Design and Analysis of a Phased-MIMIO Array Antenna with Frequency Diversity

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Abstract— Phased-Multiple-Input Multiple-Output (phased-MIMO) radar has been thoroughly investigated in the literature. This approach divides the transmit antenna array into multiple subarrays that are allowed to overlap. Each subarray coherently transmits a distinct waveform, which is orthogonal to the waveforms transmitted by other subarrays. While phased array radar enjoys the advantage of high coherence gain with no diversity gain, MIMO radar enjoys high diversity gain — whether waveform or spatial diversity gain — with no coherence gain. Phased-MIMO radar enjoys both advantages of coherence gain and diversity gain which enhance the main-to-side lobe levels but unfortunately decrease the antenna directivity.

Frequency diversity can improve antenna directivity by applying a small frequency increment between two successive elements but consequently increases average side lobes level. In this paper, we propose a new Phased-MIMO radar with frequency diversity which offers a tradeoff between main-to-side lobe levels and antenna directivity. This technique can exploit frequency diversity which is applied between subarray adjacent elements and between adjacent subarrays. The total beam pattern is an optimum case of the lowest side lobes level as phased-MIMO radar but with higher directivity.

1. INTRODUCTION

Phased-array antennas are called to play a fundamental role in high performance radar systems. Usually, electronic beam steering of transmitted or received electromagnetic signal with single frequency is made by means of phase shifters and attenuators associated to each one of the radiating elements that conforms the antenna array [1–4]. Thus, the beam can be steered to the desired direction by controlling the phase shifters across elements. It offers coherent gain which is useful for detecting/tracking targets and suppressing sidelobe interferences from other directions. The requirements of many emerging applications drive The need for new more advanced antenna array technologies [5–9].

The emerging concept of multiple-input multiple-output (MIMO) radar has been the focus of intensive research [10–12]. Currently, two types of MIMO radar systems are being investigated: MIMO radars with widely-separated antennas and MIMO radars with colocated antennas. Both have uniquely many advantages, but both also face many challenges.

MIMO radar with widely-separated antennas have two important features: space-diversity and spatial multiplexing. As a result, they can exploit radar cross section (RCS) diversity [13], they can handle slow moving targets by exploiting Doppler estimates from multiple directions [14], and they can support high resolution target localization [15]. However, they can also face problems with time and phase synchronization. As the data volume to be transmitted to the processing center in realtime is very large, these radars also face transmission and processing issues. Also, as they employ isotropic transmission antennas that transmit energy evenly to space [11], they cannot use most of their energy to illuminate the target and thus experience losses in signal-to-noise ratio (SNR).

MIMO radar with colocated antennas can exploit waveform diversity. This is important as it can: (a) significantly improve system identification; (b) increase target detection and parameter estimation performance when combined with adaptive arrays; (c) enhance transmit beam pattern design [12]; and (d) enable the design of airborne or ship-borne radars. However,because the antennas are colocated, the main drawback is the loss of space diversity which is needed to mitigate the effect of target fluctuations.

A. Hassanien, et al. [22] propose a new technique for MIMO radar with colocated antennas which called phased-MIMO radar. The new technique enjoys the advantages of the MIMO radar without sacrificing the main advantage of the phased-array radar which is the coherent processing gain at the transmitting side.

In this paper, we propose a new Phased-MIMO radar system technique that can exploit frequency diversity. This new system uses colocated Phased-MIMIO antenna which enjoys the advantage of phased array antenna, coherent gain, and the advantage of MIMO radar, waveform diversity gain, in addition to the advantage of frequency diversity gain which mitigates the effect of target fluctuations and improving the antenna beam pattern by reducing the side lobes level.

The remaining sections are organized as follows.

Section 2 introduces the new approach of phased-MIMO radar with frequency diversity model compared to other models which differ in frequency diversity existence and elements correlation. In Section 3, simulation and results are illustrated for all the above models including figures and comments. Finally this paper is concluded in Section 4.

2. NEW PHASED-MIMO WITH FREQUENCY DIVERSITY MODEL

In this section, the transmit array is divided into multiple K subarrays which can be disjoint or overlapped, as shown in Figure 1. Each transmit sub-array can be composed of any number of elements ranging from 1 to M. However, unlike the general phased-MIMO array discussed in the literature, in this paper we will partition the array into Ksub-arrays with transmitting elements in each sub-array following the rule $M_K = M - K + 1$ where M_K is the number of elements in each sub-array. A beam can be formed by each sub-array towards a certain direction. The beamforming weight vector can be properly designed to maximize the coherent processing gain. At the same time, different waveforms are transmitted by each sub-array. Each sub-array has a waveform ϕ_k orthogonal to other sub-array waveforms,in addition to a frequency shift $\pm\Delta f$ with respect to its neighbor sub-arrays. This orthogonality offers a waveforms diversity gain where the frequency shift offers a frequency diversity gain. Suppose the kth sub array consists of $M_k < M$ transmit elements, the equivalent baseband signal model of the kth sub array can be modeled as,

$$S_k(t) = \sqrt{\frac{M}{K}}\Phi_k(t)\,\tilde{w}_k^* \quad k = 1, 2, \ldots, K \tag{1}$$

where K is the number of sub-arrays, $\tilde{W}_k$ is the $M \times 1$ unit-norm complex vector which consists of M_K beamforming weights corresponding to the active antennas of the kth sub-array, that is, the number of nonzero in $\tilde{W}_k$ equals to M_K and the number of zeros equals to $M - M_K$. Note that $\sqrt{M/K}$ is used to obtain an identical transmission power constraint for subsequent comparison, which means the transmit energy within one pulse repetition interval (PRI) is given by

$$E_k = \int_{Tp} S_K^H(t)\, S_k(t)\, dt = \frac{M}{K} \tag{2}$$

This means that the total transmitted energy for the phased-MIMO radar within one radar pulse is equal to M. The signal reflected by a hypothetical target located at direction in the far-field can be then modeled

$$S_{r(t,\theta)} = \sqrt{\frac{M}{K}}\sigma_s(\theta)\sum_{k=1}^{K}\sum_{m_k=1}^{M_K}\Phi_k(t)\,a_k(\theta)\,w_{m_k}^H b_{m_k}(\theta) \tag{3}$$

$$a_k(\theta) = e^{-j\frac{2\pi}{c}(k-1)[\Delta f R_o - f_0 d\sin\theta - (k-1)\Delta f d\sin\theta]} \tag{4}$$

$$b_{m_k}(\theta) = e^{-j\frac{2\pi}{c}(m_k-1)[\Delta f R_o - f_0 d\sin\theta - (m_k-1)\Delta f d\sin\theta]} \tag{5}$$

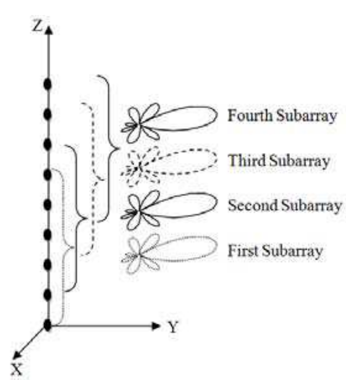

Figure 1: Illustration of the phased-MIMO array.

where σ_s is the reflection coefficient of the hypothetical target, θ is the aspect angle between target and radar boresight w_{m_k} is the complex weight of the m_kth antenna $a_k(\theta)$ is the phase shift between the signal sat the first antenna of the array and the first antenna of the kth sub-array due to both spatial displacement and frequency diversity $b_{m_k}(\theta)$ is the phase shift between the signals at the first antenna and the m_kth antenna for each subarray due to both spatial displacement and frequency diversity, R_0 is the target range measured from the first antenna of the array, d is the displacement between two successive antennas, f_0 is the carrier frequency of the 1st antenna and Δf is the increment step frequency between two successive either antennas or subarrays which equals to:

$$\Delta f = \frac{c}{2L\sin\theta} \tag{6}$$

where C is the speed of electromagnetic waves and L is the length of the target. Note that $L\sin\theta$ is the length of the target projected along the radar boresight. We assume that the first antenna is taken as a reference antenna.

It's disputable that the signal reflected $S_{r(t,\theta)}$ varies according to the existence of coherence gain, waveform diversity gain and frequency diversity gain as follows:
case (1): for phased array radar without frequency diversity ($\Delta f = 0$, $K = 1$), Equation (3) simplified to:

$$S_{r(t,\theta)} = \sqrt{M}\sigma_s(\theta)\Phi_1(t)\sum_{m_k=1}^{M} w_{m_k}^H e^{-j\frac{2\pi}{c}(m_k-1)(-f_0 d\sin\theta)} \tag{7}$$

case (2): for phased array radar with frequency diversity ($\Delta f \neq 0$, $K = 1$), Equation (3) simplified to:

$$S_{r(t,\theta)} = \sqrt{M}\sigma_s(\theta)\Phi_1(t)\sum_{m_k=1}^{M} w_{m_k}^H e^{-j\frac{2\pi}{c}(m_k-1)[\Delta f R_o - f_0 d\sin\theta-(m_k-1)\Delta f d\sin\theta]} \tag{8}$$

case (3): for MIMO radar without frequency diversity ($\Delta f = 0$, $K = M$), Equation (3) simplified to:

$$S_{r(t,\theta)} = \sigma_s(\theta) w^H \sum_{k=1}^{M} \Phi_k(t) e^{-j\frac{2\pi}{c}(k-1)(-f_0 d\sin\theta)} \tag{9}$$

case (4): for MIMO radar with frequency diversity ($\Delta f \neq 0$, $K = M$), Equation (3) simplified to:

$$S_{r(t,\theta)} = \sigma_s(\theta) w^H \sum_{k=1}^{M} \Phi_k(t) e^{-j\frac{2\pi}{c}(k-1)[\Delta f R_o - f_0 d\sin\theta-(k-1)\Delta f d\sin\theta]} \tag{10}$$

case (5): for Phased MIMO radar without frequency diversity ($\Delta f = 0$, $1 < K < M$), Equation (3) simplified to:

$$S_{r(t,\theta)} = \sqrt{\frac{M}{K}}\sigma_s(\theta)\sum_{k=1}^{K}\sum_{m_k=1}^{M_K} w_{m_k}^H \Phi_k(t) e^{-j\frac{2\pi}{c}(k-1)(-f_0 d\sin\theta)} e^{-j\frac{2\pi}{c}(m_k-1)(-f_0 d\sin\theta)} \tag{11}$$

case (6): for Phased MIMO radar with frequency diversity ($\Delta f \neq 0$, $1 < K < M$), Equation (3) express this case as:

$$S_{r(t,\theta)} = \sqrt{\frac{M}{K}}\sigma_s(\theta)\sum_{k=1}^{K}\sum_{m_k=1}^{M_K} W_k^H \Phi_k(t) e^{-j\frac{2\pi}{c}(k-1)[\Delta f R_o - f_0 d\sin\theta-(k-1)\Delta f d\sin\theta]} e^{-j\frac{2\pi}{c}(m_k-1)[\Delta f R_o - f_0 d\sin\theta-(m_k-1)\Delta f d\sin\theta]} \tag{12}$$

Now let us introduce the $K \times 1$ transmit coherent processing vector with frequency diversity

$$C(\theta) = \left[W_1^H B_1(\theta), \ldots, W_K^H B_K(\theta)\right]^T \tag{13}$$

where $B_K(\theta) = [b_1(\theta), b_2(\theta), \ldots, b_{M_K}(\theta)]^T$ is $M_K \times 1$ transmit steering vector and $W_K = [w_1, w_2, \ldots, w_{M_K}]^T$ is $M_K \times 1$ beamforming vector which contains only the elements corresponding to the active antennas of the kth sub-array.

The $K \times 1$ diversity vector (waveform diversity vector combined with frequency diversity vector)

$$D(\theta) = [a_1(\theta), a_2(\theta) \ldots, a_K(\theta)]^T \tag{14}$$

Then, the reflected signal (3) can be rewritten as,

$$S_r(t, \theta) = \sqrt{\frac{M}{K}} \sigma(\theta) (C(\theta) \odot D(\theta))^T \Phi_k(t) \tag{15}$$

where $\Phi_k(t) = [\Phi_1(t) \ldots \Phi_k(t)]^T$ is the $K \times 1$ vector of waveforms. Assuming that the target of interest is observed in the background of P interfering targets with reflection coefficient $\{\sigma_i\}_{i=1}^P$ and locations $\{\theta_i\}_{i=1}^P$, the $N \times 1$ received complex vector of array observations can be written as,

$$X(t) = S_r(t, \theta_s) G(\theta_s) + \sum_{i=1}^{P} S_r(t, \theta_i) G(\theta_i) + n(t) \tag{16}$$

where, $S_r(t, \theta_s)$ and $S_r(t, \theta_i)$ are defined as in (15), N is the number of receiving elements, and $G(\theta)$ is the received steering vector By matched-filtering $X(t)$ to each of the waveforms $\{\Phi_k\}_{k=1}^K$, we can form the $KN \times 1$ virtual data vector,

$$y = \left[x_1^T \ldots x_k^T\right]^T = \sqrt{\frac{M}{K}} \sigma_s U(\theta_s) + \sum_{i=1}^{P} \sqrt{\frac{M}{K}} \sigma_i U(\theta_i) + \tilde{n} \tag{17}$$

where the $KN \times 1$ vector

$$U(\theta) = C(\theta) \odot D(\theta)) \otimes G(\theta) \tag{18}$$

is the virtual steering vector associated with direction θ and$\tilde{n}$ is the $KN \times 1$ noise term (noise received by N receivers containing K matched filters at each receiver) whose covariance is given by $\tilde{R}_n = \sigma_n^2 I_{KN}$ where σ_n^2 is the noise power.

3. SIMULATION AND RESULTS

In this section, we use the conventional transmit/receive beamforming techniques and analyze the performance of the proposed phased-MIMO radar with frequency diversity (case 6). We also compare this case to the phased array radar (without and with frequency diversity (cases (1 and 2) respectively)), MIMO radar (without and with frequency diversity (cases (3 and 4) respectively)) and phased-MIMO radar without frequency diversity (case (5)) in terms of their transmit/receive beampatterns. We assume a hypothetical target oriented at angle $\theta = 10°$.

Comparing the received beam pattern of phased array radar (case 1), MIMO radar (case 3), and phased-MIMIO radar (case 5), it can be noticed that phased-MIMO radar enjoys the lowest side lobes level but has lower directivity than the other two cases (wider beam width).

Comparing the received beam pattern of phased array radar without frequency diversity (case 1), and with frequency diversity (case 2), it can be noticed that frequency diversity enhances the beam directivity (narrower beam width) but with higher average side lobes level.

Using the same method, comparing the received beam pattern of MIMO radar without frequency diversity (case 3), and with frequency diversity (case 4), it can be noticed that frequency diversity enhance the beam directivity (narrower beam width) but with higher average side lobes level.

Combining the advantage of phased-MIMO technique, which is suppressing side lobes level and the advantage of frequency diversity, which is improving beam directivity (by getting narrower beam width), we achieved the proposed phased-MIMO radar with frequency diversity (case 6). Comparing the all six cases mentioned above, it can be noticed that the phased-MIMO radar with frequency directivity is optimum tradeoff between main-to-side lobe levels and antenna directivity.

It's observed that both case (1) and case (3) are identical and also case (2) and case (4) are identical too. Case (6) is the optimum beam pattern which has narrow beam width compared to case (5) and lower side lobes level compared with cases (1, 2, 3, and 4).

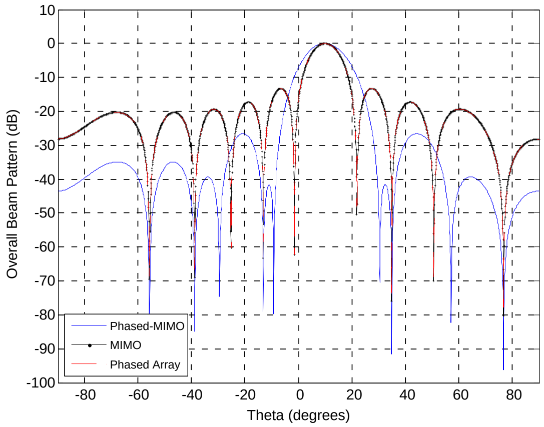

Figure 2: Overall beam pattern for phased array radar, MIMO radar, and phased-MIMO radar without frequency diversity.

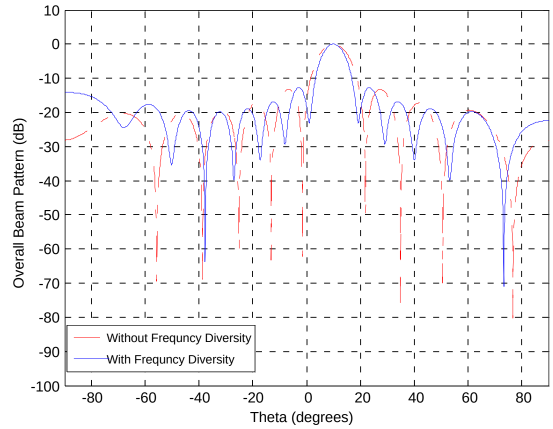

Figure 3: Overall beam pattern for phased array radar, with and without frequency diversity.

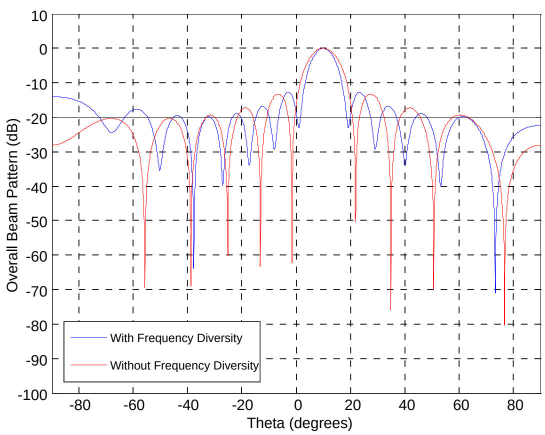

Figure 4: Overall beam pattern for MIMO radar, with and without frequency diversity.

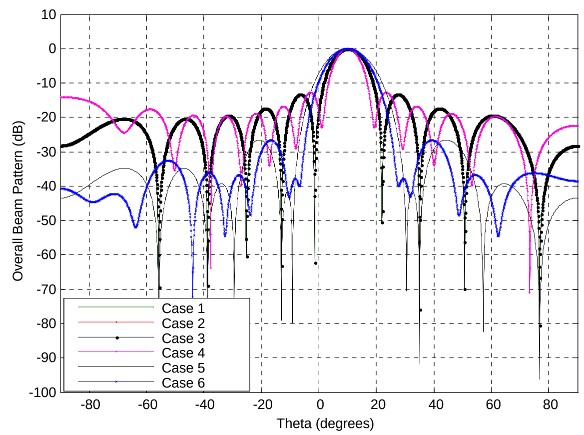

Figure 5: Overall beam pattern for the all possible six cases.

4. CONCLUSION

All possible cases of phased array, MIMO and Phased-MIMO radar in the presence and absence of frequency diversity were investigated and compared to each other. Phased-MIMO radar with frequency diversity has declared to be the optimal case of all by providing the lowest side lobes level with high directivity.

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