# Bivariate Statistical Analysis for Electromagnetic Reverberation Chamber

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**Abstract**— The probability density functions (PDFs) of electromagnetic fields inside real reverberation chambers has been categorized into eight cases and the mathematical expressions for all the cases were recently reported. In this paper, we compile the complete expressions for the magnitude and phase PDFs of bivariate normal distribution. Furthermore, we present the assessment of these PDFs using a RC located at Ajou University, South Korea. To investigate how close to the ideal case is the measured data, T-tests and F test are used to test the hypotheses for zero means, equal variances and zero correlation between the real and imaginary part of the field. The results show that the circumstance of the RC is far from the ideal case. Then, KS and AD GoF tests are applied to the magnitude and phase PDFs for investigating the agreement between the measurement data and PDFs of the eight cases. According to the GoF tests, the magnitude PDFs of the type 3 and type 7 are very highly rejected. Also, the rejection rate of magnitude PDFs of the type 4 is lower than those of most other types, and the type 8 shows the lowest rejection rate. On the other hand, the phase PDFs of even types are much less rejected than those of odd types, and the phase PDFs of the type 8 shows the lowest rejection rate. The results of the paper are helpful in better understanding the behavior of fields in RCs.

## 1. INTRODUCTION

Reverberation chambers (RCs) have been attracting attention as electromagnetic (EM) testing facilities, performance evaluations of multiple-input-multiple-output handset antennas, etc. The correct use of RCs is based on the full understanding of the field behaviors inside the RCs.

It is commonly accepted that the real and imaginary components of EM fields in RCs follow normal distributions. However, recent researches gave the PDFs of the field magnitude and phase in the viewpoint of "good-but-imperfect" approach on the basis of bivariate normal distributions (BNDs) in order to describe imperfect reverberation conditions [1, 2]. They categorized BNDs into eight cases, and suggested mathematical expressions describing the magnitude and phase of electric fields by deriving the marginal PDFs of the BNDs. Regarding the given approach to mathematically characterize EM signals, a similar but more generalized analysis was also proposed from a communication society [3]. Most recently, the analytical formulas which were not introduced in [1–3] is presented [4]. Thus, we compile the complete mathematical expressions describing the magnitude and phase of electric fields.

Moreover, we present the experimental results of electric fields in a real RC. Also, the idealness of means, variances and correlation between the real and imaginary part of the field inside the RC are investigated. Moreover, using the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) goodness-of-fit (GoF) tests, we evaluate the magnitude and phase distributions of electric fields for the eight cases of BNDs [5, 6].

#### 2. MAGNITUDE AND PHASE PDFS OF BNDS

For characterizing the electric fields of a RC, the BND of real and imaginary parts  $\xi$  and  $\eta$  is represented as,  $N(\mu_{\xi}, \mu_{\eta}; \sigma_{\xi}, \sigma_{\eta}; \rho)$  where  $\mu_{\xi}$  and  $\mu_{\eta}$  are the means for  $\xi$  and  $\eta$  respectively,  $\sigma_{\xi}$  and  $\sigma_{\eta}$  are the standard deviations and  $\rho$  is the correlation coefficient between  $\xi$  and  $\eta$ . The BNDs are categorized into eight types according to the values of  $\mu_{\xi}, \mu_{\eta}, \sigma_{\xi}, \sigma_{\eta}$  and  $\rho$ .

The magnitude r and phase  $\phi$  of the field are represented as follows:

$$r = \sqrt{\xi^2 + \eta^2}, \quad r \ge 0 \tag{1}$$

$$\psi = \frac{\pi}{2} \operatorname{sgn}(t) \cdot [1 - \operatorname{sgn}(s)] + \operatorname{sgn}(s) \cdot \tan^{-1}\left(\frac{t}{|s|}\right), \quad -\pi \le \psi \le \pi$$
(2)

where  $\operatorname{sgn}(x) = x/|x|$  for  $x \neq 0$ , and  $\operatorname{sgn}(x) = 0$  for x = 0.

According to the PDFs presented in [1, Table 1], six PDFs require numerical integrations when obtaining the MPDF of the field magnitude and phase because analytical expressions for these PDFs are not found in closed forms. In [3], the magnitude and phase PDFs of BNDs for several cases were derived in analytical forms which involve the modified Bessel function of the first kind or the complementary error function. The phase PDFs for type VI and magnitude PDFs of Type VII are presented in [4]. In reviewing the PDFs reported in [1–4], the expressions for the magnitude and phase PDFs of BNDs are completed as shown in Table 1.

Case	Phase PDF $f_{\phi}(\phi)$	Magnitude PDF $f_r(r)$
1. $N(0, 0; \sigma, \sigma; 0)$	[1, Eq. (8)]	[1, Eq. (8)]
2. $N(\mu_{\xi}, \mu_{\eta}; \sigma, \sigma; 0)$	[1, Eq. (10)]	[1, Eq. (11)]
3. $N(0, 0; \sigma_{\xi}, \sigma_{\eta}; 0)$	[1, Eq. (13)]	[1, Eq. (16)]
4. $N(\mu_{\xi}, \mu_{\eta}; \sigma_{\xi}, \sigma_{\eta}; 0)$	[4. Eq. (1)]	[1, Eq. (19)]
5. $N(0,0;\sigma,\sigma;\rho)$	[1, Eq. (21)]	[2, Eq. (6)]
6. $N(\mu_{\xi}, \mu_{\eta}; \sigma, \sigma; \rho)$	[1, Eq. (24)]	[2, Eq. (3)]
7. $N(0, 0; \sigma_{\xi}, \sigma_{\eta}; \rho)$	[1, Eq. (26)]	[4, Eq. (2)]
8. $N(\mu_{\xi}, \mu_{\eta}; \sigma_{\xi}, \sigma_{\eta}; \rho)$	[2, Eq. (1)]	[2, Eq. (2)]

Table 1: summary of the PDFS for phase and magnitude.

#### 3. MEASUREMENT AND ANALYSIS

To verify the availability of the novel BN approach, complex electric field components were measured in the RC located at Ajou University, South Korea, while the RC was performing in stirred mode. The dimension of the RC is  $2.4 \text{ m} \times 2.3 \text{ m} \times 1.6 \text{ m}$  and the vertical Z-fold shape stirrer has size of  $0.6 \text{ m} \times 0.5 \text{ m} \times 1.0 \text{ m}$ . The chamber walls are made of stainless steel and welded. The first resonant frequency is around 97 MHz, and the lowest useable frequency might occur at slightly above 291 MHz, according to [7]. In this experiment, three different frequencies of 1.8, 2.4, and 3.8 GHz were selected, and about 5,000 samples per each frequency were measured using a vector network analyzer and dipole antennas.

To evaluate how close to the ideal case is the measured data, we test the ideal hypothesis for the parameters of means, variances and correlation between the real and imaginary part of the field measured in the RC. In ideal field condition of a RC, the means are zero, variances are equal and the correlation is zero. T tests and F test are used to test the ideal hypotheses and about 50 tests per each frequency are performed with 5% level of confidence. Fig. 1 shows the rejection rates resulting from the measurements performed in the RC. The rejection rates of zero mean hypotheses are 19, 28 and 22% for 1.8, 2.4 and 3.8 GHz, respectively, which is more highly rejected than other hypotheses. The rejection rates of equal variance hypotheses are 9, 12 and 15% and those of zero covariance hypotheses are 15, 14 and 13%. The rejection rates seem non-negligibly high. Therefore, it is considered that the circumstance of the RC is far from the ideal case.



Figure 1: Ideal hypothesis Tests for means, variances and covariance.

For investigating the agreement between the measurement data and the eight cases, KS and AD GoF tests are applied to the PDFs for the magnitude and phase. KS GoF test makes an assessment of whether there is sufficient evidence to reject the null hypothesis that the measurement data and the theoretical PDF are the same. Fig. 2 presents rejection rates resulting from the KS GoF tests

with 5% of confidence level for the eight cases. The rejection rates of the magnitude PDFs of the type 4 are  $4\sim11\%$  and those of type 8 are  $1\sim4\%$ , which are less than those of other types. The magnitude PDFs of the type 3 and type 7 are very highly rejected, which have rejection rates of  $35\sim73\%$ , equally. The phase PDFs of even types are much less rejected than those of odd types. That is accord with the previous test result that the hypothesis of zero mean is highly rejected, as shown in Fig. 1. The rejection rate of phase PDFs of the type 8 is close to 0% while that of type 1 is  $79\sim100\%$ .



Figure 2: KS GoF test results for magnitudes and phases of eight cases.



Figure 3: AD GoF test results for magnitudes and phases of eight cases.

The AD GoF test generally gives more stringent results than KS test. Accordingly, the results of AD GoF test shows higher rejection rate than KS GoF test. However, the tendency is similar to KS test as shown in Fig. 3. The rejection rates of magnitude PDFs of the type 8 are  $9\sim26\%$ , while those of type 1 are  $48\sim69\%$ . The rejection rates of phase PDFs of the type 8 are  $9\sim31\%$ , while those of type 1 are  $95\sim100\%$ .

The results show that the magnitude and phase of the field inside the RC are far from the ideal distributions. Therefore, some experimental errors should be taken into consideration when applying ideal assumptions. Other PDFs of complex cases are closer accordance with the experimental data.

## 4. CONCLUSION

We compile the complete expressions for the magnitude and phase PDFs of BNDs. Also, eight cases of BNDs are assessed using the experiment and the GoF tests. In order to guarantee the accuracy of the test results from RCs, the statistical distributions of EM fields inside the real RCs should be considered. The results of the paper are helpful in better understanding the behavior of fields in RCs.

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## REFERENCES

- Serra, R. and F. G. Canavero, "Bivariate statistical approach for good-but-imperfect electromagnetic reverberation," *IEEE Trans. Electromagn. Compat.*, Vol. 53, No. 3, 554–561, Aug. 2011.
- Serra, R., F. Leferink, and F. G. Canavero, "Good-but-imperfect electromagnetic reverberation in a VIRC," *Proc. IEEE Int. Symp. Electromagn. Compat.*, 954–959, Long Beach, USA, Aug. 2011.
- 3. Dharmawansa, P., N. Rajatheva, and C. Tellambura, "Envelope and phase distribution of two correlated Gaussian variables," *IEEE Trans. Commun.*, Vol. 57, No. 4, 915–921, Apr. 2009.
- 4. Choi, S. and S. Park, "A bivariate normalization approach for characterizing reverberation chambers," *IEEE Trans. Electromagn. Compat.*, accepted.
- 5. Eadie, W. T., D. Dryard, F. E. James, M. Roos, and B. Sadoulet, *Statistical Methods in Experimental Physics*, North-Holland, Amsterdam, 1971.
- 6. Anderson, T. W., Anderson-Darling Tests of Goodness-of-fit, International Encyclopedia of Statistical Science, Springer, 2011.
- 7. IEC 61000-4-21, Electromagnetic Compatibility, Part 4: Testing and Measurement Technology, Section 21: Reverberation Chamber Test Methods.