e}{\hbar m_e c}\right)^2\sum_{\vec{k},\sigma}\frac{2\pi\hbar v_c^2}{\omega_{\vec{k}} V\Sigma(\omega_{\vec{k}})}\int_0^t \mathrm{d}t_1\int_{-t}^t \langle i|(\vec{e}_{\vec{k}\sigma}\hat{\vec{p}})\exp\left[\left(\frac{i\hbar k^2}{2m_e} - \frac{i\vec{k}\hat{\vec{p}}}{m_e} + i\omega_{\vec{k}}\right)\tau\right](\vec{e}^{\,*}_{\vec{k}\sigma}\hat{\vec{p}})|i\rangle \mathrm{d}\tau, \tag{9}$$

The second integral in this expression can be written in the form

$$\int_{-t}^t \exp\left[\left(\frac{i\hbar k^2}{2m_e} - \frac{i\vec{k}\vec{p}}{m_e} + i\omega_{\vec{k}}\right)\tau\right](\vec{e}_{\vec{k}\sigma}\vec{p})(\vec{e}^{\,*}_{\vec{k}\sigma}\vec{p})\mathrm{d}\tau,$$

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[1] In the Dirac notation the transition amplitude can be presented as [6] $a_{fi}(t) = -\frac{i}{\hbar}\int_0^t V_{fi}(t)\mathrm{d}t = -\frac{i}{\hbar}\int_0^t \langle f|\hat{V}_{int}(t)|i\rangle \mathrm{d}t$, where $|i\rangle$ and $|f\rangle$ are the initial and final states of the considered system respectively.

[2] The explicit form of the $\hat{\vec{p}}$ and $\hat{\vec{r}}$ operators in Heisenberg representation are:

$$\hat{\vec{p}}^H = \hat{\vec{p}}^S,\ \hat{\vec{r}}^H \equiv \hat{\vec{r}}^H(t) = \hat{\vec{r}}^S + \frac{\hat{\vec{p}}^S}{m_e}t, \tag{8}$$

where indexes S and H indicates the Schrödinger and Heisenberg representations respectively.

where we have used the obvious relations $\hat{\vec{p}}|i\rangle = \vec{p}|i\rangle$ and $\langle i|i\rangle = 1$.

After summation by photon polarization[3] and summation by $\vec{k}$ we obtain the following expression for $w(t)$:

$$w(t) = \frac{e^2 v_e^2}{4\pi^2 \hbar c^3} \int \omega n(\omega) \sin^2\theta \mathrm{d}\Omega \mathrm{d}\omega \int\limits_0^t \mathrm{d}t_1 \int\limits_{-t}^{t} \exp\left[i\left(\hbar \frac{n^2(\omega)\omega^2}{2m_e c^2} - \frac{v_e}{c} n(\omega)\omega\cos\theta + \omega\right)\tau\right] \mathrm{d}\tau, \quad (10)$$

where $v_e = p/m_e$ is an initial velocity of an electron. The last integral in this expression can be found using the definition of the Dirac δ-function[4].

The full radiated power can be obtained by multiplying the expression for radiation probability per time unit $\mathrm{d}w = \lim_{t\to\infty} \frac{\mathrm{d}w(t)}{t}$ on the energy of radiated photon $\hbar\omega$ and integrating over ω:

$$P_{rad} = \frac{e^2 v_e}{c^2} \int\limits_0^\infty \omega \left[1 - \left(\frac{\hbar n(\omega)\omega}{2m_e c v_e} + \frac{c}{n(\omega) v_e}\right)^2\right] \Theta\left(1 - \left|\frac{\hbar n(\omega)\omega}{2m_e c v_e} + \frac{c}{n(\omega) v_e}\right|\right) \mathrm{d}\omega. \quad (11)$$

Transition to classical limit leads to the following well-known results:

$$P_{rad} = \frac{e^2 v_e}{c^2} \int\limits_0^\infty \omega \left[1 - \left(\frac{c}{n(\omega) v_e}\right)^2\right] \Theta\left(1 - \frac{c}{n(\omega) v_e}\right) \mathrm{d}\omega. \quad (12)$$

$$\cos\theta_0(\omega) = \frac{c}{n(\omega) v_e}. \quad (13)$$

Based on the results obtained above we can take the following conclusion on the nature of Cerenkov radiation: *Cerenkov radiation is the result of interaction of charged particle with the electromagnetic vacuum which is changed by a medium, nor the interaction with a medium.*

3. CERENKOV RADIATION IN PRESENCE OF SQUEEZED ELECTROMAGNETIC VACUUM

3.1. Squeezed Electromagnetic Vacuum: Some Basic Results

Following Stoler's ideology the squeezed state is determined as an eigenstate of operator $\hat{b}$ which is connected with $\hat{a}^+$ and $\hat{a}$ as [5]:

$$\hat{b} = \mu\hat{a} + \nu\hat{a}^+,\, \hat{b}^+ = \nu^*\hat{a} + \mu^*\hat{a}^+, \quad (14)$$

where

$$|\mu|^2 - |\nu|^2 = 1,\, \mu \equiv \cosh r,\, \nu \equiv e^{i\vartheta}\sinh r, \quad (15)$$

where r and ϑ are defined below. In the calculations with the squeezed states important role plays the so-called squeezing operator (the operator of unitary transformation of coherent state to squeezed state) introduced by Stoler:

$$\hat{D}(\eta) = \exp\left[\frac{1}{2}\eta^*(\hat{a})^2 - \frac{1}{2}\eta(\hat{a}^+)^2\right], \quad (16)$$

where $\eta = re^{i\vartheta}$ is an arbitrary complex number. The state

$$|(\eta,\alpha)\rangle \equiv \hat{T}(\alpha)\hat{D}(\eta)|0\rangle \quad (17)$$

[3]Summation by photon polarization can be made using the standard procedure [6], namely, $\overline{e_i e_k^*} = \frac{1}{2}(\delta_{ik} - n_i n_k)$; $\sum_\sigma (\vec{e}_{\vec{k}\sigma}\vec{p})(\vec{e}^*_{\vec{k}\sigma}\vec{p}) = |[\vec{p}\times\vec{n}]|^2$.

[4]We use the following well-known relations: $\delta(\omega) = \lim_{t\to\infty}\frac{1}{2\pi}\int\limits_{-t}^{t} e^{-i\omega t'}\mathrm{d}t'$; $\delta(\lambda\xi) = \frac{1}{|\lambda|}\delta(\xi)$; $\int\limits_{-1}^{1}(1-x^2)\delta(x-x_0)\mathrm{d}x = (1 - x_0^2)\Theta(1-|x_0|)$, where $\Theta(\xi)$ is a standard Heaviside step-function.

is called as *an ideal squeezed state* (here $\hat{T}$ is the translation operator, $|0\rangle$ is an ordinary vacuum state).

Let us consider also the dynamics of ideal squeezed state (17) in time. The state $|(\eta,\alpha)(t)\rangle$ retains meaning as squeezed state, only the coefficients μ and ν change

$$\mu(t) = \mu e^{i\omega t},\ \nu(t) = \nu e^{i\omega t}$$

while the condition (15) still the same.

Average number of photons in ideal squeezed state $|(\eta,\alpha)\rangle$ is determined by the expression:

$$\langle(\eta,\alpha)|\hat{a}^{+}\hat{a}|(\eta,\alpha)\rangle = |\alpha|^2 + |\nu|^2. \tag{18}$$

Square of dispersion of the squeezed state, minimal and maximal values of the dispersion are:

$$D^2(t) = \frac{\hbar}{2\omega}\left[|\mu|^2 + |\nu|^2 - 2|\mu||\nu|\cos\left(\vartheta + 2\omega t\right)\right],\ D^2_{min} = \frac{\hbar}{2\omega}\left(|\mu| - |\nu|\right)^2,\ D^2_{max} = \frac{\hbar}{2\omega}\left(|\mu| + |\nu|\right)^2. \tag{19}$$

Squeezed state is characterized by the squeezing coefficient

$$K = \sqrt{\frac{D_{max}}{D_{min}}} = |\mu| + |\nu|. \tag{20}$$

This coefficient changes in the range from 1 for coherent state ($|\nu| = 0$) to big values for strongly squeezed states ($|\nu| \to \infty$). It should also be pointed out that squeezed vacuum radically differs from usual vacuum (as a lowest state of the field). Namely, the squeezed vacuum can be a high-excited and high-energy state of the field.

3.2. Cerenkov Radiation in Presence of Squeezed Electromagnetic Vacuum

Now let us generalize the problem, namely we will find the analytic expressions for probabilities per time unit for all the transitions from initial state of an electron when it moves in the medium in the presence of the squeezed vacuum.

As previously, let $|i\rangle$ and $|f\rangle$ are the initial and final states of an electron respectively. For the squeezed states we will use the following notations: initial state of electromagnetic field with mode $\vec{k}$ and polarization σ: $|0_{\vec{k}\sigma}\rangle \equiv |(0,\eta)_{\vec{k}\sigma}\rangle$; final state of electromagnetic field with mode $\vec{k}$ and polarization σ: $|\zeta_{\vec{k}\sigma}\rangle \equiv |(\alpha,\eta')_{\vec{k}\sigma}\rangle$.

Taking into account the presented notations the transition amplitude in the first order of perturbation theory can be written as:

$$a_{fi}(t) = -\frac{i}{\hbar}\int\limits_0^t \langle f|\langle\zeta_{\vec{k}\sigma}|\hat{V}_{int}(t')|0_{\vec{k}\sigma}\rangle|i\rangle \mathrm{d}t' = -\frac{i}{\hbar}\int\limits_0^t \langle f|\hat{V}_{\vec{k}\sigma}(t')|i\rangle \mathrm{d}t',\ \hat{V}_{\vec{k}\sigma}(t') = \langle\zeta_{\vec{k}\sigma}|\hat{V}_{int}(t')|0_{\vec{k}\sigma}\rangle. \tag{21}$$

As previously, we assume that the time leads to infinity $t \to \infty$.

Squaring of a_{fi} and summation by final states $|\zeta_{\vec{k}\sigma}\rangle$ and $|f\rangle$ gives:

$$\sum_{f\zeta_{\vec{k}\sigma}} a^*_{fi}(t)a_{fi}(t) = \sum_{\zeta_{\vec{k}\sigma}} \frac{1}{\hbar^2}\int\limits_0^t\int\limits_0^t \langle i|\hat{V}^{+}_{\vec{k}\sigma}(t_1)\hat{V}_{\vec{k}\sigma}(t_2)|i\rangle \mathrm{d}t_1\mathrm{d}t_2, \tag{22}$$

where $\hat{V}_{\vec{k}\sigma}$ is determined in (21) and $\hat{V}_{int}$ has the form (2). Operator of electromagnetic field has the form (3) and the fact that this operator describes the squeezed vacuum state we take into account in calculations of the matrix elements. Simplification of the matrix element in the last expression can be made using these simple relations:

$$\langle 0_{\vec{k}\sigma}|\left(\hat{a}_{\vec{k}\sigma}\right)^2|0_{\vec{k}\sigma}\rangle = -\mu^*\nu,\ \langle 0_{\vec{k}\sigma}|\left(\hat{a}^{+}_{\vec{k}\sigma}\right)^2|0_{\vec{k}\sigma}\rangle = -\mu\nu^*,\ \langle 0_{\vec{k}\sigma}|\hat{a}^{+}_{\vec{k}\sigma}\hat{a}_{\vec{k}\sigma}|0_{\vec{k}\sigma}\rangle = |\nu|^2.$$

As previously, using the expression for $\hat{\vec{r}}$ operator in Heisenberg representation (8) and making the procedures in the same way as in previous section we obtain the following expression for radiated power[5]:

$$P = P_1 + P_2,$$

$$P_1 = \left(1+|\nu|^2\right)\frac{e^2 v_e}{c^2}\int\limits_0^\infty \omega\left[1-\left(\frac{\hbar n(\omega)\omega}{2m_e c v_e}+\frac{c}{n(\omega)v_e}\right)^2\right]\Theta\left(1-\left|\frac{\hbar n(\omega)\omega}{2m_e c v_e}+\frac{c}{n(\omega)v_e}\right|\right)\mathrm{d}\omega, \tag{23}$$

$$P_2 = |\nu|^2\frac{e^2 v_e}{c^2}\int\limits_0^\infty \omega\left[1-\left(\frac{\hbar n(\omega)\omega}{2m_e c v_e}-\frac{c}{n(\omega)v_e}\right)^2\right]\Theta\left(1-\left|\frac{\hbar n(\omega)\omega}{2m_e c v_e}-\frac{c}{n(\omega)v_e}\right|\right)\mathrm{d}\omega.$$

This expression contains two terms. First term corresponds to radiation power, and the second one corresponds to absorption power.

4. CONCLUSIONS

In this paper we have considered the Cerenkov radiation in the case when the charged particle moves in the medium in the presence of squeezed electromagnetic vacuum. Because of purely quantum nature of squeezed vacuum we use the quantum-mechanical approach (time-dependent perturbation theory) for calculation of the radiated power. We obtain the expression for single-photon process in the case when an electron moves in the medium in the presence of squeezed electromagnetic vacuum. This expression shows that in this case there are two terms: first term corresponds to radiation, while the second one corresponds to absorption. This fact is connected with the complex structure of the squeezed vacuum in contrast to the ordinary vacuum state. Moreover, application of the quantum approach to the Cerenkov radiation problem allows us to formulate the following result: *Cerenkov radiation is the result of interaction of the charged particle with the electromagnetic vacuum which is changed by a medium, nor the interaction with a medium.*

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[5]We do not present step by step calculations of the matrix elements due to a large volume of intermediate transformations which, nevertheless, are simple, but tiresome.