

Nonparaxial Propagation of Complex Variable Function Cosh-Gaussian Beams

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Abstract— By applying the superposition of beams, we obtain a group of virtual sources for a complex variable function (CVF) Cosh-Gaussian wave. We also derive a closed-form expression for the CVF-Cosh-Gaussian wave that in the appropriate limit yielding the paraxial CVF-Cosh-Gaussian beam. From the spectral representation of the CVF-Cosh-Gaussian waves, we derive the j th-order nonparaxial corrections for the corresponding paraxial CVF-Cosh-Gaussian beam analytically. The angular velocity of the nonparaxial CVF-Cosh-Gaussian beams decreases with the transmission distance increasing.

1. INTRODUCTION

In recent years, the study of spiral light beams [1–8] has drawn much attention is due to their potential application in different optic technologies including the laser manipulation [6]. The spiral beams [4–11] which have singularities and are characterized by a non-zero orbital momentum, their structure of the intensity distribution are rotated during propagation.

In 1971, Deschamps [12] has presented the virtual source method. In 1976 and 1977, Felsen and his collaborators [13, 14] have systematically developed this method. By using this method, Seshadri [15–17] has derived the virtual sources of Bessel-Gaussian, elegant Hermite-Gaussian, and cylindrical symmetric elegant Laguerre-Gaussian waves. Bandres et al. [18] have introduced the higher-order complex source that generates elegant Laguerre-Gaussian waves with radial mode number n and angular mode number m in 2004. In 2007, Zhang et al. [19] have employed this method to study the propagation properties of cosh-Gaussian beams in and beyond the paraxial regime. We have obtained the virtual sources of Elegant Hermite-Laguerre Gaussian beam [20], Hollow gaussian beam [21], and Pearcey beam [22]. In 2012, Yan et al. [23] have studied the virtual source for an Airy beam. However, the virtual sources of the complex variable function (CVF) Cosh-Gaussian beams [10, 11] have not been reported.

In this paper, based on the superposition of beams, a group of virtual sources that generate the scalar CVF-Cosh-Gaussian wave are identified. The integral and the differential representation for the CVF-Cosh-Gaussian waves have been derived. From the spectral representation of the CVF-Cosh-Gaussian waves, we obtain the exact nonparaxial solution and the n th-order nonparaxial corrections for the corresponding paraxial CVF-Cosh-Gaussian beams analytically.

2. VIRTUAL SOURCE OF CVF-COSH-GAUSSIAN BEAMS

Suppose $E(r, z)$ be a monochromatic scalar wave function that describes a CVF-Cosh-Gaussian wave propagating along the positive z axis. Function $E(r, z)$ satisfies the homogeneous Helmholtz equation for $z > 0$. The CVF-Cosh-Gaussian field is assumed to have the general form

$$E(x, y, z) = q_0/q(z) \cosh(\zeta_{\pm}) \exp(ikz) \exp(v_1), \quad (1)$$

where $q(z) = z - iz_R$, $q_0 = -iz_R$, $\zeta_{\pm} = (x \pm iy)/(bw(z)) \exp[-i\vartheta(z)]$, $w(z) = w_0 \sqrt{1 + z^2/z_R^2}$, $\vartheta(z) = \pm \arctan(z/z_R)$, $v_1 = ik(x^2 + y^2)/(2q(z))$, $\sigma = (kw_0)^{-1}$, $q_{\pm}(z) = \pm z - iz_R$. The CVF-Cosh-Gaussian beam is generated by a group of sources of strength S_{cs} situated at $r = 0$ and $z = z_{cs}$. Proper choice of S_{cs} and z_{cs} yields the desired beam. The wave function $E(x, y, z)$ satisfies the inhomogeneous Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) E(x, y, z) = -S_{cs} [\delta(x - iw_0/(2b)) \delta(y \pm w_0/(2b)) \\ + \delta(x - iw_0/(2b)) \delta(y \mp w_0/(2b))] \delta(z - z_{cs}), \quad (2)$$

Applying the Fourier transform pairs,

$$E(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{E}(f_x, f_y, z) \exp(iu_1) df_x df_y, \quad (3)$$

$$\tilde{E}(f_x, f_y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(x, y, z) \exp(-iu_1) dx dy, \quad (4)$$

where f_x and f_y are the spatial frequencies in the x and y directions, $u_1 = f_x x + f_y y$. From Eq. (2), $\tilde{E}(f_x, f_y, z)$ is obtained and substituted into Eq. (3), one can find that,

$$\begin{aligned} E(x, y, z) = & \frac{i}{4\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} df_x df_y S_{cs} / \zeta [\exp(-i(f_x w_0 / (2b) \mp f_y w_0 / (2b))) \\ & + \exp(i(f_x w_0 / (2b) \mp f_y w_0 / (2b)))] \exp[i\zeta(z - z_{cs})] \exp(iu_1), \end{aligned} \quad (5)$$

where $\text{Re}(z - z_{cs}) > 0$, $\zeta = (k^2 - (f_x^2 + f_y^2))^{1/2}$. Expanding ζ into series and keeping the first and second terms, we obtain $\zeta = k(1 - (f_x^2 + f_y^2)/(2k^2))$. Under paraxial approximation, i.e., $k^2 \gg (f_x^2 + f_y^2)$, we replace ζ of the exponential part in Eq. (5) with the approximation and the other terms with k , Eq. (5) simplifies to

$$\begin{aligned} E_p(x, y, z) = & \frac{iS_{cs}}{4\pi k} \exp[ik(z - z_{cs})] \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} df_x df_y [\exp(-i(f_x w_0 / (2b) \mp f_y w_0 / (2b))) \\ & + \exp(i(f_x w_0 / (2b) \mp f_y w_0 / (2b)))] \exp[-i(f_x^2 + f_y^2)(z - z_{cs}) / (2k)] \exp(iu_1). \end{aligned} \quad (6)$$

The additional subscript p in $E_p(x, y, z)$ means the paraxial approximation. By using the relation

$$\int_{-\infty}^{+\infty} \exp(-ax^2 + bx) dx = \sqrt{\frac{\pi}{a}} \exp\left(-\frac{b^2}{4a}\right), \quad (7)$$

one can obtain the integral of Eq. (6),

$$E_p(x, y, z) = \frac{S_{cs}}{2(z - z_{cs})} \exp[ik(z - z_{cs})] \exp\left[\frac{ikr^2}{2(z - z_{cs})}\right] \cosh\left[\frac{kw_0(x \pm iy)}{2b(z - z_{cs})}\right]. \quad (8)$$

In order to obtain the CVF-Cosh-Gaussian beam for $z > 0$, we suppose the input paraxial approximation distribution is given by Eq. (1) for $z = 0$. By comparing Eq. (8) and Eq. (1) at $z = 0$, the parameters S_{cs} and z_{cs} can be given as

$$z_{cs} = ikw_0^2/2 = iz_R, S_{cs} = 2iz_R \exp(-kz_R). \quad (9)$$

Substituting Eq. (9), into (5), one can obtain the exact integral expression for $E(x, y, z)$

$$\begin{aligned} E(x, y, z) = & -\frac{z_R}{2\pi} \exp(-kz_R) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} df_x df_y 1/\zeta [\exp(-i(f_x w_0 / (2b) \mp f_y w_0 / (2b))) \\ & + \exp(i(f_x w_0 / (2b) \mp f_y w_0 / (2b)))] \exp[i\zeta(z - iz_R)] \exp(iu_1). \end{aligned} \quad (10)$$

From Eq. (9), the source in Eq. (2) lies external to $z > 0$, and the solution given by Eq. (10) is the exact solution to the homogeneous Helmholtz equation.

3. EXACT NONPARAXIAL PROPAGATION OF A COMPLEX VARIABLE FUNCTION COSH-GAUSSIAN BEAM

Making use of the Green-function approach, we can obtain the differential representation of the CVF-Cosh-Gaussian waves. The solution of the differential equation

$$\left(\nabla_r^2 + \frac{\partial^2}{\partial z^2} + k^2\right) G(r, z) = -S_{cs}/(2\pi) \left(\frac{\delta(r_1)}{r_1} + \frac{\delta(r_2)}{r_2}\right) \delta(z - z_{cs}), \quad (11)$$

is given by [16–18] $G(r, z) = S_{cs} \exp(ikR)/(4\pi R)$, where $r_1 = \sqrt{\frac{(x - iw_0/(2b))^2 + (y \pm w_0/(2b))^2}{(z - iz_R)^2}}$, $r_2 = \sqrt{\frac{(x + iw_0/(2b))^2 + (y \mp w_0/(2b))^2 + (z - iz_R)^2}{(z - iz_R)^2}}$. $R = [x^2 + y^2 + (z - iz_R)^2]^{1/2}$. Comparing Eq. (11) and Eq. (2) and applying Eq. (9), the differential or multipole representation of the CVF-Cosh-Gaussian waves is found to be

$$E(x, y, z) = \frac{iz_R}{2} \exp(-kz_R) \left[\frac{\exp(ikr_1)}{r_1} + \frac{\exp(ikr_2)}{r_2} \right]. \quad (12)$$

From Eq. (12), we see that CVF-Cosh-Gaussian modes can be generated by superposing two complex-source-point spherical wave $S_{cs} \exp(ikR)/(4\pi R)$ corresponding to the paraxial Gaussian beam.

By using the power expansion $1/\zeta$ and $\exp[i\zeta(z - z_{cs})]$ in Eq. (5) and keeping the product of both series terms up to order σ^{2j} ($\sigma = 1/(kw_0)$), we can obtain the j th-order corrections

$$E^{(0)}(x, y, z) = E_1^{(0)}(x, y, z) + E_2^{(0)}(x, y, z) = -\frac{i}{4}Q \exp(ikz) \times \exp[iQ(x^2 + y^2)/w_0^2] \cosh[w_0Q(x \pm iy)/b], \quad (13)$$

$$E^{(2j)}(x, y, z) = (-1)^j \left[(Q\rho_1)^{2j} L_j^j(-iQ\rho_1^2) E_1^{(0)}(x, y, z) + (Q\rho_2)^{2j} L_j^j(-iQ\rho_2^2) E_2^{(0)}(x, y, z) \right], \quad (14)$$

where $E^{(0)}(x, y, z)$ is the paraxial CVF-Cosh-Gaussian beam, $E^{(2j)}(x, y, z)$ is denoting the j th-order correction to the CVF-Cosh-Gaussian beam, $Q = (z/z_R - i)^{-1}$, $E_1^{(0)}(x, y, z) = -\frac{i}{4}Q \exp(iQ\rho_1^2)$, $E_2^{(0)}(x, y, z) = -\frac{i}{4}Q \exp(iQ\rho_2^2)$, $\rho_1 = \sqrt{(x - iw_0/(2b))^2 + (y \pm w_0/(2b))^2}/w_0$, $\rho_2 = \sqrt{\frac{(x + iw_0/(2b))^2 + (y \mp w_0/(2b))^2}{(z - iz_R)^2}}/w_0$, and $L_j^j(\cdot)$ is the associated Laguerre polynomial of order j and degree j . It is easy to find that Eq. (13) is the same as Eq. (1). Eq. (14) is the nonparaxial expression of the CVF-Cosh-Gaussian beams for the j th nonparaxial corrections. We can find that the nonparaxial solution approaches the exact solution Eq. (12) with the increase of the parameter j . In particular, when j becomes infinite, the nonparaxial solution becomes the exact solution.

4. CONCLUSION

In conclusion, a group of complex sources that generate the CVF-Cosh-Gaussian waves has been presented. We have obtained the integral and the differential representation for the CVF-Cosh-Gaussian waves. From the integral representation of the CVF-Cosh-Gaussian waves, we derive the first three orders of nonparaxial corrections for the corresponding paraxial CVF-Cosh-Gaussian beams analytically and numerically. The angular velocity of the paraxial and nonparaxial CVF-Cosh-Gaussian beams decreases with z increasing.

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