Time Domain Transient Analysis for Ellipsoidal and Hyperbolic Reflector Antennas

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Abstract— A closed-form analytical solution is developed for predicting the early-time transient electromagnetic fields which are generated by a perfectly conducting elliptical reflector antenna when it is illuminated by a transient step spherical wave due to an elemental Huygen's source located at the focus near the surface. Furthermore we discussed a transient analysis of a hyperbolic reflector antenna is performed based on a mathematic continuation of surface curvatures of an ellipsoidal reflector antenna. This work makes the time-domain (TD) analysis useful for the design of reflector antennas since both ellipsoidal and hyperbolic reflectors are widely used as sub-reflectors for dual-reflector antenna system.

1. INTRODUCTION

This work is motivated by the potential applications of elliptical reflectors. In the past, elliptical reflectors had been widely used as a sub-reflector to a parabolic reflector antenna in a Gregorian dual-reflector antenna system. The characteristics of dual-focuses on the elliptical surface allow it to be used as a measure to have a longer focal length for the parabolic main reflector while, in the mean time, retaining a compact size for the dual-reflector. Recently it is also found very useful in the design of near-field focused antennas as interested in the RFID and non-contacted microwave inspection system. It is expected that it will attract more attention in the future phase of antenna applications.

The transient solution is developed here by analytically inverting, in closed form, the corresponding frequency-domain solution in terms of a radiation integral that employs an asymptotic high-frequency geometrical optics (GO)-based approximation for the fields in an equal-phase surface aperture which appears to be a sphere centered at the second focus of the ellipse. This equal-phase and spherical aperture surface allows the integration of radiation integral to be significantly simplified, and results a closed-form formulation. And then this paper we will extends the transient analysis of electromagnetic (EM) fields scattering from a perfectly conducting ellipsoidal reflector, and presents the corresponding analysis for the case of a hyperbolic reflector. This work is motivated by the fact that both ellipsoidal and hyperbolic reflectors are widely used as sub-reflectors in the dual-reflector antenna systems [1]. The transient analysis presented in this paper makes the time-domain (TD) analysis of sub-reflector antennas more complete for practical applications. In particular, the analysis is performed based on a mathematic continuation of surface curvatures of an ellipsoidal reflector, and as a result, the analysis of such ellipsoidal reflectors can be applied straightforwardly. Furthermore, the analysis successfully decomposes the scattering fields into components of reflection and edge diffractions, and allows one to interpret the scattering phenomena within the frameworks of geometrical theory of diffraction (GTD) [2, 3] and its wave propagation mechanisms. Examples are presented to demonstrate the wave scattering phenomena.

2. FORMULATION DEVELOPMENT

2.1. Near Field Focused Antenna

In the past, there experienced some applications of NF focused antennas, which were not very popular if a counterpart of far field focused antenna is compared. In the current application, it appears application advantages. Note that a "NF focused antenna" is referred because it is implemented by using an aperture antenna that is sufficiently large to radiate fields focused at a spot in the near field region of the antenna. Thus the focused fields may increase the energy to be received by the tags, and also reduce the interference caused by the scattering from adjacent portals of tags because the tags are usually located at 1–2 m from the reader's antenna aperture. This paper presents a realization of NF focused antenna using a reflector such that the number of circuit elements may be minimized to also minimize the energy loss. Usually only a single element is sufficient to provide a good feed to the reflector. In particular, an elliptic reflector will be realized.

Note that an elliptic reflector has two focal points. The radiation from a feed located at a focus will experience reflections from the reflector surface, and concentrates its energy at another focus which is usually located in the near-zone of the reader antenna. Thus it may be effectively employed to build the NF focused antenna.

2.2. Elliptic Reflector's Characteristics

An elliptic reflector is formed by taking a part of a rotationally symmetric ellipse described by the following equation:

$$\frac{z^2}{a^2} + \frac{x^2 + y^2}{b^2} = 1 \tag{1}$$

where a and b ($a > b$) are the radii of its axes This ellipse has two focuses (referred as F_1 and F_2, respectively, hereafter) located on z-axis at $z = \pm c$ where $c = \sqrt{a^2 - b^2}$. Note that as $a = b$, $c = 0$, and the ellipse becomes a sphere. The two focuses coincide at the center of the sphere. A good characteristics of this ellipse is that any ray launched from a focus (say F_1) will experience a reflection from the ellipse surface and coincide at the other focus (say F_2) as also illustrated in Fig. 1. In particular, given a point, Q, on the ellipse surface, it can be shown that

$$\overline{F_1Q} + \overline{QF_2} = 2a \tag{2}$$

which assures an in-phase superposition at F_2 if a feed's phase center is located at F_1.

2.3. Formulations

Based on these design criterions, the fields at the NF focused region may be approximately found in the following. Let the radiation, ($\vec{E}_a$, $\vec{H}_a$) is Huygen's source, of the feed's antenna represented by

$$\vec{E}_a = V_o \left[\hat{\theta}_1 f_{\theta_f} - \hat{\phi}_1 f_{\phi_f}\right] \frac{1}{r_f} e^{-\frac{s}{v}(2a - r_1)} \tag{3}$$

$$\vec{H}_a = \hat{r}_f \times \frac{\vec{E}_a}{Z_o} \tag{4}$$

where $f_{\theta_f} = \cos\phi_1(\cos\theta_f)^q$, $f_{\phi_f} = \sin\phi_1(\cos\theta_f)^q$ represented by E-plane and H-plane pattern respectively where (r_f, θ_f, ϕ_f) is defined in the feed's coordinate system with $\hat{z}_f$ pointing to $\overrightarrow{F_1Q_c}$. Note that $r_f = a + c\cos\theta$ if the feed is located at F_1. The scattering near fields can be found, via Geometric Optics (GO) approximation, by

$$\vec{E}_r(\vec{r}, s) \cong \frac{Z_o}{4\pi v} \iint_{s_a} \left[\hat{R} \times \hat{R} \times \hat{z}_p \times \vec{H}_a + Y_o \hat{R} \times \left(\vec{E}_a \times \hat{z}_p\right)\right] \frac{e^{-\frac{s}{v}R}}{R} ds' \tag{5}$$

The time domain solution can be obtained via an inverse Laplace transform and the characteristic of delta function

$$\delta\left(t - \frac{R + 2a - r_1}{v}\right) = \frac{\delta(\rho - \rho_t)}{\left|\frac{\partial}{\partial_\rho}\left(t - \frac{R+2a-r_1}{v}\right)\right|_{\rho=\rho_t}} \tag{6}$$

which is a response to a transient-step incident wave and can be founded by performing a numerical integration over S_a. It can be simplified to become a line integral by considering the situations where the delta function has nonzero values at t by $\delta(t - \frac{R+2a-r_1}{v}) \neq 0$ which occurs at $vt - 2a + r_1 = R$ that corresponds to a line contour $C_t(t)$ on the radius r_1 of spherical and shown in Fig. 1. It projection on the x_p-y_p plane is a circular contour described by $x_1'^2 + y_1'^2 = \rho^2$ where $\rho^2 = r_1^2 - \frac{Q^2}{4z_p^2}$, thus one considers a parameter transformation by using $x' = \rho\cos\phi$, $y' = \rho\sin\phi$ and $\iint\limits_S \ldots ds' \to \iint \ldots J_A(\theta, \phi) d\phi \cdot d\theta$ where $J_A(\theta, \phi)$ is the transformation Jacobian

$$J_A(\rho, \phi) = \frac{ds'}{d\rho d\phi} = \left|\bar{r}'_\rho \times \bar{r}'_\phi\right|, \quad \hat{n} = \pm\frac{\bar{r}'_\rho \times \bar{r}'_\phi}{\left|\bar{r}'_\rho \times \bar{r}'_\phi\right|} \tag{7}$$

Allows one to simplify the double integration into a line integral by

$$\vec{E}_r(\vec{r}) \cong \frac{r_1}{4\pi z_p}\left\{\int \hat{R}\times\hat{R}\times V_o\left[-\hat{\theta}_1 f_{\theta_f}-\hat{\phi}_1 f_{\phi_f}\right]\frac{1}{r_f}d\phi - \int \hat{R}\times V_o\left[\hat{\phi}_1 f_{\theta_f}-\hat{\theta}_1 f_{\phi_f}\right]\frac{1}{r_f}d\phi\right\} \tag{8}$$

the integral in (8) can be found in a closed-form.

2.4. Analysis Description for Hyperbolic Surface

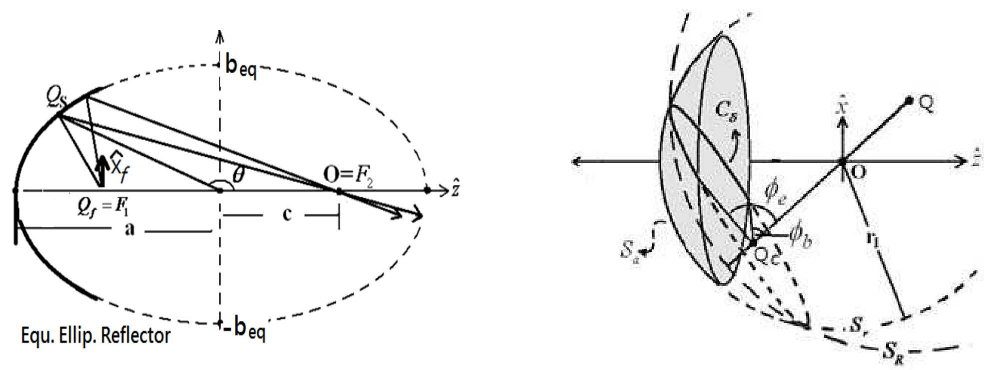
The hyperbolic reflector, illustrated in Fig. 1(a), is described by a part of the following surface

$$\frac{(z+c)^2}{a^2} - \frac{x^2+y^2}{b^2} = 1, \tag{9}$$

and truncated at $z = z_a$, where a and b ($a > b$) are related to the radii of its two principal axes with $c = \sqrt{a^2+b^2}$. The two focuses, F_1 and F_2, are located at $z = -c$ and $z = c$, where F_2 was selected as the origin of a global coordinate system I the following analysis while the feed is located at F_1 for a focus feeding. Given an arbitrary point, Q_s, on the hyperbolic surface, it is straightforward to show that

$$|F_1 Q_s - Q_s F_2| = 2a. \tag{10}$$

The feed's radiation is a $\hat{x}_f$-polarized spherical wave with a cosine-taper and a transient step function, and is described by (9)–(10) in [4]. Here (r_f, θ_f, ϕ_f) is defined in the feed's spherical coordinate system with $\hat{z}_f = +\hat{z}$ and $\hat{x}_f = \hat{x}$ as illustrated in Fig. 1(a).



(a) (b)

Figure 1: A rotationally symmetric hyperbolic and a near-field focused reflector which is taken from a part of hyperbolic.

2.5. The Continuation of Surface Curvature

The hyperbolic surface in (9) can be analogously described as

$$\frac{(z+c)^2}{a^2} + \frac{x^2+y^2}{b_{eq}^2} = 1 \tag{11}$$

where $b_{eq} = jb$ is a complex radii of surface curvature, and makes the surface an ellipsoidal one. Based on this curvature continuation into a complex space, the solution in [4] can be extended for the analysis of hyperbolic reflector.

2.6. Resulting Solution

The scattering field at $\bar{r}_o$ due to the illumination of a TD step-function feed radiation can be expressed in a closed-form as

$$\bar{E}_s(\bar{r}_o,t) = \left(\bar{E}_r(\bar{r}_o,t)\cdot T(\phi_e-\phi_b) + \bar{E}_e(\bar{r}_o,t)\big|_{\phi_p''=\phi_e} - \bar{E}_e(\bar{r}_o,t)\big|_{\phi_p''=\phi_b}\right)\Pi(t) \tag{12}$$

where $T(\varphi) = \varphi/(2\pi)$ and $\Pi(t) \equiv u(t-t_1) - u(t-t_2)$ with $u(t)$ is a Heaviside step function. In (12), t_1 and t_2 are the starting and ending time that the contributing contour, C_δ, overlaps with the reflecting surface.

3. NUMERICAL EXAMPLES

Numerical examples case 1 are presented ellipsoidal reflector to demonstrate the characteristics of scattering fields as shown Fig. 2. In this case, the reflector has a diameter of 2 m (i.e., $z_a = -7.84\,\text{m}$), and $z_p = 1.06\,\text{m}$. The case 2 examples consider a reflector size with the parameters in (11) given by $a = 4\,\text{m}$, $b = 3\,\text{m}$ and $c = 5\,\text{m}$. The reflector in Fig. 1 is rotationally symmetric with a radius $r_a = 1\,\text{m}$. Fig. 3(a) shows the TD response on the z-axis with z_p being the distance measured from F_2 toward the — z direction. In contrast to the radiation of an elliptical reflector, this reflector radiates defocusing fields. The time duration becomes shorter at the observation of farer distance. Fig. 3(b) shows the TD response at $r_o = 15\,\text{m}$ with various observation angles. In this case, the field at the axis corresponds to the main beam while that at the far away angles are the sidelobes.

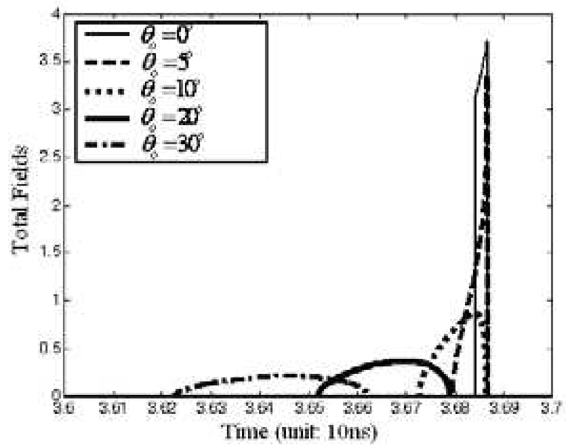
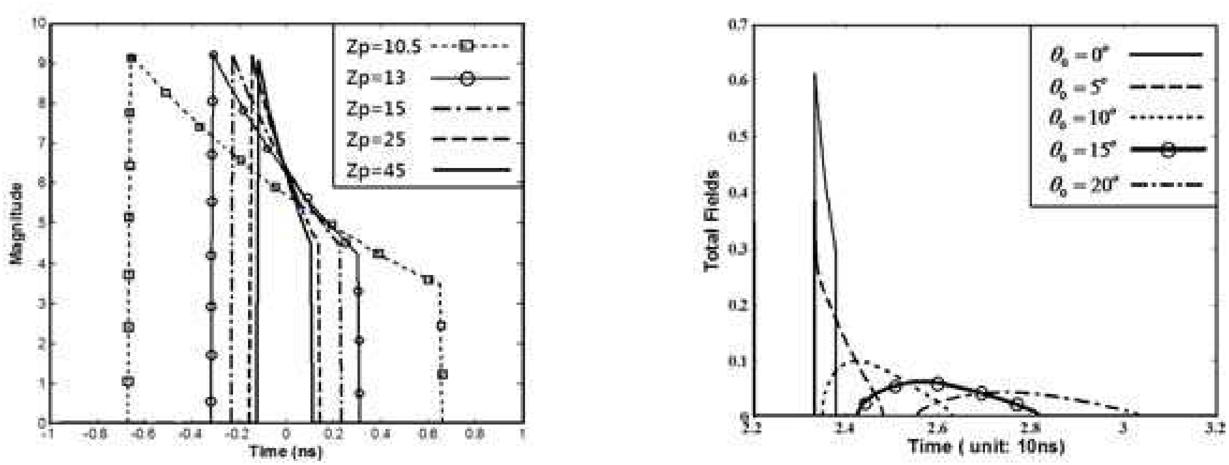


Figure 2: Transient scattering field with respect to varying observation angles.



(a) (b)

Figure 3: TD field response of scattering fields. (a) Observation on the axis. (b) Observation at $r_o = 15\,\text{m}$.

4. CONCLUSIONS

This paper will present the near field time domain transient closed-form solution for elliptic reflector illuminated by Huygen's Source at the focus. That employs an asymptotic high-frequency geometrical optics (GO)-based approximation for the fields in an equal-phase surface aperture which appears to be a sphere centered at the second focus of the ellipse. Then we discuss the TD analysis of a hyperbolic reflector based on the mathematic continuation of an ellipsoidal reflector antenna. The formulation has been developed with numerical results presented to validate the results.

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