Multilayer Antenna with Metamaterial and Arbitrary Substrate

H. C. C. Fernandes, R. R. C. França, and A. F. Gomes
Department of Electrical Engineering, Federal University of Rio Grande of Norte, Brazil

Abstract—This paper presents a study and development of equations consisting metamaterials of negative refraction index. The equations developed for this structure are based in full wave Transverse Transmission Line (TTL) method. This method possibility a significant algebraic simplification of the equations involved in the process. In order to analyze this structure the complex resonant frequency were determined. The results obtained for this application are presented.

1. INTRODUCTION

Microstrip antennas are widely used due to its various advantages such as reducing size, weight, and cost and in addition because it interfaces easily with other microwave circuits, compatibility with integrated hybrid circuits and the possibility of acting with dual frequency [1]. They can be used in several systems, such as: radars, wireless, mobile telephony and communication by satellites. This paper presents an application of multilayer antenna with metamaterial. The analysis is made using the concise full wave TTL method.

However, most of the designs are still in the stage of theory and there is a big challenge to integrate antennas with dielectric multilayer has advantages, such as, the flexibility in the operation frequency band and a smaller physical size.

The TTL — Transverse Transmission Line method is used in the determination of the electromagnetic field components in the Fourier transform domain (FTD), for the three regions of the structures shown in the Fig. 1. The moment method is applied and adequate basis functions are used to expand the current densities in the metallic strip.

![Figure 1: Multilayer microstrip antennas.](image1)

![Figure 2: TW-SRR Structure compost with SRR and TW metamaterial.](image2)

The metamaterial substrate depends on the spatial coordinates and this causes spatial dispersion. As a result the medium will be no homogenous. For a no homogeneous structure, the incident wave undergoes a process of multiple scattering. The substrate shown in region 1 of Fig. 1 is modeled by utilizing the anisotropic tensor properties, which are expressed as [2]:

\[
\begin{align*}
\mu &= \mu_0 \begin{bmatrix}
\mu_{xx} & 0 & 0 \\
0 & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix} \\
\varepsilon &= \varepsilon_0 \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\end{align*}
\] (1) (2)
2. METAMATERIAL THEORY

The term electromagnetic metamaterial has been applied to artificial structures, that possess properties beyond those available in naturally materials. The identification of a permittivity and permeability with an inhomogeneous structure allows the properties of the metamaterial to be expressed conveniently, often in analytic form. The ability to describe an otherwise complicated inhomogeneous structure as a continuous material characterized by homogenized material parameters is the heart of the metamaterial concept. Although the terms photonic crystal, artificial material or metamaterial are often used interchangeably, it is the key term that best fits the type of structures presented here in Fig. 2 [1].

3. FIELDS THEORY

The TTL — Transverse Transmission Line method in the Fourier transform domain, uses a component of propagation in the y direction, treating the general equations of electric and magnetic field as functions of ˜Ey and ˜Hy [3–5].

Starting from the Maxwell equations, and after several algebraic manipulations, the equations that represent the electromagnetic fields in the x and z directions are obtained as a function of the electromagnetic fields the in the y direction.

The metamaterial substrate shown in region 1 of Fig. 1 is modeled by utilizing bianisotropic tensor properties Equations (2.6) and (2.7), together with wave equations which are expressed as [2, 3]:

\[
\frac{\partial^2 \tilde{E}_y}{\partial y^2} - \gamma^2 \tilde{E}_y = 0 \\
\frac{\partial^2 \tilde{H}_y}{\partial y^2} - \gamma^2 \tilde{H}_y = 0
\]

After using the Maxwell’s equations in the spectral domain and the previous definitions above, the general equations of the electric and magnetic fields to the resonator, are obtained as:

\begin{align*}
\tilde{E}_x &= \frac{1}{\gamma_y^2 + k_0^2 \varepsilon_r} \left[ -j \alpha_n \frac{\partial}{\partial y} \tilde{E}_y + \omega \mu_0 \mu_{xx} \beta_k \tilde{H}_y + \omega \mu_0 \mu_{zz} \beta_k \tilde{H}_y \right] \\
\tilde{E}_z &= \frac{1}{\gamma_y^2 + k_0^2 \varepsilon_r} \left[ -j \beta_k \frac{\partial}{\partial y} \tilde{E}_y - \omega \mu_0 \mu_{xx} \alpha_n \tilde{H}_y - \omega \mu_0 \mu_{zz} \alpha_n \tilde{H}_y \right] \\
\tilde{H}_x &= \frac{1}{\gamma_y^2 + k_0^2 \varepsilon_r} \left[ -j \alpha_n \frac{\partial}{\partial y} \tilde{H}_y - \omega \varepsilon_0 \varepsilon_{xx} \beta_k \tilde{E}_y - \omega \varepsilon_0 \varepsilon_{zz} \beta_k \tilde{E}_y \right] \\
\tilde{H}_z &= \frac{1}{\gamma_y^2 + k_0^2 \varepsilon_r} \left[ -j \beta_k \frac{\partial}{\partial y} \tilde{H}_y + \omega \varepsilon_0 \varepsilon_{xx} \alpha_n \tilde{E}_y + \omega \varepsilon_0 \varepsilon_{zz} \alpha_n \tilde{E}_y \right]
\end{align*}

where \( i = 1, 2, 3 \), represent the dielectrics regions of the structure; \( \gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \) is the constant of the propagation in y direction; \( \alpha_n \) is the spectral variable in “x” direction and \( \beta_k \) the spectral variable in “z” direction; \( k_i^2 = \omega^2 \mu \varepsilon = k_0^2 \varepsilon_{ri} \) is the wave number of dielectric region; \( \varepsilon_{ri} = \varepsilon_{ri} - j \frac{\varepsilon_r}{\varepsilon_0} \) is the dielectric constant relative of the material with losses; \( \omega = \omega_r + j \omega_i \) is the complex angular frequency; \( \varepsilon_i = \varepsilon_{ri} \cdot \varepsilon_0 \) is the dielectric constant of the material.

The solutions of Helmoltz’s equations for the three regions of the antenna are given by:

For region 1:

\[
\tilde{E}_{y1} = A_{1e} \cdot \cosh(\gamma_1 y) \\
\tilde{H}_{y1} = A_{1h} \cdot \cosh(\gamma_1 y)
\]

For region 2:

\[
\tilde{E}_{y2} = A_{2e} \cdot \sinh(\gamma_2 y) + B_{2e} \cdot \cosh(\gamma_2 y) \\
\tilde{H}_{y2} = A_{2h} \cdot \sinh(\gamma_2 y) + B_{2h} \cdot \cosh(\gamma_2 y)
\]
For region 3:

\[ \tilde{E}_{y3} = A_{3e} \cdot \cosh[\gamma_3(da - y)] \]  
\[ \tilde{H}_{y3} = A_{3h} \cdot \cosh[\gamma_3(da - y)] \]  

Substituting these solutions in the equations of the fields, in function of the unknown constants \( A_{1e} \), \( A_{1h} \), \( A_{2e} \) and \( A_{2h} \) is obtained, for example, for the region 1 [3]:

\[ \tilde{E}_{x1} = \frac{-j}{k_1^2 + \gamma_1^2} [\alpha_n \gamma_1 A_{1e} \sinh(\gamma_1 y) + (\mu_{xx} + \mu_{zz})] \]  
\[ \tilde{E}_{z1} = \frac{-j}{\mu_0} [\beta_k \gamma_1 A_{1e} \sinh(\gamma_1 y)] \]  
\[ \tilde{H}_{x1} = \frac{-j}{k_1^2 + \gamma_1^2} [\alpha_n \gamma_1 A_{1h} \cosh(\gamma_1 y) - (\varepsilon_{xx} + \varepsilon_{zz})] \]  
\[ \tilde{H}_{z1} = \frac{-j}{k_1^2 + \gamma_1^2} [\beta_k \gamma_1 A_{1h} \cosh(\gamma_1 y) + (\varepsilon_{xx} + \varepsilon_{zz})] \]  

To the determination of the unknown constants described above the boundary conditions are applied in the structure shown in Fig. 1, in \( y = h_1 \) and \( y = h = h_1 + h_2 \). The electromagnetic fields general equations as function of \( \tilde{E}_{xh} \) and \( \tilde{E}_{zh} \), which are the tangential components of the electric fields are obtained to calculate the propagation constant:

\[ \tilde{E}_{x1} = \tilde{E}_{x2}, \quad \tilde{H}_{x1} = \tilde{H}_{x2} \]  
\[ \tilde{E}_{z1} = \tilde{E}_{z2}, \quad \tilde{H}_{z1} = \tilde{H}_{z2} \]  

After several calculations are obtained the following constant values:

\[ A_{1h} = \frac{\beta_k \tilde{E}_{xy} - \alpha_n \tilde{E}_{zy}}{\sinh(\gamma_1 y) \mu_0 (\mu_{xx} + 2 \mu_{zz})} \]  
\[ A_{1e} = \frac{j (\alpha_n \tilde{E}_{xy} + \beta_k \tilde{E}_{zy})}{\gamma_1 \sinh(\gamma_1 y)} \]  
\[ A_{2e} = \frac{j (\alpha_n \tilde{E}_{xh} + \beta_k \tilde{E}_{zh})}{A} \left[ \gamma_1 \cosh(\gamma_2 h_2) \sinh(\gamma_1 h_1) - \frac{\varepsilon_1}{\varepsilon_2} \sinh(\gamma_2 h_2) \cosh(\gamma_1 h_1) \right] \]  
\[ A_{2h} = \frac{(\beta_k \tilde{E}_{xh} - \alpha_n \tilde{E}_{zh})}{\omega \mu A} \left[ \gamma_1 \cosh(\gamma_1 h_1) \cosh(\gamma_2 h_2) - \sinh(\gamma_1 h_1) \sinh(\gamma_2 h_2) \right] \]  
\[ B_{2e} = \frac{j (\alpha_n \tilde{E}_{xh} + \beta_k \tilde{E}_{zh})}{A} \left[ \frac{\varepsilon_1}{\varepsilon_2} \cosh(\gamma_1 h_1) \cosh(\gamma_2 h_2) - \frac{\gamma_1}{\gamma_2} \sinh(\gamma_1 h_1) \sinh(\gamma_2 h_2) \right] \]  
\[ B_{2h} = \frac{(\beta_k \tilde{E}_{xh} - \alpha_n \tilde{E}_{zh})}{\omega \mu A} \left[ \sinh(\gamma_1 h_1) \cosh(\gamma_2 h_2) - \frac{\gamma_1}{\gamma_2} \cosh(\gamma_1 h_1) \sinh(\gamma_2 h_2) \right] \]  
\[ A_{3h} = \frac{\beta_k \tilde{E}_{xy} - \alpha_n \tilde{E}_{zy}}{\omega \mu} \]  
\[ A_{3e} = \frac{-j (\alpha_n \tilde{E}_{xy} + \beta_k \tilde{E}_{zy})}{\gamma_3} \]  

where

\[ A = \gamma_1 \sinh(\gamma_1 h_1) \cosh(\gamma_2 h_2) + \frac{\varepsilon_1}{\varepsilon_2} \cosh(\gamma_1 h_1) \sinh(\gamma_2 h_2) \]  
\[ B = \sinh(\gamma_1 h_1) \cosh(\gamma_2 h_2) + \frac{\gamma_1}{\gamma_2} \cosh(\gamma_1 h_1) \sinh(\gamma_2 h_2) \]
4. ADMITANCE MATRIX

The following equations relate the current densities in the sheets ($\tilde{J}_{xy}$ and $\tilde{J}_{zy}$) and the magnetic fields in the interface $y = h_1 + h_2$:

$$\tilde{H}_{x2} - \tilde{H}_{x3} = \tilde{J}_z$$  \hspace{1cm} (31)
$$\tilde{H}_{z2} - \tilde{H}_{z3} = -\tilde{J}_x$$  \hspace{1cm} (32)

After the obtaining of the electromagnetic fields components, the magnetic boundary conditions are applied, being $[Y]$ the dyadic Green matrix.

$$\begin{bmatrix} Y_{xx} & Y_{xz} \\ Y_{zx} & Y_{zz} \end{bmatrix} \begin{bmatrix} \tilde{E}_{z} \\ \tilde{E}_{x} \end{bmatrix} = \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_z \end{bmatrix}$$  \hspace{1cm} (33)

The matrix inversion is used and the current densities in the interface are expanded using base functions [4]:

$$\begin{bmatrix} Z_{xx} & Z_{xz} \\ Z_{zx} & Z_{zz} \end{bmatrix} \begin{bmatrix} \tilde{J}_{zh} \\ \tilde{J}_{xh} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{xh} \\ \tilde{E}_{zh} \end{bmatrix}$$  \hspace{1cm} (34a)

$$\tilde{J}_{xh} = \sum_{i=1}^{n} a_{xi} \cdot \tilde{f}_{x_i}(\alpha_n, \beta_k)$$  \hspace{1cm} (34b)

$$\tilde{J}_{zh} = \sum_{j=1}^{m} a_{zj} \cdot \tilde{f}_{z_j}(\alpha_n, \beta_k)$$  \hspace{1cm} (34c)

Applying the Galerkin technique, the electric fields out of the metallic strip are eliminated. The current densities are expanded in terms of appropriate basis functions, and become a homogeneous complex matrix as shown in (35).

$$\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix} \begin{bmatrix} a_x \\ a_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (35)

Each element of the $[K]$ characteristic matrix is shown in (36)–(39):

$$K_{xx} = \sum_{-\infty}^{\infty} \tilde{f}_x(x, z)Z_{xx}\tilde{f}_x^*(x, z)$$  \hspace{1cm} (36)

$$K_{xz} = \sum_{-\infty}^{\infty} \tilde{f}_z(x, z)Z_{xz}\tilde{f}_x^*(x, z)$$  \hspace{1cm} (37)

$$K_{zx} = \sum_{-\infty}^{\infty} \tilde{f}_x(x, z)Z_{zx}\tilde{f}_z^*(x, z)$$  \hspace{1cm} (38)

$$K_{zz} = \sum_{-\infty}^{\infty} \tilde{f}_z(x, z)Z_{zz}\tilde{f}_z^*(x, z)$$  \hspace{1cm} (39)

5. RESULTS

Figure 3 shows the results of the resonance frequency depending on the length $L$ of the patch resonator, simulated for two cases.

- Case 1: 1 layer dielectric substrate comprised of PBG 2D considering focusing on the wave polarization $s$, 2 dielectric layer composed of RT Duroid 5880 and dielectric layer 3 is the air;
- Case 2: 1 layer dielectric substrate comprised of PBG 2D considering focusing on the wave polarization $p$, 2 dielectric layers composed of RT Duroid 5880 and dielectric layer 3 is the air.

The analysis of Fig. 3, we can conclude that when the material PBG is the first layer beneath the patch irradiator, the frequency of resonance does not suffer significant changes with the change in the wave incidence in the substrate (polarization $s$ or $p$). This effect is due to difference in heights of layers, as the $\varepsilon_r = 2.2$ predominates on the PBG material.

Comparing the first two layers of the structure mentioned in Fig. 1, with another one antenna structure of one layer, we see that the results are similar, as shown in Fig. 4.
Figure 3: Frequency of resonance as a function on the length of patch material for PBG 2D for the polarization s or p.

Figure 4: Radiation pattern E-plane.

6. CONCLUSIONS
The results obtained can be used to the design of novel meta-materials with potential applications in wireless systems.

The full wave Transverse Transmission Line — TTL method, was used to obtain the numeric results of the planar antennas with three metamaterial layers.

In this work, the insertion of the metamaterial influence was realized through of the use in tensor permeability and permittivity, and good results were obtained.

REFERENCES