On the Theory of Transition Radiation in the Anisotropic Magneto Dielectric Plate in a Waveguide

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Abstract— The transition radiation of charged particle in a regular waveguide of arbitrary cross section filled with anisotropic magneto dielectric plate of finite length is considered. It is assumed that the charged particle crosses the plate perpendicular to the waveguide axis. The wave equations and the analytical expressions for the transverse-electric (TE) field in various regions in the waveguide are obtained. Taking into account the obtained analytical expressions for TE field and using the Poynting vector the energy of transition radiation of moving particle is calculated. The case of rectangular waveguide is considered and the energy of transition radiation is analyzed in this case. The analytical expression of the transition radiation of charged particle for the case of thin plate in the waveguide (the wavelength in the plate is much grater than the length of the plate) is found. The possibility of appearance of Vavilov-Cerenkov radiation is analyzed. The conditions under which occurs the Cerenkov radiation are found. The frequency of Cerenkov radiation is determined too.

1. INTRODUCTION

The study of transition radiation of charged particle in the anisotropic magneto dielectric plate in the waveguide presents the great interest both from point of view of development of the theory and possibilities of wide application of the effect of transition radiation in practice [1]. In [2, 3] the transition radiation of charged particle moving perpendicular to the waveguide axis with anisotropic magneto dielectric filling was considered. In this article the transition radiation of charged particle in the anisotropic magneto dielectric plate in the waveguide is considered, when the particle crosses the plate perpendicular to the waveguide axis.

2. STATEMENT OF THE PROBLEM AND ITS SOLUTION

Consider the regular waveguide of arbitrary cross-section which axis coincides with oz axis of some rectangular coordinate system. Let in the waveguide placed the anisotropic magneto dielectric plate which occupies the region $-d \le z \le d$. Suppose that the permittivity and permeability of the plate have the form

$$\widehat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}, \tag{1}$$

where $\varepsilon_1 = const$, $\varepsilon_2 = const$, $\mu_1 = const$, $\mu_2 = const$.

Let the particle with charge q moves with constant velocity $\vec{v} = \{v, 0, 0\}$ in the direction perpendicular to the axis of the waveguide and crosses the surface of the plate in the points $A_1(x_1, y_0, 0)$ and $A_2(x_2, y_0, 0)$. In this case the charged density the current density of the particle are described with help of Dirac's δ function in he form [4, 5]

$$\rho = q \cdot \delta(x - vt) \cdot \delta(y - y_0) \cdot \delta(y), \quad j = j_x = q \cdot v \cdot \delta(x - vt) \cdot \delta(y - y_0) \cdot \delta(y). \tag{2}$$

Transverse-electric (TE) filed, as in our early articles (see [6, 7]), we shall describe by longitudinal component of magnetic vector $H_z(x, y, z, t)$. The wave equation for H_z is received from Maxwell equations

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad rot\vec{H} = j + \frac{\partial \vec{D}}{\partial t}, \quad div\vec{D} = \rho, \quad div\vec{B} = 0, \quad D = \varepsilon_0 \widehat{\varepsilon} \vec{E}, \quad \vec{B} = \mu_0 \mu \vec{H}, \tag{3}$$

where $\varepsilon_0 = (36\pi \cdot 10^3)^{-1} F/m$ is dielectric constant, $\mu_0 = 4\pi \cdot 10^{-7} H/m$ is magnetic constant. In Fourier representation in different regions of the waveguide its has a form:

I and III regions ($z \geq d$ and $z \leq -d$)

$$\Delta_\perp H_{\omega z} + \frac{\partial^2 H_{\omega z}}{\partial z^2} + \varepsilon_0^2 \mu_0^2 \omega^2 H_{\omega z} = 0, \tag{4}$$

II region ($-d \leq z \leq d$)

$$\Delta_\perp H_{\omega z} + \frac{\mu_2}{\mu_1} \cdot \frac{\partial^2 H_{\omega z}}{\partial z^2} + \varepsilon_0 \mu_0 \varepsilon_1 \mu_2 \omega^2 H_{\omega z} = \frac{\partial j_\omega}{\partial y}, \tag{5}$$

where

$$j_\omega = \frac{1}{\sqrt{2\pi}} \cdot q \cdot e^{-i\omega \frac{x}{v}} \cdot \delta(z) \cdot \delta(y - y_0), \tag{6}$$

$\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is a two-dimensional Laplace operator.

The solutions of the Equations (4) and (5) we shall look in the form

$$H_{\omega z} = \sum_{n=0}^{\infty} H_n(z) \cdot \widehat{\psi}_n(x, y), \tag{7}$$

where the orthonormal eigenfunctions $\widehat{\psi}_n(x, y)$ of the second boundary-value problem for the cross-section of the waveguide (the Neumann problem) satisfy the Helmholtz's equation with corresponding boundary condition

$$\Delta_\perp \widehat{\psi}_n(x, y) + \widehat{\lambda}_n^2 \widehat{\psi}_n(x, y) = 0, \left. \frac{\partial \widehat{\psi}_n(x, y)}{\partial \vec{n}} \right|_\Sigma = 0. \tag{8}$$

Note that in (8) Σ is the counter of the cross-section of the waveguide, $\vec{n}$ is the normal to Σ, $\widehat{\lambda}_n$ are the eigenvalues of the Neumann problem. The transverse components of the TE filed are obtained from Maxwell's equations and have a form:

I and III regions ($z \geq d$and$z \leq -d$)

$$\vec{H}_{\omega\tau} = \sum_{n=0}^{\infty} \widehat{\lambda}_n^{-2} \frac{dH_n(z)}{dz} \nabla \widehat{\psi}_n(x, y), \quad \vec{E}_{\omega\tau} = -i\mu_0^2 \omega \sum_{n=0}^{\infty} \widehat{\lambda}_n^{-2} H_n(z) \left[\vec{z}_0 \nabla \widehat{\psi}_n(x, y) \right], \tag{9}$$

where τ refers to the transverse components, $\nabla = \vec{i}(\partial/\partial x) + \vec{j}(\partial/\partial y)$ is the operator nabla, $\vec{z}_0$ is the unit vector of the oz axis.

II region ($-d \leq z \leq d$)

$$\vec{H}_{\omega\tau} = \frac{\mu_2}{\mu_1} \sum_{n=0}^{\infty} \widehat{\lambda}_n^{-2} \frac{dH_n(z)}{dz} \nabla \widehat{\psi}_n(x, y), \quad \vec{E}_{\omega\tau} = -i\mu_0 \mu_2 \omega \sum_{n=0}^{\infty} \widehat{\lambda}_n^{-2} H_n(z) \left[\vec{z}_0 \nabla \widehat{\psi}_n(x, y) \right], \tag{10}$$

Substituting (7) in (4) and (5) in view of (8) and making integration over the cross-section of the waveguide for $H_n(z)$ in different regions of the waveguide we obtain the following equations:

I and II regions ($z \geq d$ and $z \leq -d$)

$$\frac{d^2 H_n(z)}{dz^2} + \widehat{\gamma}_n^2 H_n(z) = 0, \tag{11}$$

where

$$\widehat{\gamma}_n^2 = \varepsilon_0^2 \mu_0^2 \omega^2 - \widehat{\lambda}_n^2. \tag{12}$$

II region ($-d \leq z \leq d$)

$$\frac{d^2 H_n(z)}{dz^2} + \widehat{\Gamma}_n^2 H_n(z) = -\frac{\mu_1}{\mu_2} \cdot \frac{q\delta(z)}{\sqrt{2\pi}} \cdot \widehat{B}_n, \tag{13}$$

where

$$\widehat{\Gamma}_n^2 = \frac{\mu_1}{\mu_2}\left(\varepsilon_0\mu_0\varepsilon_1\mu_2\omega^2 - \widehat{\lambda}_n^2\right), \quad \widehat{B}_n = \int\limits_{x_1}^{x_2} \left.\frac{\partial\widehat{\psi}_n(x,y)}{\partial y}\right|_{y=y_0} \cdot e^{-i\omega\frac{x}{v}}dx. \tag{14}$$

The solution of differential Equations (11) and (13) in view of a condition of radiation (there are no waves running to the plate from infinity) and the fact, that the directions $z \geq d$ and $z \leq -d$ are equivalent, results to the following expressions:

$$\begin{aligned} &H_{n1}(z) = \widehat{a}_n \cdot e^{-i\widehat{\gamma}_n z}, \quad H_{n3}(z) = \widehat{b}_n \cdot e^{i\widehat{\gamma}_n z}, \\ &H_{n2}(z) = \widehat{c}_n \cdot e^{i\widehat{\Gamma}_n z} + \widehat{d}_n \cdot e^{-i\widehat{\Gamma}_n z} - \frac{iq\mu_1}{2\mu_2} \cdot \frac{e^{i\widehat{\Gamma}_n|z|}}{\widehat{\Gamma}_n} \cdot \widehat{B}_n. \end{aligned} \tag{15}$$

The unknown coefficients $\widehat{a}_n$, $\widehat{b}_n$, $\widehat{c}_n$ and $\widehat{d}_n$ we shall obtain from the boundary conditions at the $z = \pm d$. They have a form

$$\begin{aligned} &z = -d \quad \mu_0 H_{n3}(z) = \mu_2 H_{n2}(z), \quad \frac{dH_{n3}(z)}{dz} = \frac{\mu_2}{\mu_1} \cdot \frac{dH_{n2}(z)}{dz}, \\ &z = d \quad \mu_2 H_{n2}(z) = \mu_0 H_{n1}(z), \quad \frac{\mu_2}{\mu_1} \cdot \frac{dH_{n2}(z)}{dz} = \frac{dH_{n1}(z)}{dz}, \end{aligned} \tag{16}$$

Substituting (15) in (16) and solving the resulting system of equations for the unknown coefficients we find

$$\widehat{c}_n = \widehat{d}_n = \frac{\widehat{\beta}_n}{\widehat{\beta}_n^{+} \cdot e^{i\widehat{\Gamma}_n d} + \widehat{\beta}_n^{-} \cdot e^{-i\widehat{\Gamma}_n d}}, \tag{17}$$

$$\widehat{a}_n = \widehat{b}_n = \frac{\mu_2}{\mu_0} \cdot \frac{\widehat{\beta}_n}{\widehat{\beta}_n^{+} \cdot e^{i\widehat{\Gamma}_n d} + \widehat{\beta}_n^{-} \cdot e^{i\widehat{\Gamma}_n d}} \cdot \left[e^{i\left(\widehat{\Gamma}_n + \widehat{\gamma}_n\right)d} + e^{-i\left(\widehat{\Gamma}_n - \widehat{\gamma}_n\right)d}\right] - i\frac{q\mu_1}{2\mu_0} \cdot \frac{\widehat{B}_n}{\widehat{\Gamma}_n} \cdot e^{i\left(\widehat{\Gamma}_n + \widehat{\gamma}_n\right)d}, \tag{18}$$

where

$$\widehat{\beta}_n = i\frac{q\mu_1}{2\mu_2} \cdot \widehat{B}_n \cdot \left(\frac{\mu_1\widehat{\gamma}_n}{\widehat{\Gamma}_n} + \mu_0\right) \cdot e^{i\widehat{\Gamma}_n d}, \quad \widehat{\beta}_n^{\pm} = \mu_1\widehat{\gamma}_n \pm \mu_0\widehat{\Gamma}_n. \tag{19}$$

The energy of transition radiation for $z > d$ can be calculated using the Poynting vector according to the formula

$$S_{zn}^{(1)} = \frac{1}{2}\mathrm{Re}\int\limits_{-\infty}^{\infty} \iint\limits_{s} \left(E_{n1x} \cdot H_{n1y}^{*} - H_{n1x}^{*} \cdot E_{n1y}\right) ds dt, \tag{20}$$

where s is a cross-sectional area of the waveguide, H_{n1y}^{*} and H_{n1x}^{*} are the complex conjugate quantities of H_{n1y} and H_{n1x}. Then from (20) with help of (9) and having lead integration on s we shall receive

$$S_{zn}^{(1)} = \frac{\mu_0^2}{\sqrt{2\pi}\widehat{\lambda}_n^2}\mathrm{Re}\int\limits_{0}^{\infty} \widehat{\gamma}_n \cdot |H_{n1}(z)|^2 \omega d\omega. \tag{21}$$

If we now take into account the expressions for $H_{n1}(z)$ (see (15)) and $\widehat{a}_n$ (see (18)) then (21) can be converted to the form

$$S_{zn}^{(1)} = \frac{q^2\mu_0^2\mu_1^2}{16\sqrt{2\pi}\widehat{\lambda}_n^2}\mathrm{Re}\int\limits_{0}^{\infty} \frac{\widehat{\gamma}_n \left|\widehat{B}_n\right|^2 \omega}{\mu_1^2\widehat{\gamma}_n^2\cos^2\widehat{\Gamma}_n d + \mu_0^2\widehat{\Gamma}_n^2\sin^2\widehat{\Gamma}_n d} \cdot d\omega. \tag{22}$$

The total energy of transition radiation for $z > d$ has the form $S^{(1)} = \sum\limits_{n} S_{zn}^{(1)}$, where the summation is carried out by propagating in the waveguide modes.

Let us consider the case of rectangular waveguide, assuming that the charged particle in its motion crosses the surface of the waveguide at the points $A_1(0;a)$ and $A_2(0;b)$. Caring out the integration on x from 0 to a in expression for $\widehat{B}_n$ (see (14)) taking into account, that $\widehat{\psi}_n(x,y)$ expressed by the formula

$$\begin{gathered}\widehat{\psi}_n\left(x,y\right)=\widehat{\psi}_{n,m}\left(x,y\right)=\sqrt{\frac{\delta_n\delta_m}{ab}}\cdot\cos\frac{\pi m}{a}x\cdot\cos\frac{\pi n}{b}y,\\ \delta_p=2,\quad p\neq 0,\quad \delta_0=1,\quad \widehat{\lambda}_n=\widehat{\lambda}_{n,m}=\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}\end{gathered}\tag{23}$$

and substituting in (22), for $S_{zn,m}^{(1)}$ we shall receive

$$S_{zn,m}^{(1)}=\frac{q^2\mu_0^2\mu_1^2\sqrt{\pi^3}n^2\delta_n\delta_m\sin^2\frac{\pi n}{b}y_0}{4\sqrt{2}\,\widehat{\lambda}_{n,m}^2ab^3v^2}\cdot\int\limits_{\omega_1}^{\infty}\frac{\widehat{\gamma}_{n,m}\omega^3}{\mu_1^2\widehat{\gamma}_{n,m}^2\cos^2\widehat{\Gamma}_{n,m}d+\mu_0^2\widehat{\Gamma}_{n,m}^2\sin^2\widehat{\Gamma}_{n,m}d}\cdot\frac{\sin^2\left(\frac{\pi m}{a}-\frac{\omega}{v}\right)\frac{a}{2}}{\left[\left(\frac{\pi m}{a}\right)^2-\frac{\omega^2}{v^2}\right]^2}d\omega,\tag{24}$$

where $\omega_1=\widehat{\lambda}_{n,m}/\varepsilon_0\mu_0$ is the solution of the equation $\widehat{\gamma}_{n,m}=0$.

From the received expression (24) follows that at performance of conditions

$$\frac{\pi n y_0}{b}=k\pi,\quad k=1,2,3,\ldots,\quad \left(\frac{\pi m}{a}-\frac{\omega}{v}\right)\frac{a}{2}=l\pi,\quad l=1,2,3,\ldots\tag{25}$$

in the transition radiation will be no corresponding modes and corresponding frequencies.

As shown in [2–5] under the realization of the condition $\varepsilon_0\mu_0\varepsilon_2\mu_1v^2>1$ at the frequency $\omega_m=\pi mv/a$, which is determined from the equation $(\pi m/a)-(\omega/v)=0$, in the spectrum of the transition radiation appears the sharp peak of the Vavilov-Cerenkov radiation. The width of Cerenkov radiation is determined by the formula $\Delta\omega_{n,m,cer.}=4\pi v/a$. Note that the Cerenkov radiation of the charged particle is locked in the plate and is not propagated in the waveguide.

3. CONCLUSION

The received expression of the energy of transition radiation of the charged particle shows that firstly, under the certain conditions in the radiation will be absent some modes and some frequencies, secondly, under the condition of the appearance of Cerenkov radiation around a certain frequency occurs the Cerenkov radiation, which is not coming out of the plate.

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