A Finite Difference Frequency Domain Study of Curvature Lifted Modes Degeneration

C. S. Lavranos and G. A. Kyriacou
Microwaves Lab., Department of Electrical & Computer Engineering
Democritus University of Thrace, 12 Vas. Sofias St., Xanthi 67100, Greece

Abstract — The curvature induced degenerated modes separation in curved waveguides is studied in this paper. The analysis of curved waveguides is performed by using a two dimensional (2-D) Finite Difference Frequency Domain (FDFD) eigenvalue method employing orthogonal curvilinear coordinates. The eigenvalues frequency spectrum (propagation constants dispersion curves) of curved circular waveguides is considered. Curvature lifted modes degeneration is observed in these numerical results. The accuracy of the method is examined through comparisons with already published results.

1. INTRODUCTION
Curved waveguides have become a significant key in the design of several microwave circuits and systems. A variety of applications like moving airborne platforms or modern phased arrays carrying microwave transmit-receive systems require their antennas to be made conformal to the objects surface. The recent adoption of “smart skin” ideas asks for the whole RF front end to be made conformal to the host object’s surface. This approach enforces printing or integrating microwave devices on curved surfaces. The curvature varies from canonical objects surfaces as cylinders, to almost arbitrary (usually aerodynamic) curvatures. In turn, a plethora of microwave devices can be considered as comprised of curved waveguiding sections. Therefore, the accurate design of conformal systems requires the knowledge of curved waveguides and particularly printed transmission lines characteristics. This paper contributes exactly to this specific research field.

Despite its apparent simplicity, the analysis of propagation in a curved waveguide continues to be a challenging electromagnetic problem. Since the publication of Lewin et al book in 1977, [1], an extensive research on curved waveguides has been carried out, e.g., [2–4]. However, these works were mainly restricted to the study of canonical geometries. In particular, Lewin et al., [1], investigated E- and H- plane bends of rectangular or circular waveguides with a perturbation method. Besides, other perturbation methods were employed for the analysis of curved waveguides mainly in the optical spectrum, for example Xi Sheng Fang, [2]. Within this effort the “expansion of the bend modes into straight waveguide modes” was also employed by many researchers, e.g., [3]. Experiments with bends in nonradiative dielectric waveguides were also performed, [4].

Concerning the analysis of practical devices, numerical methods like the two dimensional Finite Element Method (FEM) or the Method of Moments (MoM) are in principle capable of handling curved geometries e.g., [5–6]. Yet, FEM is unable to handle waveguide curvatures in the propagation direction and MoM involves the Green’s functions of the structure which are not usually available for arbitrary shaped curved geometries. Hence, these methods by no means can serve as a general tool for the analysis of arbitrary curved waveguides.

Our research effort is based on a two-dimensional Finite Difference Frequency Domain (2-D FDFD) eigenvalue method formulated in orthogonal curvilinear coordinates. The theoretical basics have been presented in our previous works, [7–8]. This analysis is formulated as an eigenvalue problem for the complex propagation constants. It is restricted to structures uniform along the propagation direction. The waveguiding structure can be curved in all directions and this constitutes its main advantage. Besides that, it retains the classical FDFD method capability of handling arbitrary shaped geometries loaded with either isotropic or anisotropic materials.

The present work mainly focuses on the degenerated modes separation. For instance, in straight circular waveguides the TE_{01} and TM_{11} modes are degenerative, namely they present the same dispersion curves. Besides that, all right and left hand circularly polarized modes are also degenerative. When the waveguide is curved - bend the degeneration is lifted and the dispersion curves are separated. This phenomenon must be distinguished from the “birefringence”, where a new mode (not existed before) is generated due to some perturbation, for example a material anisotropy. In the following sections this phenomenon will be studied for a curved circular waveguide.
2. GEOMETRY OF A CURVED CIRCULAR WAVEGUIDE

The curved circular waveguide geometry shown in Fig. 1(a) will be studied, where the modified cylindrical coordinate system (\(\hat{\rho}, \hat{\varphi}, \hat{s}\)) introduced in [1] will be employed. This geometry is obtained by bending a straight circular waveguide, so that its symmetry axis-\(z\) forms an arc with radius \(R\). The unit vector \(\hat{s}\) is tangential to this arc, which also defines the propagation direction.

\[ E(\rho, \varphi, s) = E(\rho, \varphi) \cdot e^{-j\beta s} \quad \text{and} \quad H(\rho, \varphi, s) = H(\rho, \varphi) \cdot e^{-j\beta s} \]  

(1)

3. THE 2-D FDFD EIGENVALUE METHOD FOR CURVED WAVEGUIDES

In order to analyze the specific geometry of Fig. 1(a), the general purpose 2-D FDFD method established in our previous works, [7–8] is employed. This approach considers an orthogonal curvilinear coordinate system (\(\hat{u}_1, \hat{u}_2, \hat{u}_3\)) as shown in Fig. 1(b). The next paragraphs give a short description of the method.

Maxwell’s curl equations for the electric and magnetic field are expressed in \((\hat{u}_1, \hat{u}_2, \hat{u}_3)\) coordinates system according to [9]. The wave is assumed to propagate along the \(\hat{u}_3\)-direction similar to equation (1), while the cross section \((\hat{u}_1, \hat{u}_2)\) of the waveguide structure can be of arbitrary geometry loaded with inhomogeneous and in general anisotropic materials. In order to ensure the problem’s correct establishment the curvature of the guide axis is restricted to constant curvatures, [9]. A key point of the analysis is the separation of the field into axial and transverse components. In this manner the electric fields curl equation reads:

\[
\nabla \times E = -j\omega \mu H \rightarrow 
\begin{bmatrix}
-(j\beta) \cdot (\frac{1}{h_3}) \cdot \hat{a}_3 \times (\cdot) & -\hat{a}_3 \times \left(\left(\frac{1}{h_3}\right) \cdot \nabla_{lc}(\cdot)\right)
\end{bmatrix}
\begin{bmatrix}
E_t \\
h_3 E_3
\end{bmatrix}
= -j\omega
\begin{bmatrix}
\bar{p}_{tt} & \bar{p}_{tu} \\
\bar{p}_{ut} & \bar{p}_{uu}
\end{bmatrix}
\begin{bmatrix}
H_t \\
h_3 H_3
\end{bmatrix}
\]  

(2)

where \(\nabla_{lc} = \hat{a}_1 \left(\frac{1}{h_1}\right) \cdot \frac{\partial}{\partial u_1} + \hat{a}_2 \left(\frac{1}{h_2}\right) \cdot \frac{\partial}{\partial u_2}\) and \(h_1, h_2, h_3\) are the scale (or metric) factors. These metric factors in the modified cylindrical coordinate system \((\hat{\rho}, \hat{\varphi}, \hat{s})\), according to [1], are:

\[
h_\rho = 1, \quad h_\varphi = \rho, \quad h_s = 1 - \rho \cos \varphi / R
\]  

(3)

Likewise, its dyadic expression representing the curl of magnetic field is obtained. These two expressions are in turn discretized with the aid of a curvilinear grid over the whole solution domain, according to the basic principles of Yee’s cell, [10]. The discretized form of (2) and its dyadic are then formulated in a non-deterministic eigenproblem of the form \([A][u] = \beta[u]\). Vector \([u]\) is the eigenvector and \(\beta\) the sought eigenvalue. Matrix \([A]\) consists of sub-matrices, which represent the discrete form of the basic operators, such as the gradient and the divergence. Every single operator is descretized with respect to the curvilinear mesh. The boundary conditions are incorporated into this system by direct modification of the matrices involved. Due to the sparcity of these matrices, the final eigenvalue problem is solved using the Arnoldi Algorithm, [11].
4. NUMERICAL RESULTS & DISCUSSIONS

Numerical tests are carried out for two curved circular geometries. First, an empty curved circular waveguide of radius 0.02 m as shown in Fig. 1(a) is simulated. Dispersion curves for two indicative curvature radii are given in Fig. 2. In this case, the TE_{01} and TM_{11} mode degeneration lifting is studied. In straight waveguides this degeneration becomes triple for the TE_{01} and TM_{11} modes. Namely, TE_{01}, left hand circularly polarized and right hand circularly polarized TM_{11} modes have the same dispersion curves. The waveguide curvature breaks this degeneracy and separates these three modes as shown in Fig. 2. However, these modes are not completely independent. Lewin et al., [1], studied this case and distinguished these three separated modes of the curved circular waveguide as one pure TM_{11} and two distinct modes similar to the sum and difference of TE_{01} and TM_{11} mode. These two modes seem to be resulting from a coupling between TE_{01} and TM_{11} modes of the straight waveguide. Lewin et al. spotted on these two mixed modes and gave the propagation constants correction formula for a curved circular waveguide with inner radius \( \alpha \) and curvature radius \( R \) as, [1]:

\[
\Delta \beta = j k \sqrt{2/\Gamma_0 R} \quad \text{where} \quad \Gamma_0 = j_{11}/\alpha, \quad J_1(j_{11}) = 0
\]  

(4)

As explained by Lewin and also noticed in Fig. 2, these two mixed modes dispersion curves are shifted symmetrically upwards and downwards with respect to those of the straight case. In addition, the curved TM_{11} mode’s dispersion curve is slightly shifted downwards comparing also to that of the straight case. Our numerical results are mainly focused on the mixed modes; let’s denote them as TE_{01a} and TE_{01b}. These are compared with the propagation constants correction formula (4). In Fig. 2(a) the curvature radius is 25 times the waveguide dimension while in Fig. 2(b) this ratio is 10. Fig. 2(a) shows that the propagation constants change for the two mixed modes is about to 1% upwards and downwards. On the other hand, for a smaller curvature radius as shown in Fig. 2(b), the propagation constants change rises to 3%. The deviation between our results and (4) in both cases is smaller than 0.07%, so a good agreement is observed. Moreover, the curved TM_{11} mode’s dispersion curve shown in Fig. 2(a) is almost the same with that of TM_{11} mode in the straight waveguide, while in Fig. 2(b) a difference of about 1% is observed. However, propagation constants correction formula for curved TM11 mode is not given in [1], so comparison can not be done. It is important to note, that when the curvature radius becomes significantly small the agreement between our results and (4) breaks down. That’s because Lewin’s theory as well as our method are not accurate for significantly small curvature radii, due to the restrictions that are imposed.

![Figure 2](image_url)

Figure 2: Dispersion curves of coupled TE_{01a} and TE_{01b} modes in a curved circular waveguide of radius 0.02 m for two curvature radii: a) 0.5 m, b) 0.2 m, compared with those given by Lewin, [1]. The dispersion curves of degenerated TM_{11}/TE_{01} mode in a straight circular waveguide with the same dimensions along with those of TM_{11} mode in the curved circular waveguide are depicted.

Since the method is validated in the first example, the second example refers to a partially loaded curved circular waveguide as shown in Fig. 3(a). In the corresponding straight waveguide all right and left hand circularly polarized modes are degenerative. On the other hand, in the
curved waveguide all these degenerative modes are separated. As shown in Fig. 3(b) the first mode of the straight waveguide which was degenerative is spitted now to two distinct modes. In contrary to the first example, the two modes dispersion curves are both shifted upwards comparing to that of the straight waveguide after 7 GHz. But, at lower frequencies the dispersion curve of mode 1a was shifted downwards.

Figure 3: a) A curved circular partially loaded waveguide, b) Dispersion Curves of mode 1a and mode 1b in the curved circular partially loaded waveguide compared with that of mode 1 in the corresponding straight waveguide.

5. CONCLUSIONS

Numerical results for the curvature induced degenerated modes separation in a curved circular waveguide were presented in this paper. The propagation constants dispersion curves for two curved circular geometries were computed with our already proposed two dimensional (2-D) Finite Difference Frequency Domain (FDFD) curvilinear eigenvalue method. The results were validated by comparison to already published ones. An important conclusion is that all these studied phenomena can not be characterized as birefringence phenomena, because all the “new” separated modes were existed as degenerative in the straight waveguide. A variety of simulations for different curved structures will be presented at the conference along with a discussion on a number of new subjects that need investigation.

ACKNOWLEDGMENT

This work is implemented in the framework of Measure 8.3 through the O.P. Competitiveness 3rd Community Support Programme and is co-funded by: 75% of the Public Expenditure from the European Union — European Social Fund, 25% of the Public Expenditure from the Hellenic State — Ministry of Development — General Secretariat for Research and Technology, and Private Sector (INTRACOM SA).

REFERENCES


