Dissipation-induced Super Scattering from $\boldsymbol{PT}$-synthetic Plasmonic Metafilms

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Abstract— We found that the extraordinary transmission and reflection of a finite bandwidth can occur at the same wavelength when the electromagnetic wave is incident on a periodic array of $\mathcal{PT}$-symmetric dimers embedded in a metallic film. Remarkably, this phenomenon vanishes if the metallic substrate is lossless while keeping other parameters unchanged. When the metafilm is adjusted to the vicinity of a spectral singularity, tuning the substrate dissipation to a critical value can lead to supper scattering in stark contrast to what would be expected in conventional systems. This phenomenon implies that strong coherent radiation may be able to generate from a cavity having gain elements by tuning cavity dissipation to a critical value.

1. INTRODUCTION

In a pioneering work, Bender and colleagues proved that non-Hermitian Hamiltonian with parity-time ($\mathcal{PT}$) symmetry may exhibit entirely real spectrum below a phase transition (symmetry breaking) point [1]. Inspired by this emerging concept, in the past decade there has been a growing interest in studying $\mathcal{PT}$-symmetric Hamiltonian in the framework of optics [2–19] where the $\mathcal{PT}$ complex potential in quantum mechanics is translated into a complex permittivity profile satisfying $\epsilon(r) = \epsilon^*(-r)$ in optical systems. In optics, most of the $\mathcal{PT}$-symmetric structures are realized by parallel waveguides or media with alternating gain and loss either along or across the propagation direction. The $\mathcal{PT}$-symmetric systems are a subset of open quantum systems for which Hamiltonian is non-Hermitian and the eigenvalues are complex in general [20]. The unique properties associated with non-Hermitian Hamiltonian are exceptional points and spectral singularities. An exceptional point is associated with level repulsion [21] and symmetry breaking [14, 19, 22]. Spectral singularity is related to scattering resonance of non-Hermitian Hamiltonian and manifest itself as giant transmission and reflection with vanishing bandwidth [23, 24].

Parity-time synthetic materials represent a new class of metamaterials with novel electromagnetic properties arising from a delicate balance between loss and gain elements. Global $\mathcal{PT}$ symmetry is a demanding condition. Systems with local $\mathcal{PT}$-symmetry are easier to implement. Array of $\mathcal{PT}$-symmetric dimers where each gain-loss pair in itself possesses local $\mathcal{PT}$-symmetry with respect to its own center allows for the real spectrum in the right parameter region [2]. Except for compensating loss with gain, active plasmonic materials offer an ideal platform for studying non-Hermitian Hamiltonian in the electromagnetic domain at the subwavelength scale. Most studies on $\mathcal{PT}$-symmetric structures use analytical models based on either one dimensional scalar Helmholtz equation or two dimensional scalar paraxial wave equation. For plasmonic metamaterials having subwavelength "meta-atom" as resonators, above analytical descriptions are not applicable. In this paper, we investigate electromagnetic properties of $\mathcal{PT}$-synthetic plasmonic metafilm which is composed of a planar array of coupled $\mathcal{PT}$-symmetric dimers. We have found that this structure can display super scattering by control of plasmonic substrate dissipation. When the metafilm is steered to the vicinity of a singular region of the system, tuning the substrate dissipation to a critical value can lead to substantially amplified waves radiated from both sides of the metafilm.

2. THEORETICAL APPROACH

Figure 1 schematically depicts a unit cell in a planar subwavelength square array of gain-loss elements embedded in a ultra-thin metallic film. The gain-loss dimer repeats in the x-y plane. The plasmonic metafilm satisfies the local $\mathcal{PT}$ symmetry with respect to the x directions, i.e., $\epsilon(x, y, z) = \epsilon^*(-x, y, z)$ for $\Delta x/2 < |x| < b + \Delta x/2$. This structure cannot be described by the paraxial wave equation due to the abrupt change of electromagnetic (EM) field at the metal-dielectric interfaces. We numerically solve Maxwell's equations based on rigorous coupled-wave analysis [25]. Numerical approaches can handle more complicated structures and guide engineering designs to search for the right parameter combination. This is important for practical implementation of the extraordinary properties predicted by analytical theory.

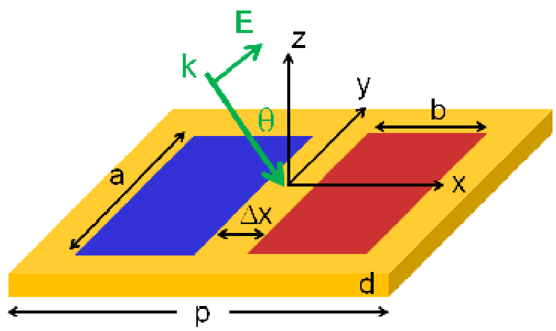

Figure 1: (Color online) A schematic showing a $\mathcal{PT}$-symmetric unit cell composed of loss (blue) and gain (red) subwavelength elements embedded within an aluminum thin-film. The dimers and the aluminum film have the same thickness, i.e., the metallic mesh is filled with gain-loss elements. The unit cell repeats in the x-y plane with the same period in both directions. The real part of the relative permittivity of the loss and gain elements is fixed at 3.6 through out this work. The imaginary part varies, but satisfies $\epsilon''_{gain} = -\epsilon''_{loss}$ to ensure the local $\mathcal{PT}$ symmetry. The permeability is unit for all the materials. The period $p = 3.5\,\mu$m, the dimer length $a = 2.5\,\mu$m and width $b = 1.0\,\mu$m are fixed throughout the paper. The incidence wave is p-polarized with the electric field parallel to the x-z plane.

For the infrared $\mathcal{PT}$-synthetic materials, the dispersion of metal, which is aluminum (Al) in our case, cannot be neglected. Assume a harmonic time dependence $\exp(-i\omega t)$ for electromagnetic field, the permittivity of Al was obtained by curve-fitting experimental data [26] with a Drude model,

$$\epsilon_m = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{1}$$

where the "plasma frequency" $\omega_p = 9.38\,\mu\mathrm{m}^{-1}$ and the damping constant $\gamma = 0.048\,\mu\mathrm{m}^{-1}$. Maxwell's equations can be recast in a Schrödinger-type form:

$$i\frac{\partial}{\partial z}\begin{pmatrix} E_t \\ \hat{\mathbf{z}} \times H_t \end{pmatrix} = \widetilde{H} \cdot \begin{pmatrix} E_t \\ \hat{\mathbf{z}} \times H_t \end{pmatrix}, \tag{2}$$

where the subscript 't' refers to the transverse (x, y) components of the EM field on the meta-surface; and $\hat{\mathbf{z}}$ is the unit vector along the z direction. The Hamiltonian is given by

$$\widetilde{H} = \begin{pmatrix} 0 & k_0\mu_t\hat{\mathbf{I}}_t + \frac{1}{k_0}\nabla_t\frac{1}{\epsilon_z}\nabla_t \\ k_0\epsilon_t\hat{\mathbf{I}}_t + \frac{1}{k_0}\hat{\mathbf{z}} \times \nabla_t\frac{1}{\mu_z}\hat{\mathbf{z}} \times \nabla_t & 0 \end{pmatrix}, \tag{3}$$

where $k_0 = \omega/c$, and the c is the speed of light in vacuum. The subscript 'z' refers to the component in the z direction. $\hat{\mathbf{I}}_t = \hat{\mathbf{I}} - \hat{\mathbf{z}}\hat{\mathbf{z}}$ is the two-dimensional unit dyadic; and

$$\nabla_t \equiv \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y}. \tag{4}$$

The form of Eq. (3) can handle uniaxial anisotropic materials with the optical axis along the z direction. The Hamiltonian given by Eq. (3) is non-Hermitian. The scattering and transfer matrices, as well as the transmittance and the reflectance are calculated numerically. The transfer matrix which connects the field at the output and the input surfaces is defined as

$$|\Psi_i\rangle = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}|\Psi_o\rangle, \quad |\Psi\rangle \equiv \begin{pmatrix} E_t \\ \hat{\mathbf{z}} \times H_t \end{pmatrix}. \tag{5}$$

where the subscripts 'o' and 'i' refer to the EM field at the output and input surfaces, respectively. The relationship between the transfer and scattering matrices in our case is given by

$$\begin{aligned} S_{11} &= M_{21}M_{11}^{-1}, \quad S_{21} = M_{11}^{-1}, \\ S_{22} &= -M_{11}^{-1}M_{12}, \quad S_{12} = M_{22} - M_{21}M_{11}^{-1}M_{12}, \end{aligned} \tag{6}$$

where the S_{11} and S_{21} are, respectively, the reflection and transmission coefficients of the electric field. In general, the transfer and scattering matrices are multidimensional due to multiple scattering channels. We have conducted extensive numerical studies and confirmed that for our geometry the magnitudes of the higher order and cross-polarization scatterings are much smaller than that of the first order event due to the subwavelength nature of the films. Therefore, the transfer and scattering matrices can be reduced to 2×2 matrices. The problem can be effectively described in a two-dimensional space.

3. SUPER SCATTERING

In the geometry of Fig. 1, many parameters can be changed. In this work, the period ($p = 3.5\,\mu\text{m}$) of the square array, the size ($a \times b = 2.5 \times 1.0\,\mu\text{m}^2$) of the dimers, and the real part of the relative permittivity ($\epsilon'_r = 3.6$) of the dimers are fixed throughout the paper. For a comparison we show in Fig. 2 the transmittance (T) and reflectance (R) of a normal incident electromagnetic wave onto a lossless metafilm with lossless dimers. The frequency dependent permittivity of aluminum was taken from the real part of Drude model given by Eq. (1). In the lossless case we have $T + R = 1$ (energy conservation) which is clearly demonstrated in Fig. 2. An increase of the transmission is accompanied by a decrease of the reflection and vice versa. The peaks and valleys of the transmittance and the reflectance repeat periodically with the variation of the thickness of the film. The metafilm behaves like a low-Q Fabry-Pérot cavity below $9\,\mu\text{m}$. Above $12\,\mu\text{m}$, the metafilm behaves towards a perfect electric conductor (PEC).

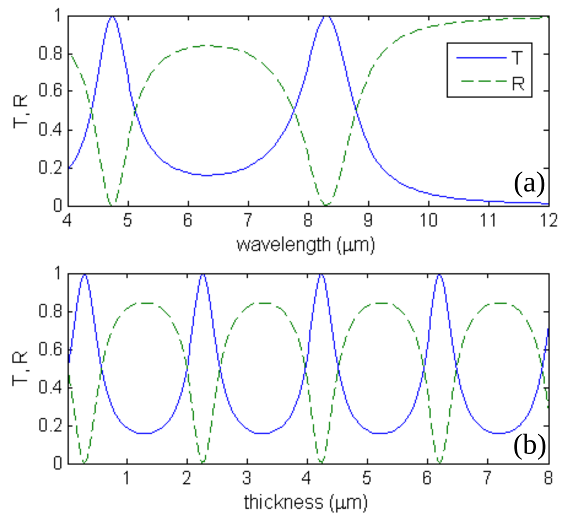

Figure 2: (Color online) Transmittance (blue solid) and reflectance (green dashed) of the normal incidence wave on a lossless metafilm versus (a) wavelength and (b) thickness with the electric field parallel to the shorter edge of the dimers. The separation of the two dimers $\Delta x = 0.5\,\mu\text{m}$ for both cases. The thickness of the metafilm $d = 1.5\,\mu\text{m}$ for the upper panel and the wavelength $\lambda = 6\,\mu\text{m}$ for the lower panel. The relative permittivity of the dimers is real and given by $\epsilon_r = 3.6$.

Now we proceed to investigate the EM scattering from a plasmonic thin-film having balanced gain-loss elements. The $\mathcal{PT}$-synthetic plasmonic metafilm no longer behaves as a Fabry-Pérot cavity. To demonstrate the effect of the substrate dissipation, we investigate the transmittance and reflectance of the metafilm with and without plasmonic dissipation in the presence of the balanced gain-loss dimers. The Fig. 3(a) shows the transmittance and reflectance of the EM wave normally incident on the $\mathcal{PT}$-synthetic lossless plasmonic metafilm of which the dispersion is given by the real part of Drude model in Eq. (1). The relative permittivity of the dimers is given by $\epsilon = 3.6(1 \pm 0.06i)$. Clearly, the sum of the transmittance and reflectance is close to unit, i.e., $T + R = 1$. Overall the metafilm performs as a conventional lossless medium where the maximum transmission is accompanied by the minimum reflection, and vice versa. This behavior can be understood as the subwavelength dimers have balanced gain-loss profile and are embedded in the lossless substrate. The situation changes dramatically when the plasmonic substrate dissipation is taken into account as shown in the Fig. 3(b) where the giant transmittance and reflectance occur at the same wavelength, unlike conventional media where the increase of one at the expense

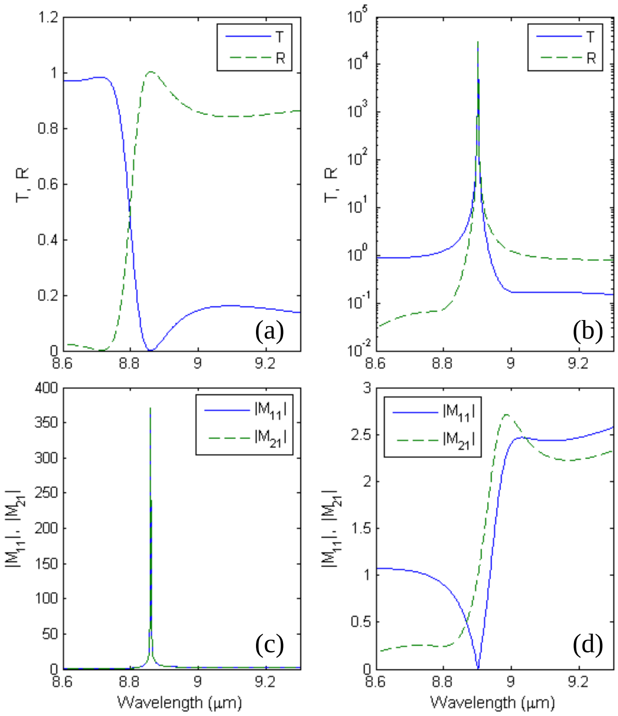

Figure 3: (Color online) Simulation (a), (c) without and (b), (d) with metallic substrate dissipation. (a), (b) Transmittance (blue solid) and reflectance (green dashed) for the normal incidence with the electric field parallel to the shorter edge of the dimers. (c), (d) The magnitude of M_{11} (blue solid) and M_{21} (green dashed). The separation of the two dimers $\Delta x = 0.5\,\mu\text{m}$; and the thickness of the mesh $d = 1.5\,\mu\text{m}$. The relative permittivity of the gain/loss elements $\epsilon = 3.6\ (1 \pm 0.06i)$.

of the other. This peculiar property can be analyzed through the scattering parameters S_{11} and S_{21} that have a common denominator M_{11} as shown by Eq. (6). When the denominator of S_{11} and S_{21} vanishes, both transmittance and reflectance approach infinity as long as M_{21} is finite. Figs. 3(c) and 3(d) show the magnitude of M_{11} and M_{21} with and without the substrate dissipation described by the Drude model in Eq. (1). Without the substrate dissipation (Fig. 3(c)), both magnitude of M_{11} and M_{21} are large at the wavelength about $8.88\,\mu\text{m}$ which explains the null in the transmittance and the peak in the reflectance (see Fig. 3(a)). When turn on the substrate dissipation (Figs. 3(d)), the $|M_{11}|$ vanishes at the wavelength about $8.92\,\mu\text{m}$ whereas the $|M_{21}|$ is finite. Thus, both transmission and reflection approach infinity. Here the spectral singularities are manifested as the dissipation-induced super scattering.

4. CONCLUSION

We have numerically demonstrated the dissipation-induced super scattering phenomenon when the system parameters are steered to the vicinity of the spectral singularity. This phenomenon implies that tuning cavity dissipation can lead to a super radiation. Our numerical experiment with the dispersive material parameters is one step further toward practical applications of $\mathcal{PT}$-synthetic materials. The existence of the narrow-band giant scattering phenomenon reveals a new type of resonant effect with potential applications in many areas, such as scattering cross sections, notch filters, and directional couplers for highly sensitive target identifications of biological and chemical agents.

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