A Time Domain Analytic Solution to Predict the Transient Radiation for Phased Periodic Array

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Abstract— Most recently the applications have been extended to treat the problems arising in the near zone of electrically large antennas such as the vital life-detection systems and noncontact microwave detection systems, where the objects under detection may locate in the near zone of antenna. A TD analytic solution to predict the transient radiation from a phased periodic array of elemental antennas is thus developed. The TD phenomena are investigated in this paper. The TD phenomena of Floquet modes in the quantity of field potentials with a transient impulse excitation in the current moments were examined. In this generalized analysis, one first considers a two dimensional finite array of current moments with phases impressed to radiate fields focused in the near zone of array aperture, where the focal field point can be arbitrarily selected.

1. INTRODUCTION

The increasing interest in the time domain (TD) analysis of ultra wideband or short pulse target identification and remote sensing applications has resulted in the development of new TD techniques to analyze the antenna radiation, which provides more physically appealing interpretation of wave phenomena. Most recently the applications have been extended to treat the problems arising in the near zone of electrically large antennas such as the vital life-detection systems and noncontact microwave detection systems, where the objects under detection may locate in the near zone of antenna. The potential applications of near-field antennas continue to grow dramatically and desire more exploration in the near future. A TD analytic solution to predict the transient radiation from a time-delayed periodic array of elemental antennas is thus developed. In this analysis, the signals of array excitations are properly delayed to radiate electromagnetic (EM) fields focused in the near zone of array aperture [1, 2]. Examples include the developments of TD uniform geometrical theory of diffraction (TD-UTD) [3, 4], physical theory of diffraction (TD-PTD) [5], physical optics (TD-PO) [6] and TD aperture integration (TD-AI) [7] techniques that were obtained by using either a direct inverse Laplace transform or an analytical time transform (ATT) [8] of the corresponding FD formulations [9]. These solutions are limited to the transient analysis of antenna radiation with scattering mechanisms such as the reflector antennas, and are not applicable to the current situation of direct antenna radiation from a time-delayed array. This paper presents an analytical transient analysis of electromagnetic field radiation from a time-delayed and finite periodic array of antennas for the near- and far-field focused applications. The elemental current moments of array are assumed with a transient impulse input for the excitations whose signals are delayed to radiate near-zone focused fields. The transient field phenomena for each of the Floquet mode expansion were analyzed. The solution reduces to the case of far-zone field radiation by moving the focus point to the far zone. The analysis shows that the radiation exhibits an impulse field at the focused point, and finite pulses at locations away from the focus point. Phenomena of partial cylindrical wave functions have been observed.

2. TIME DOMAIN IMPULSE RESPONSES WITH RESPECT TO EACH OF THE FLQUET MODE EXPANSION

2.1. Transient Phenomena of an Unit Current Moment

Let the array be planar, rectangular and periodic with $(2N_x + 1) \times (2N_y + 1)$ identical elements of infinitesimal magnetic current moment, $d\bar{p}(\bar{r}', t)$, with periods, d_x and d_y, along the x- and y-coordinates, respectively, which is illustrated in Figure 1. In particular, this current moment has a transient behavior of Dirac delta function (impulse function) by

$$d\bar{p}\left(\bar{r}', t\right) = d\bar{P}\left(\bar{r}'\right) \cdot \delta(t - t') \tag{1}$$

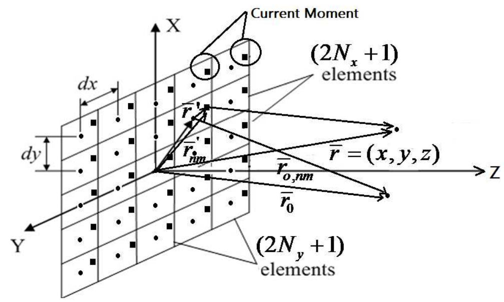

Figure 1: A two dimensional periodic array of current moments induced on the array antenna elements to radiate near-zone focused field at $\bar{r}_o$.

where $\delta(\cdot)$ is the Dirac delta function. The nmth element of the array is located at $\bar{r}'_{nm} = (nd_x, md_y, 0) + \bar{r}'(-N_x \leq n \leq N_x,\ -N_y \leq m \leq N_y)$, and has a FD potential, $d\bar{F}_{nm}(\bar{r}, s)$ by

$$d\bar{F}_{nm}(\bar{r}, s) = \frac{1}{4\pi} d\bar{P}(\bar{r}') \frac{e^{-\frac{s r_{nm}}{c}}}{r_{nm}}, \tag{2}$$

where $s = j\omega$ with ω being the angular frequency, and c is the speed of light in free space. In (2), $\bar{r}_{nm} = \bar{r} - \bar{r}'_{nm}$ with $\bar{r} = (x, y, z)$ being the coordinates of observation point. This radiation exhibits a transient behavior of impulse in TD by

$$d\bar{F}_{nm}(\bar{r}, t) = \frac{1}{4\pi} d\bar{P}(\bar{r}') \frac{\delta(t - \frac{r_{nm}}{c})}{r_{nm}}. \tag{3}$$

2.2. Transient Phenomena of an Array of Time-delayed Unit Current Moments

The net potential in FD from this array is given by

$$d\bar{F}(\bar{r}, s) = \sum_{m=-N_y}^{N_y} \sum_{m=-N_x}^{N_x} A(n, m) \frac{1}{r_{o,nm}} e^{\frac{s}{c}\phi(n,m)} d\bar{F}_{nm}\left(\bar{r}, s\right), \tag{4}$$

where $A(n, m)$ is an additional amplitude taper, which can be used to reduce the diffraction effects of a finite array. In TD, gives

$$d\bar{F}(\bar{r}, t) = \frac{d\bar{P}\left(\bar{r}'\right)}{4\pi} \left[\sum_{m=-N_y}^{N_y} \sum_{m=-N_x}^{N_x} A(n, m) \frac{\delta\left(t - \frac{r_{nm} - \phi(n,m)}{c}\right)}{r_{o,nm} r_{nm}} \right]. \tag{5}$$

It is noted that a convolution in time with the impulse response in (5) can be performed to obtain the response of a general transient inputs for the elements' excitations.

2.3. Transient Floquet Mode Representation

The element-by-element summation in (5) is time-consuming and does not provide radiation phenomena with physical meanings. It is thus alternatively expressed in terms of Floquet modes by using the Poisson sum formula of a periodic structure:

$$\sum_{n=-N_1}^{N_2} f(n) = \frac{f(N_1^+)}{2} + \frac{f(N_2^-)}{2} + \sum_{p=-\infty}^{\infty} \int_{N_1^+}^{N_2^-} f(v) e^{-j2\pi p v} dv. \tag{6}$$

Thus referring to the illustration in Figure 2 and applying (6) over the double sum formulation in (5) will make (5) becoming

$$d\bar{F}(r, t) = \frac{1}{4} \sum_{\ell=1}^{4} \bar{C}_\ell\left(\bar{r}, t\right) + \frac{1}{2} \sum_{\alpha=1}^{4} \bar{G}_\alpha\left(\bar{r}, t\right) + \sum_{q=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} \bar{F}_{pq}^{w}\left(\bar{r}, t\right), \tag{7}$$

which transforms the discrete source radiations into that of continuous aperture sources. The phenomena and analysis of each term are interpreted as corner, edge and Floquet wave contributions similar to these previously identified in [10]. In particular, $\bar{C}_\ell(\bar{r},t)$ are related to the corner effects, $\bar{G}_\alpha(\bar{r},t)$ are related to the four finite edge effects, and $\bar{F}^w_{pq}(\bar{r},t)$ are related to the effects of a finite aperture, which are discussed in the following sections.

3. AN INTEGRATION CONTOUR AND ITS CHARACTERISTICS

The surface integral can be reduced to a line integral at the conditions that the Dirac delta function has a nonzero value. It occurs at $r_{uv} - \phi(u,v) = ct$, and with $(n,m) \rightarrow (u,v)$, which results in a line contour $C_t(t)$ on S_a as illustrated in Figure 2. It is noted that $C_t(t)$ is the intersection of a hyperbolic surface and $z = 0$ plane as illustrated in Figure 2, where the two focused points of the hyperbolic surface are located at $F_1 = (x - x', y - y', z - z')$ and $F_2 = (x_0, y_0, z_0)$, respectively. As shown in $C_t(t)$ can be described by

$$\frac{(x_{i,d} - e_{1,d})^2}{A^2} + \frac{(y_{i,d} - e_{2,d})^2}{B^2} = \rho_t^2 \tag{8}$$

4. NUMERICAL EXAMPLES

The array of antennas has a period of 0.1 m in both x- and y-dimensions. The focus point is at $(x_o, y_o, z_o) = (0, 0, 1.5\,\text{m})$, which is relatively far away from the array aperture. The observation point is at, and is in $(0, 0, 30\,\text{m})$ the near zone. In this examination, one considers the behaviors of $\bar{F}_{pq}(\bar{r},t)$, which represents the dominating contributions in the array radiation for the impulse

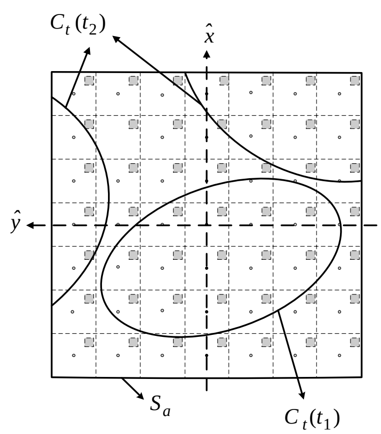

Figure 2: Illustration of S_a and $C_t(t)$, where $C_t(t_1)$ and $C_t(t_2)$ on the figure are used to indicate different types occurring at t_1 and t_2 according to which part of the hyperbolic surface intersects with S_a.

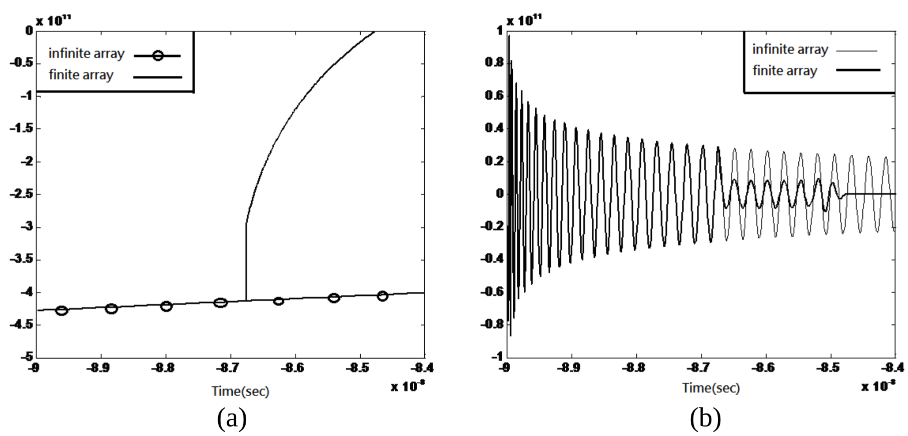

Figure 3: Transient responses of various Floquet modes for an infinite and a finite array of current sources with impulse excitations. The periods are 0.1 m in both x- and y-dimensions. The focus and observation points are at $(0, 0, 1.5\,\text{m})$ and $(0, 0, 30\,\text{m})$, respectively. (a) $(p, q) = (0, 0)$. (b) $(p, q) = (-1, 0)$.

current distributions for every element. One first considers the case of an infinite and finite array, and examines the fundamental $(p, q) = (0, 0)$ and $(-1, 0)$ modes, whose results are shown in Figures 3(a)–(b).

5. CONCLUSIONS

This paper presents a quasi-analytic TD solution to perform a transient analysis of radiation from an array of current sources. The solutions for electromagnetic radiation potential are obtained for the array with a finite size and impulse transient excitations. The array is excited to radiate fields focused in the near zone of array apertures. It is also found that each Floquet mode exhibits the characteristics of different time harmonic orders.

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