

Through-Silicon-Via Pairs Modelling via Compressed Sensing

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Abstract— Through-Silicon-Vias (TSVs) are the critical enabling technique for three-dimensional integrated circuits (3D ICs). While there are a few existing works in literature to model the electrical performance of TSVs, they are either for fixed geometry or in lack of accuracy. In this paper, we use compressed sensing technique to model the electrical performance of TSV pairs. Experimental results indicate that with an exceptionally small number of samples, our model has a maximum relative error of 3.70% compared with full-wave simulations over a wide range of geometry parameters and frequencies.

1. INTRODUCTION

System-in-Package (SiP) technology is widely used today in high-density and small mobile devices such as smart phones, digital cameras, media players, and so on. Three-dimensional Integrated Circuits (3D ICs) technology is being considered as a real breakthrough along this direction, which can be used to achieve higher integration density and higher performance by vertically stacking ICs. Through-Silicon-Via (TSV) is the most critical enabling technique in 3D ICs, which routes the electrical path through all stacked chips. TSV shortens the connection between vertical stacking chips which leads to better electrical performance and more compact size of the system [1–4]. A typical TSV structure can be found in Figure 1.

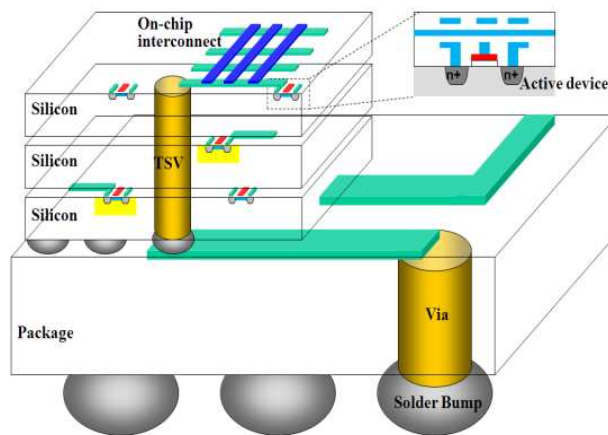


Figure 1: TSV structure in SiP.

It is important to model the TSV structure to estimate the delay, coupling noise and many other electrical performance metrics. In literature, this has been done analytically and numerically. A three dimensional full-wave solver can be used to analyse the TSV structure, but it takes a considerable amount of simulation time for each different geometrical dimension and frequency. In addition, it cannot produce a parameterized model which is desired for design optimization. Towards this, physics-based analytical models have also been developed (e.g., [4, 5]). However, the TSVs are embedded in the semi-conductive silicon substrate, so the behaviour of wave transmission is quite different from that in typical waveguide structures. Also, the properties of semiconductor devices make the rigorous analytical approach even more difficult. Accordingly, the physics-based circuit models proposed are only approximate, and are not quite accurate in a wide range of TSV geometries and frequencies.

A straightforward approach for analytical modeling of TSVs might be to build a look-up-table based on the samples with different geometry parameters and frequencies. However, the full wave simulation to obtain one sample takes a long time. Accordingly, it is not possible to build a model based on a large number of samples. With limited samples, the accuracy of the look-up-table approach is questionable.

In this paper, we propose to build the analytical model for a TSV pair based on compressed sensing technique. Compressed sensing is a recently developed technique to recover signals with randomly distributed samples. It is fast, stable and yields high reconstruction accuracy using an exceptionally small number of samples. Experimental results show that our model has a maximum relative error of 3.70% compared with full-wave simulations over a wide range of geometry parameters and frequencies.

The remainder of the paper is organized as follows: we briefly review the physics-based models in Section 2. Section 3 gives a brief review of the compressed sensing techniques and adapts the technique to the electrical modelling of TSV pairs. Experimental results are presented in Section 4 and concluding remarks are given in Section 5.

2. REVIEW OF PHYSICS-BASED MODEL (PHYSICAL BASED MODELS IS A BETTER NAME. PLEASE MAKE CORRECTIONS IN THE REMAINING OF THE PAPER)

In this paper, we focus on the structure of a pair of signal and ground TSVs, as shown in Figure 2(a). Previously, such a structure inside a lightly-doped silicon substrate has been investigated and a lumped circuit has been proposed, as shown in Figure 2(b) [5].

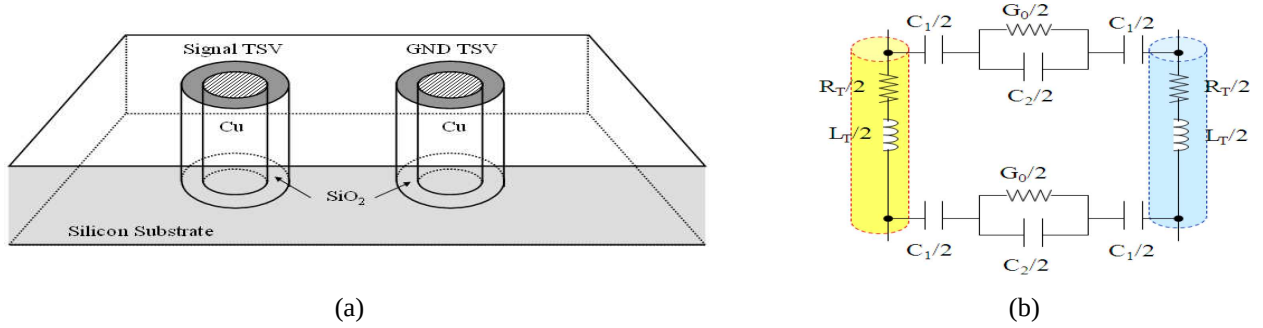


Figure 2: (a) A pair of signal and ground TSVs in a silicon substrate. (b) Lumped circuit model of the TSVs for lightly-doped case.

This structure was modeled with a 3D full-wave simulator, and the E -field distribution was analyzed to investigate a physics-based circuit model. In the model of Figure 2(b), the equations for C_1 , C_2 , G_0 , R_T , and L_T have been derived as

$$C_1 = \frac{2\pi\epsilon_{\text{SiO}_2}T_L}{\ln\left(\frac{T_D+2T_T}{T_D}\right)}, \quad (1)$$

$$C_2 = \frac{\epsilon_{\text{Si}}\pi T_L}{\ln\left(\frac{T_P}{T_D} + \sqrt{\left(\frac{T_P}{T_D}\right)^2 - 1}\right)}, \quad \text{and} \quad (2)$$

$$G_0 = \frac{\sigma_{\text{Si}}C_2}{\epsilon_{\text{Si}}}, \quad (3)$$

$$R_T = R_{dc} + R_{ac}, \quad \text{and} \quad (4)$$

$$L_T = L_{ext} + 2L_{int}, \quad (5)$$

where $R_{dc} = \frac{2T_L}{\sigma_{\text{Cu}}\pi(T_D/2)^2}$, $R_{ac} = \frac{2T_L\sqrt{\pi\mu_0\sigma_{\text{Cu}}f}}{\sigma_{\text{Cu}}\pi T_D}$ and $L_{ext} = \frac{\mu_0 T_L}{\pi \cos h^{-1}(T_P/T_D)}$, $L_{int} = \frac{T_L}{2\pi T_D} \sqrt{\frac{\mu_0}{\pi\sigma_{\text{Cu}}f}}$.

The above equations were extracted from a TSV structure, where the pitch between the two TSVs (T_P) is 20 μm , the TSV length (T_L) is 20 μm , the TSV diameter (T_D) is 10 μm , and the thickness of SiO₂ (T_T) is 0.2 μm .

For this given TSV geometry, the S -parameters extracted from the circuit model agree well with those obtained from the full-wave simulation up to 20 GHz. However, we notice that the difference becomes quite large (up to about 30%) when the TSV geometries deviates significantly from the settings listed above, or at high frequencies.

3. COMPRESSIVE SENSING BASED MODELS

3.1. Overview of Compressed Sensing

Compressed sensing is a recently developed technique in the field of signal processing. Its key idea is to use an exceptionally small number of samples to recover a desired signal, under the assumption that the signal has sparse representation in certain basis functions. In this section, we will use a one-dimensional signal to briefly review the technique.

Consider a signal $f(t)$ in the t -domain, which can be represented as

$$f(t) = \sum_{i=1}^N \psi_i \alpha_i \quad (6)$$

where ψ_i ($i = 1, \dots, N$) are the basis functions in a Hilbert space. α_i are the coefficients and can be calculated as

$$\alpha_i = \langle f, \psi_i \rangle, \quad i = 1, \dots, N \quad (7)$$

where $\langle \rangle$ operation is the inner product defined in the space spanned by the basis functions. If the basis functions are chosen properly, many of the coefficients can be zero. Specifically, if the vector α ($\alpha \in \mathbb{R}^N$) formed by the coefficients has at most k non-zero entries, we call it k -sparse.

Under the assumption that we are able to find a set of basis functions to represent $f(t)$ with k -sparse coefficients, compressed sensing enables us to accurately recover f with $M = O(k \log(N/k))$ samples, when the sampling are “random” enough to follow certain properties.

There are many different ways to choose the basis functions and to reconstruct the signal. In Section 3, we will describe one that best fits the electrical performance of TSV pairs. A more detailed description of the compressed sensing techniques, including these conditions required to apply the technique, is beyond the scope of the paper. Interested readers are referred to [6, 7] for more details.

3.2. Application to Electrical Modeling of TSV Pairs

To apply the compressed sensing technique to the electrical modelling of TSV pairs, we will first need to select a proper set of basis functions such that the S parameter, as a function of T_P , T_L , T_D , T_T and f has a sparse representation. While there are many possible candidates such as wavelet functions and polynomials, in our experiments we find that discrete cosine functions offer the best sparsity and accuracy.

In this section, we use bold to indicate a vector (e.g., $\mathbf{f}$), bold capitalization to indicate a matrix (e.g., $\mathbf{A}$), and subscript to denote the element-wise index (e.g., $\mathbf{f}_i$).

Without loss of generality, we discretize the range of interest for T_P , T_L , T_D , T_T and f and label them in integers, i.e., $T_P = \{1, 2, \dots, P\}$, $T_L = \{1, 2, \dots, L\}$, $T_D = \{1, 2, \dots, D\}$, $T_T = \{1, 2, \dots, T\}$ and $f = \{1, 2, \dots, F\}$. As such, the basis functions we selected are

$$g_{i,j,k,l,m}(T_P, T_L, T_D, T_T, f) = \cos\left(\frac{\pi(2T_P - 1)(i - 1)}{2T}\right) \dots \cos\left(\frac{\pi(2f - 1)(m - 1)}{2F}\right)$$

where $1 \leq i \leq P$, $1 \leq j \leq L$, $1 \leq k \leq D$, $1 \leq l \leq T$, $1 \leq m \leq F$.

These basis functions need some constant coefficients to normalize, but omitting them does not affect our algorithm. The S parameter function $f(T_P, T_L, T_D, T_T, f)$ can be represented using these basis functions as

$$f(T_P, T_L, T_D, T_T, f) = \sum_{i=1}^P \sum_{j=1}^L \sum_{k=1}^D \sum_{l=1}^T \sum_{m=1}^F \alpha(i, j, k, l, m) g_{i,j,k,l,m}(T_P, T_L, T_D, T_T, f) \quad (8)$$

where $\alpha(i, j, k, l, m)$ are the coefficients. (8) it is in fact the discrete cosine transform (DCT).

Next, we randomly select M ($M \ll PLDTF$) samples with geometry parameters ($T_{P_u}, T_{L_u}, T_{D_u}, T_{T_u}, f_u$) ($u = 1, 2, \dots, M$) and measure the corresponding crosstalk f_u . As such, we can obtain a set of equations based on these sampling points, i.e.,

$$f_u(T_P, T_L, T_D, T_T, f) = \sum_{i=1}^P \sum_{j=1}^L \sum_{k=1}^D \sum_{l=1}^T \sum_{m=1}^F \alpha(i, j, k, l, m) g_{i,j,k,l,m}(T_{P_u}, T_{L_u}, T_{D_u}, T_{T_u}, f_u) \quad (9)$$

It is worthwhile to note here, that in (4), the only unknowns are the coefficients $\alpha(i, j, k, l, m)$. And we can re-cast it in a compact form as

$$\mathbf{f} = \mathbf{A}\boldsymbol{\alpha} \quad (10)$$

where $\mathbf{A}$ is a constant matrix formed by $g_{i,j,k,l,m}(T_{P_u}, T_{L_u}, T_{D_u}, T_{T_u}, f_u)$. $\boldsymbol{\alpha}$ is a vector formed by $\alpha(i, j, k, l, m)$. And $\mathbf{f}$ is a vector formed by f_u .

If we can get the coefficients by directly solving (10), and insert them back to (8), we will have an analytical expression for the crosstalk estimation. Unfortunately, we will not be able to do so, because the number of equations (M), which is equal to the number of samples available, is much smaller than the number of variables ($N = PLDTF$). In other words, (10) is an underdetermined equation.

With the assumption that the coefficients $\alpha(i, j, k, l, m)$ are sparse, however, we can *approximately* solve it using an optimization. Specifically, we can solve

$$\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \quad \text{subject to } \mathbf{f} = \mathbf{A}\boldsymbol{\alpha} \quad (11)$$

where $\|\boldsymbol{\alpha}\|_0$ is the zero norm (the number of non-zeros in $\boldsymbol{\alpha}$). The meaning of such an optimization is to minimize the non-zeros in the coefficients subject to the measurement data available.

Zero-norm is a nonlinear function, and thus (11) is still very difficult to solve. Accordingly, we resort to an approximate version of (11), by replacing the zero-norm with one-norm, i.e.,

$$\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_1 \quad \text{subject to } \mathbf{f} = \mathbf{A}\boldsymbol{\alpha} \quad (12)$$

It is well established in literature that the optimal solution (12) is also sparse.

It is obvious that the quality of the compressed sensing algorithm depends on how to efficiently solve (12). While many different methods can be used such as the interior point methods [8] and the homotopy method [9], in this paper we choose to use the iteratively-weighted least squares (IRLS) method [10], as in the experiments we find that it leads to the most accurate results, with a minimum runtime.

Finally, we would like to present an extra benefit of our method. With the analytical expression (8), we are able to easily calculate the sensitivity information with respect to each geometry parameters accurately. Such information is extremely valuable to guide the design optimization.

4. EXPERIENTIAL RESULTS

To verify how the above discussed compressed sensing technique works in the electrical modelling of TSV pairs, we randomly select 400 samples in the range of interest as summarized in Table 1. HFSS is used to conduct full-wave simulation for those samples [11] to obtain the S -parameters. We then randomly select 380 of them to build the model using compressed sensing, and reserve the remaining 20 to verify the model accuracy.

Table 1: Range of the 400 samples.

Parameter		Range
T_P	TSV pitch	5 μm $\sim$ 25 μm
T_L	TSV length	10 μm $\sim$ 100 μm
T_D	TSV diameter	5 μm $\sim$ 25 μm
T_T	SiO ₂ thickness	0.1 μm $\sim$ 1 μm
f	frequency	1 GHz $\sim$ 20 GHz

We first verify that the basis functions (DCT) we used actually lead to a sparse representation of the S parameter. For this purpose, we construct a model for the magnitude of S_{11} based on the 380 sampling points, and calculate the coefficients by solving the one-norm problem (12). The results are depicted in Figure 3. This indicates that the problem is indeed suitable for our compressed sensing based technique.

To visualize where these non-zero coefficients are in the DCT basis, we fix T_D , T_T and f , and plot the coefficients with various T_P and T_L in Figure 4. As we can see from the figure, many of the non-zero coefficients are clustered near the low frequency region (close to zero), with only a few in the high-frequency region. This reflects that the function is smooth and has little sharp transitions.

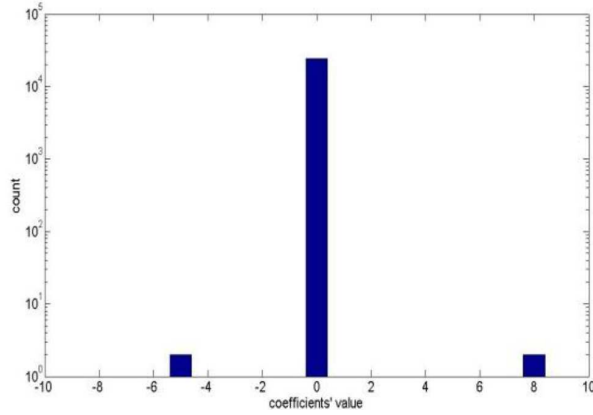


Figure 3: Histogram of the coefficients $\alpha(i, j, k, l, m)$. Note that log-scale is used for y -axis.

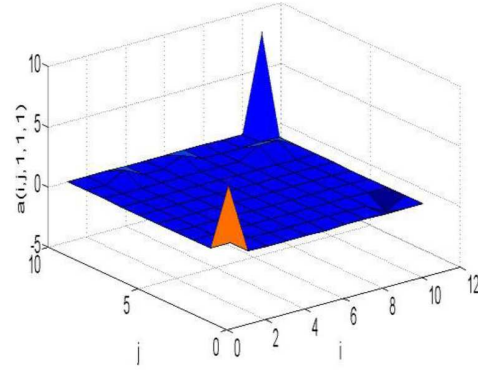


Figure 4: Plot of the coefficients $\alpha(i, j, 1, 1, 1)$.

In addition, we verify the accuracy of our model (CS) and compare it with the physics-based models [5] on the 20 reserved samples. The results are shown in Table 2. From the table we can see that the S -parameters predicted by our model closely match the results from HFSS, with an average error of 0.25% and maximum error of 0.83%, while that from [5] has an average error of 11.2% and maximum error of 29.93%.

Finally, we compare the phase and magnitude of S_{11} , S_{21} and S_{22} from our model and from the full-wave simulations. From the table we can see that our method has a maximum error of 3.94% over the 20 samples. In addition, it is also interesting to note that in general our model results in more error in phase than in magnitude. This is probably due to the existence of sharper transitions in the phase of those S -parameters.

Table 2: Accuracy verification ($S_{11}(\text{mag})$).

Parameters					HFSS	[5]		CS	
T_T (μm)	T_D (μm)	T_P (μm)	T_L (μm)	f (μm)	$S_{11}(\text{mag})$	$S_{11}(\text{mag})$	relative error	$S_{11}(\text{mag})$	relative error
0.8	23	9	90	3.0	0.0599	0.0542	9.48%	0.0597	0.26%
0.7	17	9	30	6.0	0.009	0.0092	1.95%	0.0090	0.09%
0.9	9	5	20	7.0	0.0047	0.004	15.04%	0.0047	0.09%
0.3	25	21	70	8.0	0.0379	0.0389	2.74%	0.0378	0.29%
1	9	5	40	9.0	0.0167	0.012	28.00%	0.0166	0.25%
0.7	25	11	60	10.0	0.0354	0.0363	2.66%	0.0352	0.40%
0.4	7	7	100	11.0	0.0328	0.0345	5.13%	0.0327	0.32%
0.9	13	17	20	12.0	0.0063	0.0066	4.31%	0.0064	0.80%
0.6	19	5	70	13.0	0.0503	0.0491	2.35%	0.0503	0.02%
0.9	13	17	20	14.0	0.0071	0.0069	2.86%	0.0071	0.11%
0.6	15	7	70	15.0	0.0437	0.0392	10.24%	0.0437	0.03%
0.8	23	9	90	17.0	0.0658	0.0565	14.09%	0.0656	0.31%
0.7	11	15	30	19.0	0.0107	0.0107	0.07%	0.0107	0.05%
1	9	5	40	20.0	0.0157	0.0117	25.48%	0.0157	0.02%
0.2	17	23	80	15.0	0.0387	0.0406	4.82%	0.0387	0.02%
0.3	5	11	40	13.0	0.016	0.0114	28.96%	0.0160	0.17%
0.1	15	25	50	19.0	0.0303	0.025	17.46%	0.0303	0.19%
0.9	13	17	20	7.0	0.0091	0.0076	16.66%	0.0091	0.12%
0.4	11	21	40	13.0	0.0231	0.0162	29.93%	0.0230	0.68%
0.9	13	17	20	17.0	0.0066	0.0067	1.78%	0.0066	0.83%

Table 3: Relative error (CS compared to HFSS).

	Minimum	average	maximum
S_{11} (mag)	0.02%	0.25%	0.83%
S_{11} (phase)	0.09%	0.78%	2.44%
S_{21} (mag)	0.04%	0.15%	0.73%
S_{21} (phase)	0.01%	0.76%	3.70%
S_{22} (mag)	0.00%	0.40%	1.97%
S_{22} (phase)	0.03%	1.13%	3.94%

5. CONCLUSION

In this paper, a compressed sensing based model is established to predict the S parameter of TSV pairs, as a function of the pitch of two TSVs, TSV length, TSV diameter, the thickness of SiO_2 and the frequency. It only requires a small number of samples, yet it has a maximum error of 3.70% over a wide range of geometry parameters and frequencies.

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