MUSIC-type Imaging of Dielectric Spheres from Single-Frequency, Asymptotic and Exact Array Data

S. Gdoura, D. Lesselier, G. Perrusson, and P. C. Chaumet

1Département de Recherche en Électromagnétisme, Laboratoire des Signaux et Systèmes CNRS-SUPÉLEC-UPS 11, 91192 Gif-sur-Yvette cedex, France
2Institut Fresnel, Université Aix-Marseille III, 13397 Marseille cedex 20, France

Abstract — Imaging of a dielectric sphere from its Multi-Static Response (MSR) matrix at a single frequency of operation is considered herein via a MUSIC-type, non-iterative method. Synthetic data are both asymptotic ones and data calculated by the Coupled Dipole Method (CDM) which, in contrast, models the wavefield in exact fashion. Comparisons of scattered fields, distributions of singular values, and MUSIC images are carried out. In particular, even far beyond the domain of application of the asymptotic modeling (on which the analysis of the MSR matrix is based), it is shown that fair localization of the sphere is achieved from CDM data.

1. INTRODUCTION

In recent works, an involved analysis of a 3-D MUSIC-type imaging of a small volumetric, dielectric and/or magnetic scatterer (or a set of such scatterers), based on an asymptotic formulation of the electromagnetic wavefield in a full Maxwell setting, has been proposed, refer to [1] in a free space configuration, to [2] with focus onto back-propagation, and to [3] for generalization to a half space. Yet, most of the numerical illustrations so far are from data calculated according to the asymptotic formulation itself (with addition of noise). Here the Coupled Dipole Method (CDM) [4], which involves no approximation, is used as the main calculation tool of the data to be inverted. The paper is organized as follows: The asymptotic method and the CDM are summarized first, before outlining the imaging procedure. Then, fields, distributions of singular values, and MUSIC images simulated according either method are proposed and discussed. A short conclusion follows. (Preliminary comparison results are in [5].)

2. MODELING OF THE SCATTERED FIELD

Let us consider in free space (permittivity \(\varepsilon_0\), permeability \(\mu_0\)) a planar (horizontal) array within the plane \(z = h\), which is made of \(N\) ideal electric dipoles all oriented (for simplicity) in the vertical \(\hat{z}\) direction. The array is operated at a single frequency \(\omega\) (wavelength \(\lambda\), wavenumber \(k\)) and it illuminates a collection of \(m\) non-magnetic spherical scatterers located in a prescribed search box somewhere below it (usually in the near-field of the array). The spheres are of radius \(a_j = \alpha d_j\), where \(\alpha\) is the order of magnitude of their size and \(d_j\) are multiplicative scale factors; their permittivities are \(\varepsilon_j\), centers are at \(x_j\), and their volumes read as \(V_j\), \(j = 1, \ldots, m\). Let \(E_0^{(n)}(r)\) be the primary electric field at location \(r (r \in \mathbb{R}^3)\) radiated from the \(n\)th dipole with amplitude \(I_n\), and let \(E^{(n)}(r)\) be the total electric field in the presence of the scatterers. One has

\[
E_0^{(n)}(r) = i\omega\mu_0 \mathbf{G} (r, r_n) \cdot \hat{z} I_n
\]

where \(\mathbf{G} (r, r_n)\) is the Green’s dyad in free space (reciprocity \(\mathbf{G} (r, r_n) = \mathbf{G} (r_n, r)^t\) holds as usual). Then, the Lippman-Schwinger vector integral formulation of the field reads as

\[
E^{(n)}(r) - E_0^{(n)}(r) = \sum_{j=1}^{m} \int_{V_j} dr' \left[ \omega^2 \mu_0 (\varepsilon_j - \varepsilon_0) \mathbf{G} (r, r') \cdot E^{(n)}(r') \right].
\]

From that point, two solution methods can be employed to calculate the electric field at an arbitrary receiver location (e.g., at the nodes of an array, the same as the source one, or another one).
2.1. The Asymptotic Formulation of the Scattered Field

Assuming that $\alpha \ll \lambda$, a rigorous asymptotic field formulation (refer to aforementioned references) holds:

$$E^{(n)}(r) - E_0^{(n)}(r) = \sum_{j=1}^M \left[ (i\omega \mu_0)^{-1} G(r, x_j) \cdot M_j \cdot E_0^{(n)}(x_j) \right] + [o((k\alpha)^3)].$$  \hfill (3)

$M_j = \frac{k^3\alpha^3 \mu_0 c^2}{\epsilon_0 (\epsilon_j - \epsilon_0)} M(\epsilon_j/\epsilon_0; V_j)$ is the generalized polarization tensor, letting $M(\epsilon_j/\epsilon_0; V_j)$ be the polarization tensor associated to the scatterer of volume $V_j$ and contrast $\epsilon_j/\epsilon_0$, and $c$ is the speed of light. Let us notice that, since the scatterer is spherical, its polarization tensor $M(\epsilon_j/\epsilon_0; V_j)$ has explicit form $\frac{2\pi}{\epsilon_j + 2\epsilon_0} |V_j| I_3$, where $I_3$ is the identity matrix in $\mathbb{R}^3$.

2.2. Calculation of the Scattered Field by CDM

The Coupled Dipole Method is based on the same integral formulation (2). But now, the scatterer under study is discretized into a set of $L$ subunits arranged on a cubic lattice. If the size of the subunit is small enough vs. the wavelength of the illumination, the electromagnetic field is accurately assumed to be uniform over each subunit. Hence, the field at each subunit (here, $V_j$ as the volume of subunit $j$), for $i = 1, \ldots, L$, reads as

$$E(r_i) = E_0(r_i) + \sum_{j=1}^L \omega^2 \mu_0 (\epsilon_j - \epsilon_0) \int_{V_j} \left[ G(r_i, r') \right] dr' \cdot E(r'_j).$$ \hfill (4)

If $i \neq j$ one can approximate $\int_{V_j} \left[ G(r_i, r') \right] dr' = V_j G(r_i, r'_j)$, which holds for the scatterers studied herein. The computation of the self term, i.e., $\int_{V_j} \left[ G(r_i, r') \right] dr'$, is given in [4]. Then, the field at each subunit is obtained by solving the linear system (4). The scattered field at each position of observation follows as

$$E(r) = \sum_{j=1}^L \omega^2 \mu_0 (\epsilon_j - \epsilon_0) V_j \left[ G(r, r'_j) \right] \cdot E(r'_j).$$ \hfill (5)

3. MUSIC-TYPE IMAGING METHOD

Let us assume now that the receiver array is also made of ideal electric dipoles enabling us to collect the scattered electric field, and that it is coincident with the source array. Those $N$ vertical electric dipoles are at $\{r_1, \ldots, r_N\}$. Transmitted amplitudes are $I_n$, $n = 1, \ldots, N$. For any $x$ in $\mathbb{R}^3 \backslash \{r_1, \ldots, r_N\}$, matrices $G^e(x) \in \mathbb{C}^{N \times 3}$ read as

$$G^e(x) = [G(x, r_1) \cdot \hat{z}, \ldots, G(x, r_N) \cdot \hat{z}]^T.$$ \hfill (6)

In the asymptotic framework, the Multi-Static Response (MSR) matrix $A \in \mathbb{C}^{N \times N}$, which is made of the scattered electric fields collected at each (vertical) receiver location in the array, each (vertical) dipole source of the said array radiating successively, can be decomposed as

$$A = \sum_{j=1}^M G^e(x_j) M_j \left[ G^e(x_j) \right]^T.$$ \hfill (7)

It has been shown that the rank of $G^e(x)$ does not depend upon $x$ in $\mathbb{R}^3 \backslash \{r_1, \ldots, r_N\}$, and is equal to 3 in the present configuration (refer also to [6]). Also, for $m$ well-resolved scatterers, i.e., whenever the inner products (* as transpose conjugation) $G^e(x_j)$ are close to 0 for $i, j = 1, \ldots, m$, each scatterer can be imaged independently, and rank is $3m$. Now, if the dimension $s$ of the signal space is known or estimated in the absence of information on the number of scatterers, from the singular value decomposition $A = U \Sigma V^*$, the MUSIC algorithm applies: For any vector $e \in \mathbb{R}^3$, such as $\|G(x) \cdot e\| \neq 0$, and any $x$ within the search domain, the estimator

$$W(x) = 1/ \sum_{i=s+1}^N \left| \langle U_i, G(x) \cdot e \rangle \right|^2$$

peaks (to infinity, in theory) at the scatterers’ centers. (This algorithm implies that $N > s$.)
4. NUMERICAL EXAMPLES

The frequency of operation is set to $f = 500$ MHz, all lengths henceforth being given in meters. The planar transmitter/receiver array consists of $21 \times 21$ vertical electric dipoles distributed at the nodes of a regular mesh with a half-a-wavelength step size (here, $\lambda = 0.6$), and is placed at $h = 5\lambda$ symmetrically about the axis $z$. A single dielectric sphere with permittivity $\varepsilon_j = 5\varepsilon_0$ is centered at $x_j = (-0.15, 0.15, 0.175)$. In each numerical example a different radius of the sphere is chosen.

4.1. Comparisons of the Scattered Field

In the first example the sphere is of radius $0.06 (= \lambda/10)$ at $x_j$. It is illuminated by one vertical ($\hat{z}$-orientated) electric dipole at $(−3, −3, 3)$. One displays the normalized $E_x$, $E_y$ and $E_z$ components of the scattered field at the position of the $21 \times 21$ dipoles. Figure 1 shows a comparison of the results provided by the asymptotic approach and by CDM. Both methods provide almost the same scattered field at the array location for this small $a = \lambda/10$ radius (its electric size, since its relative permittivity is 5, is however more than twice larger). Other simulations (not shown for lack of space) for other radii of the dielectric sphere have been carried out, and as expected, for larger and larger radii, the asymptotic formula (where the scatterer size only matters as a factor $a^3$) becomes more and more inaccurate.

Figure 1: Comparison of asymptotic and CDM fields for a dielectric sphere of radius $\lambda/10$.

Figure 2: Distribution of the singular values (the first 75) of the MSR matrix calculated with the asymptotic method and 3-D representation of the MUSIC functional (isosurface 20% of the max value) calculated using the first three ones.
4.2. Distribution of Singular Values and Imaging by the MUSIC Procedure

4.2.1. Imaging a Sphere of Radius $a = \lambda/10$

In the second example one is considering the same dielectric sphere at $x_j$ with radius $\lambda/10$. For each method (asymptotic one and CDM), the Multistatic Response matrix is constructed from the scattered field computed at the array. After singular value decomposition of this matrix via a standard code, the MUSIC algorithm as sketched before is applied within the search box (here a cube of side $2\lambda$ is chosen). In Figure 2 the results obtained in the asymptotic framework are

Figure 3: Same as in Figure 2 with CDM.

Figure 4: MUSIC images (isosurface 20% of the max value) calculated from CDM data with reference to the exact scatterer. Clockwise from top left, the sphere radius is $\lambda/10$, $2\lambda/10$, $3\lambda/10$, and $4\lambda/10$; 55, 75, 100, and 125 singular vectors are used in the procedure, respectively.
displayed, and in Figure 3 those resulting from the CDM. Let us observe that, as expected, only the three first singular values are significant in the asymptotic framework, all others being valued almost to zero. But with using CDM, even though the three first values are much larger than the others, and can safely be separated from them, a number of non-zero ones is appearing with slowly decreasing amplitudes. However, the two methods provide a similar, and excellent, image when using the corresponding 3 singular vectors only — using all vectors associated to most non-zero singular values from data provided by CDM, here the first 55 ones, one would still get a good estimate of the location of the sphere. Let us notice that the difference of behavior of the singular spectrum between asymptotic and exact approaches comes from the truncation at order \( o ((ka)^3) \) inherent to the asymptotic modeling, which at first order does not involve any multipole contribution (modeled at the next order and further on).

4.2.2. Imaging of Spheres of Various Radii from CDM Data

Images obtained for larger radii than \( \lambda/10 \), i.e., \( 2\lambda/10, 3\lambda/10 \) and \( 4\lambda/10 \), are shown in Figure 4. The singular spectrum of the multistatic response matrix computed from CDM data is obviously more complicated now, when \( a > \lambda/10 \), than in the case \( a = \lambda/10 \), and many more singular values of significant amplitude are observed and have to be accounted for in the imaging procedure, in tune with the higher complexity of the scattering phenomenon itself. That is, using the first three singular values does not yield the sphere location. But, by taking all (or at best most) non-zero ones, the location of the sphere is well retrieved, even for a large electrical size (about one-wavelength radius for the largest sphere) whilst it appears, further investigation pending, that one can get at least some estimate of the scatterer volume itself.

5. CONCLUSION

In this paper, one has investigated the robustness of the MUSIC-type imaging method against data acquired outside the asymptotic framework wherein the analysis of the Multi-Static Response matrix is carried out. In particular, one has exhibited that when the radius of the scattering sphere is of the order of \( a \leq \lambda/10 \), and twice more at least in terms of equivalent electrical size, the asymptotic data and the exact ones (those calculated by the Coupled Dipole Method) yield the same result. But, for larger and larger radii, the asymptotic formula becomes less and less valid, the imaging algorithm still working fairly well even though the scatterer is far from punctual.

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REFERENCES