# Monitoring for Resonant Mode of High-way System at Impulsive Impact

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**Abstract**— This note concerns on monitoring for a resonant mode of a high-way system at an impulsive impact. This work might be helpful for elementary finding of a key to show an advanced techniques at design, constructing, and maintaining of the life line systems which should be in a resonant mode for safety of public use even under any severe seismic condition on the surface of the earth.

## 1. INTRODUCTION

This note concerns on monitoring for a resonant mode of a high-way system at an impulsive impact. This work might be helpful for elementary finding of a key to show an advanced techniques at design, constructing, and maintaining of the life line system which should be in a resonant mode for public use even under any severe seismic condition on the surface of the earth.

## 2. A REVERSED PENDULUM UPSIDE DOWN

The most elementary system of the high-way system is a reversed pendulum upside down on the flat ground surface.

Now, assume a pole is set vertically on a flat surface to be horizontal in the gravity field. This pole must be stiffly fixed on the ground. Then, the pole can be an example of the reversed pendulum. If the string is made of a straight steel line, dynamics of this line can be formulated to have an simple harmonic oscillation. The frequency of the oscillation is determined by the size of the line and the essential elastic properties of the material which is consisting in the interested line.

When a weight is placed to set and fix at the top of the line. This line with the weight forms an elastic pendulum system.

In a case of a cylindrical pole made of concrete, this pole is equivalent to the steel line for a small amplitude though the concrete is not resistive to any forcing of tension, generally. Hence, a concrete pole with a buried steel skeleton is widely used in practice.

Then, the concrete pole can be equivalent under a support of a steel skeleton system to resisting external bending force which cause a tension and compression to the pole though the critical of the pole is taken in account of consideration not to breaking the pole.

Nevertheless, each of these poles made by a reinforced concrete cannot be resistive against some strong shock or an impulsive impact beyond its elastic critical.

## 3. A HIGH-WAY SYSTEM

A high-way system is constructed often using the space produced above an arranged linage of the multiple reinforced poles (each of poles with a top weight).

It is important to take care of the resonant mode of oscillations in practice. If not, this arrangement cannot be effective for the function of the aimed high-way system construction.

## 4. EXTERNAL FORCING

As for a possible external forcing to the high-way system noted above, it is essential to consider loading on the traffic above the top of the pole arrangement. In addition, some consideration should be required to consider about the local conditions of the ground surface.

Locality affects to the safety of the high-way system. Especially, the traffic in the seismic zones where some dangerous factor of accidents should be considered.

Concerning to a reinforced pole with a loading at the top, it is hard to construct the footing never yield to be in strength enough against a vertical impulsive impact as well as that for a possible horizontal forcing.

Especially, the reinforced pole structure is fragile at a vertical impulsive impact.

A specific manner of formulation for dynamics on a reinforced pole can be described as that might be seen in the field of structural mechanics for civil engineering or architectural engineering.

## 5. MATHEMATICAL DESCRIPTION

When a mathematical expression of dynamics of a reinforced pole with a loading at the top, could be reduced for a vertical impulsive impact, then, the vertical response $\mathbf{V}$ must be,

$$\mathbf{V}(x,y,z,t) = \int f(x,y,z,t;\, U(x,t), V(y,t), W(z,t))dxdydzdt, \tag{1}$$

where, the notation $U$, $V$, and $W$ in a Cartesian co-ordinate $O$-$xyz$, and the notation $f$ is a simplified expression for denoting a complicated mathematical solution. This can be written as follow. For the author's convenience,

$$\mathbf{V} = \int f(\mathbf{x}, \mathbf{U}, t)d\mathbf{x}d\mathbf{U}dt, \quad \text{or,} \quad V = \int f(\mathbf{x}, t)d\mathbf{x}dt, \tag{2}$$

where, $\mathbf{x} = \mathbf{x}(x,y,z,t)$, and, $\mathbf{U} = \mathbf{U}(U,V,W,t)$.

Generally, such the solution of the dynamics could be in a form of an ensemble of suffix "$i$" as a series of integer. Then, the above expression for $\mathbf{V}$ can be rewritten as,

$$\mathbf{V} = \int \mathbf{V}_o \exp(\mathbf{k}_i\mathbf{x} - \omega_i t)d\mathbf{x}dt. \tag{3}$$

## 6. IMPULSIVE IMPACT

As for a vertical impulsive impact, it can be expressed in a mathematical form by using a delta function, that is, for example,

$$\mathbf{I}_j = \mathbf{I}_j(\mathbf{x}, t) = \delta(\mathbf{I}_j; t = 0), \tag{4}$$

where, $\mathbf{I}_j = \mathbf{I}_j(\mathbf{I_x}, \mathbf{I_y}, \mathbf{I_z})$.

Then, a dynamical response $\mathbf{R}$ of the pole as a reversed pendulum with a top weight can be written as, for the vertical external impulsive impact, i.e.,

$$\mathbf{R} = \mathbf{I}_j \times \mathbf{V} = \int \mathbf{I}_j V_o \exp(\mathbf{k}_i\mathbf{x} - \omega_i t)d\mathbf{k}_i dt. \tag{5}$$

Then, the above expression can be rewritten in a more simple form, i.e.,

$$\mathbf{R} = \int \delta(\mathbf{I}_j; t = 0) V_o \exp(\mathbf{k}_j\mathbf{x} - \omega_i t)d\mathbf{k}_j dt. \tag{6}$$

Now, a resonant mode of the reinforced pole with a top weight for the given external forcing in a form of an impulsive impact can be found.

As for a set of arranged poles in a formation of a high-way net work system, the resonant mode can be found after a mathematical reduction to obtain a solution in a quite similar manner as seen above.

Figure 1: A pattern similar to "Dominoes game" found in the arranged reinforced poles under the high-way system for serving traffics and lifeline net-works in Kobe in 1995 January 17 (courtesy of Kobe City).

## 7. DISCUSSIONS AND CONCLUSIONS

In a case of 1995 earthquake hit Kobe, there is kept their reminding memory that a water filled in a bath tub shed out just after the impulsive impact because the bath tub landed to leave out the tap at the bath tub flat bottom.

A scientific estimation of the acceleration at the forcing of the seismic shock in a form of an impulsive impact was evaluated to produce an acceleration of more than six gal or more. Maximum must be about ten gal or more so that the bath tub must be in non-gravity forcing for a short instance.

The episode of the bath tub in a non-gravity forcing was seen just neighbor the seismic fault formation along the seismic fault at the event crossing the urban area in Kobe.

When any one of the pole supporting the high-way system is crushed in the resonant mode after yielding or creeping beyond the elastic critical, then, the arranged reinforced poles start a dominoes game to result the lifeline system's crush (cf. Figure 1).

In memory of the hazards at the Nepal earthquake on 25 April 2015.