Magnetic Field Approximation in MR Tomography

Michal Hadinec¹, Pavel Fiala¹, Eva Kroutilová¹, Miloslav Steinbauer¹, and Karel Bartušek²

¹Institute of Scientific Instruments of the ASCR, v.v.i
Králůvápolská 147, 612 64, Brno, Czech Republic

²Department of Theoretical and Experimental Electrical Engineering
Faculty of Electrical Engineering and Communication, Brno University of Technology
Kolejní 2906/4, 612 00, Brno, Czech Republic

Abstract — This paper describes a method, which can be used for creating map of magnetic field. Method has a great usage in magnetic resonance tomography, when we need to get information about homogeneity and characteristics of magnetic field inside the working space of the MR tomograph. The main purpose of this article is to describe basic principles of magnetic resonance phenomenon and mathematical method of Legendre polynoms which can be used for signal processing of FID (Free Induction Decay) signal obtained from tomograph detection coils. In the end of my article is experimental solution of magnetic field and models of magnetic field created by Matlab.

1. INTRODUCTION

Magnetic resonance tomography is an imaging technique used primarily in medical setting to produce high quality images of the human body. Magnetic resonance imaging is based on the principles of nuclear magnetic resonance (NMR) and at the present time it is the most developed imaging technique at biomedical imaging [2]. Lately, medical science lays stress on the measuring of exactly defined parts of human body, especially human brain. If we want to obtain the best quality images we have to pay attention to homogeneity of magnetic fields, which are used to scan desired samples inside the tomograph. We should know how to reduce inhomogeneity, which can cause misleading information at the final images of samples. Generally, inhomogeneity of magnetic fields at magnetic resonance imaging cause contour distortion of images. To eliminate these inhomogeneity correctly, we need to know the map of the magnetic field and we also need to have an exact information about parameters of the magnetic field. This paper presents the experimental method, which can easily create the map of electromagnetic induction at any defined area inside the tomograph. This method uses mathematical theory of Legendre polynoms, which are used for approximation of magnetic field, if we know specific coefficients. The coefficients of Legendre polynoms, which are computed using measured values of magnetic induction at exactly defined discrete points are used for creating map of magnetic field. If we know these coefficients, we are able to compute magnetic induction at any point of defined area. At the ideal case, there should be no difference between measured data and approximated data.

2. MR PRINCIPLES

In quantum mechanics, spin [2] is important for systems at atomic length scales, such as individual atoms, protons or electrons. One of the most remarkable discoveries associated with quantum physics is the fact, that elementary particles can possess non zero spin. Elementary particles are particles that cannot be divided into any smaller units, such as the photon, the electron and the various quarks. The spin carried by each elementary particle has a fixed value that depends only on the type of particle, and cannot be altered in any known way. Particles with spin can possess a magnetic dipole moment, just like a rotating electrically charged body in classical electrodynamics. The main principle of magnetic resonance spectroscopy and magnetic resonance imagining is, that radiofrequency fields (RF pulses) excite transitions between different spin states in a magnetic field. The information content can be retrieved as resonance frequency, spin to spin couplings and relaxation rates. We can imagine, that protons are rotating along their axes and there is also a wobbling motion called precession, that occurs when a spinning object is the subject of an external force. Thanks to the positive charge of protons and its spin, protons generate a magnetic field and gets a magnetic dipole moment. If the protons are placed in a magnetic field, the magnetic moment
will precess about the direction of magnetic field with specific frequency. This frequency is called Larmor frequency and can be described by the Larmor equation [5]

\[ \Omega = \gamma \cdot B \]  

(1)

where \( \Omega \) [MHz] is the frequency of precession, \([MHz/T]\) is the gyromagnetic ratio and \( B \) is strength of external magnetic field. In ordinary materials, the magnetic dipole moments of individual atoms produce magnetic fields that cancel one another, because each dipole points in a random direction. In ferromagnetic materials however, the dipole moments are all lined up with another, producing a macroscopic, non-zero magnetic field. If there is no external magnetic field, magnetic moments of atoms are chaotically spread and there is nearly no resulting magnetization vector \( M_0 \). If we place a sample into the stationary magnetic field \( B_0 \), we realize, that there is a vector of magnetization \( M_0 \) which is created as a sum of magnetic moments of each atom. The direction of this vector is the same as the direction of external magnetic field \( B_0 \). This state is called longitudinal magnetization. Now we apply a high frequency magnetic field of induction \( B_1 \), which is vertical to stationary magnetic field \( B_0 \). This high-frequency magnetic field causes resonance effect and magnetization vector \( M_0 \) starts to rotate with specific angular frequency. To measure vector \( M_0 \), we need to drop it into the xy plain (on condition that \( B_0 \) has direction of \( z \) axes). This dropping is done by a high-frequency excitation pulse \( B_1 \), which has a proper shape. This state is called transversal magnetization. Set of these pulses is called pulse sequence. Pulse sequence is a pre-selected set of defined RF and gradient pulses, usually repeated many times during a scan. Pulse sequences control all hardware aspects of the measurement process. At the x-y plain, there is scanning coil, which is used for scanning of FID signal.

After excitation pulses, the spins has tendency to minimize transverse magnetization and to maximize longitudinal magnetization. The transverse magnetization decays toward zero with characteristic time constant \( T_2 \) and the longitudinal magnetization returns towards maximum with a characteristic time constant \( T_1 \).

![Fid signal and MR spectrum.](image)

Figure 1: Fid signal and MR spectrum.

3. LEGENDRE POLYNOMS THEORY

If we want to determine the magnetic induction values in the specific points of measured area, we should use Legendre polynomials [4] which are defined according to equation (2)

\[ P_n(z) = \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{n!} \left[ z^n - \frac{n(n-1)}{2(2n-1)} z^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 4(2n-1)(2n-3)} z^{n-4} - \ldots \right] \]  

(2)

Legendre polynomials of zero and first order

\[ P_0(z) = 1 \]  

(3)

\[ P_1(z) = z = \cos v \]  

(4)

Legendre polynomials of of higher order are defined according to recursion formula

\[ P_{n+1}(z) = [(2n+1) \cdot z \cdot P_n(z) - n \cdot P_{n-1}(z)] / (n + 1) \]  

(5)

Now we can define functions associated to Legendre polynomials, which are derivation of Legendre
polynomials

\[ P_0^1(z) = 0 \] (6)
\[ P_2^1(z) = \frac{1}{2}(3z^2 - 1) \] (7)
\[ P_{n+1}^m(z) = \left[ (2n+1) \cdot z \cdot P_n^m(z) - (n+m) \cdot P_{n-1}^m(z) \right] / (n-m+1) \] (8)
\[ P_{n+2}^m(z) = \frac{2 \cdot (m+1) \cdot z \cdot \sqrt{1-z^2} \cdot P_n^{m+1} - (n-m) \cdot (n+m+1) \cdot P_n^m(z)}{n-m+1} \] (9)

Magnetic field induction can be approximated at any point of measured area. These points can be selected using spherical coordinates \( [r, \theta, \varphi] \), so we can define approximation formula as follows

\[ B_a(r, \theta, \varphi) = \sum_{k=0}^{N_K} \sum_{m=0}^{m=K} r^k \cdot P_{m,k}(\cos \theta) \cdot [A_{m,k} \cos m \cdot \varphi + B_{m,k} \sin m \cdot \varphi] \] (10)

where \( N_K \) is the highest order of Legendre polynom for chosen approximation, \( A_{mk} \) and \( B_{mk} \) are unknown coefficients. \( N_K \) is defined according to sampling theorem and depends on number of measured points \( N_b \):

\[ N_k = \frac{N_b}{2} - 1 \] (11)

Coefficients \( A_{mk} \) and \( B_{mk} \) is then possible to find like the minimum value of this formula:

\[ \Psi = \min \sum_{i=1}^{N_m} (B_{im} - B_{ia})^2 \] (12)

where \( B_m \) are measured values of magnetic induction at the desired area (circle, sphere, cylinder) and \( B_a \) are approximated values of magnetic induction. This method is known as Least Square method.

4. EXPERIMENTAL RESULTS

All values of magnetic induction on following figures are presented at [\( \mu T \)] unit. Fig. 2 shows three-dimensional map of the field on the surface of sphere, which is created only from measured values. Fig. 3 is map created from computed values, it means values which were computed during minimum searching (least square method) in Matlab. At the Fig. 4 and Fig. 5 is a comparison using 2D contour plot.

![Figure 2: 3D map on the surface (measured values).](image)

![Figure 3: 3D map on the surface (computed values).](image)

Finally if we want to get values of magnetic field induction inside the sphere, we use computed coefficients. Then we generate new coordinates of desired points inside the sphere and compute map of the field. This map can be any slice through the sphere as we can see in the Fig. 6.
5. CONCLUSION

We proposed a method for magnetic field mapping and approximation on the basis of measured values along specific area. As we can see from results, the map created from measured and computed values are quite similar. Future work can be directed towards minimization of differences between measured values of magnetic induction and approximated values, so we can obtain exact coefficients for approximation inside the sphere.

REFERENCES