A Novel Linear Beamforming Algorithm

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Abstract — Quantum Signal Processing (QSP) beamformer, which is based on Quantum Signal Processing framework, is presented in this paper. This new beamformer can realize the same output gain and signal-to-interference-and-noise ratio (SINR) as conventional beamformers through adjusting impact factors without pre-estimating the covariance matrix of received signals. Finally, the effectiveness and feasibility of the new beamformer are verified by simulation results.

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1. INTRODUCTION

Beamforming, which is widely used in the fields of radar [1], communications [2] and sonar [3], is a key technique in array signal processing. It can be classified into two main categories, namely conventional beamforming [2, 4] and blind beamforming [5, 6], depending on the availability of the source location information. Conventional beamforming is widely used since it needs only direction of arrival (DOA) of the interested signal and can be easily implemented. However, estimation of covariance matrix of received signals is indispensable in conventional beamforming, which restricts its application when the number of data samples is limited. A novel linear beamformer, which is defined as QSP beamformer, is presented in this paper by using ideas from Quantum Signal Processing framework [7]. The performance of the new beamformer approaches to that of conventional beamformer, and only the information of DOA is needed in this method. And for the reason that the covariance matrix of received signals does not need to be pre-estimated, the new beamformer can be used in fast fading channels.

The remainder of this paper is organized as follows. In Section 2, basic signal model of array processing and mathematical expressions of the new beamformer are introduced. In Section 3, the simulations of QSP beamformer are carried out and the results are analyzed respectively. Finally, concluding remarks are drawn in Section 4.

2. SIGNAL MODEL AND QSP BEAMFORMER

2.1. Signal Model [8]

Assume that there are J narrow-band stationary signals in far field \( \{i_j(t), j = 1, 2, \ldots, J\} \), and the DOAs of the signals are \( \{\theta_j, j = 1, 2, \ldots, J\} \) respectively. At the receiver antenna array is composed of M elements. Suppose that the additive white Gaussian noises at each array element are \( \{n_k(t), k = 1, 2, \ldots, M\} \) with the same variance \( \sigma^2 \), then the received signal of the kth array element can be denoted as follows:

\[
x_k(t) = \sum_{j=1}^{J} a(\theta_j) i_j(t) + n_k(t)
\]

And the vector form of the received signals is:

\[
x(t) = As(t) + n(t) = \sum_{j=1}^{J} a(\theta_j) i_j(t) + n(t)
\]

where \( a(\theta_j) = [a_1(\theta_j) \ldots a_M(\theta_j)] \) is the steering vector corresponding to signals with DOAs \( \{\theta_j, j = 1, \ldots, J\} \), \( x(t) = [x_1(t) \ldots x_M(t)]^T \), \( A = [(a(\theta_1) \ldots a(\theta_J))^T] \), \( s(t) = [i_1(t) \ldots i_J(t)]^T \), and \( n(t) = [n_1(t) \ldots n_M(t)]^T \).

Equation (2) is the matrix form of received signals. According to the principle of adaptive beamforming, the beam pattern can be obtained through adjusting the weight coefficient of each
array element. All the weight coefficients form the weight vector \( \mathbf{w} \). The output signal of the beamformer is denoted as:

\[
y(t) = \mathbf{w}^H \mathbf{x}(t)
\]  

(3)

2.2. QSP Beamformer

As discussed above, the search for optimal weight vector \( \mathbf{w} \) is to achieve both a high output gain in the direction of interested signal and a low gain in the direction of interference signals, which means that the interference is suppressed to the maximum extent. Meanwhile, since the signals are always exposed to influence of noises, an ideal beamformer should handle influences and noises effectively at the same time. In order to meet the requirement mentioned above, we set the principle of new exposed to influence of noises, an ideal beamformer should handle influences and noises effectively at the same time. In order to meet the requirement mentioned above, we set the principle of new beamformer as follows:

Assume that DOA \( \theta_j \) is given. If we expect that high signal-to-noise ratio (SNR) can be acquired in this DOA, the weight vector \( \mathbf{w}(\theta_j) \) of the signal with DOA \( \theta_j \) should either equal or approach to the steering vector \( \mathbf{a}(\theta_j) \) under some constraint. Similarly, the same way can be used to achieve high SNR for other users. Thus we summarize the discussion above as follows: when a group of steering vectors \( \{\mathbf{a}(\theta_j), 1 \leq j \leq J\} \) are given, we could look for a set of vectors \( \{\mathbf{w}(\theta_j), 1 \leq j \leq J\} \) which equal or approach to \( \{\mathbf{a}(\theta_j), 1 \leq j \leq J\} \) with some constraint to achieve high SNR in each DOA of the interested signals.

Meanwhile, the maximum signal-to-interference ratio (SIR) is also expected in beamforming, so any weight vector \( \mathbf{w}(\theta_j) \) should be orthogonal to steering vectors \( \{\mathbf{a}(\theta_j'), 1 \leq j' \leq J, j' \neq j\} \) to the greatest extent. And according to the discussion above the weight vectors \( \{\mathbf{w}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \) should approach to the steering vectors \( \{\mathbf{a}(\theta_j'), 1 \leq j' \leq J, j' \neq j\} \) to the greatest extent, so \( \mathbf{w}(\theta_j) \) should be orthogonal to \( \{\mathbf{w}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \) to the greatest extent.

Based on the discussion above and under suitable constraint, the realization of an ideal beamformer can be expressed as follows:

Assume that there are \( J \) steering vectors \( \{\mathbf{a}(\theta_j), 1 \leq j \leq J\} \) in Hilbert space \( \mathcal{H} \), when a group of vectors \( \{\mathbf{a}(\theta_j), 1 \leq j \leq J\} \) are given, we can construct a group of vectors \( \{\mathbf{w}(\theta_j), 1 \leq j \leq J\} \) whose element is orthogonal to each other and which approach to the given group of vectors \( \{\mathbf{a}(\theta_j), 1 \leq j \leq J\} \) under Least-Square constraint as close as possible. Furthermore, we introduce different impact factors \( \{q_j, 1 \leq j \leq J\} \) for different signals, which satisfy \( q_1 + q_2 + \ldots + q_j = 1 \), then the vectors \( \{\mathbf{w}(\theta_j), 1 \leq j \leq J\} \) should have the following equation minimum \[9\]:

\[
\varepsilon_{LS} = \sum_{j=1}^{J} q_j \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j) \mathbf{a}(\theta_j) - \mathbf{w}(\theta_j) \leq J, j' \neq j \}
\]

(4)

Meanwhile subject to:

\[
\langle \mathbf{w}(\theta_j), \mathbf{w}(\theta_{j'}) \rangle = c^2 \delta_{jj'}
\]

(5)

where \( c \) is a constant value greater than zero, \( \delta_{jj'} \) indicates that the vectors should be orthogonal to each other and the constant \( q_j \) denotes the impact factor of the \( j \)-th signal. After calculation we can get:

\[
\mathbf{w}(\theta_j) = c \mathbf{a}(\theta_j) \mathbf{Q} \left( (\mathbf{Q} \mathbf{A}^* \mathbf{A} \mathbf{Q})^{1/2} \right)^+ \quad \{1 < j < J\}
\]

(6)

where \( (\cdot)^+ \) denotes the Moore-Penrose pseudo-inverse of \( (\cdot) \) and \( \mathbf{Q} \) is a diagonal matrix \( \mathbf{Q} = \text{diag}(q_1, q_2, \ldots, q_J) \).

The beamformer defined by Equation (6) is called QSP beamformer in this paper.

According to Equation (6), different performance outputs can be obtained when different impact factors chosen. If the impact factors of all \( J \) signals are equal to each other, each signal makes the same contribution to \( \varepsilon_{LS} \) in Equation (4). Now without loss of generality DOA of the interested signal is \( \theta_j \), if its impact factor \( q_j \) approximates to 1 and impact factors \( \{q_{j'}, 1 \leq j' \leq J, j' \neq j\} \) from other DOAs equal to each other, the weight vector \( \mathbf{w}(\theta_j) \) of the signal from DOA \( \theta_j \) should approach to its steering vector \( \mathbf{a}(\theta_j) \) more than the weight vectors \( \{\mathbf{w}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \) of other signals in the solving process of minimizing Equation (4), therefore the interested signal obtains high SNR. Similarly, if the impact factor \( q_j \) approximates to 0 and the impact factors \( \{q_{j'}, 1 \leq j' \leq J, j' \neq j\} \) are the same, the vectors \( \{\mathbf{w}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \) will be much closer to vectors \( \{\mathbf{a}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \), that means the weight vector \( \mathbf{w}(\theta_j) \) will be approximately orthogonal to vectors \( \{\mathbf{a}(\theta_j), 1 \leq j' \leq J, j' \neq j\} \) subject to the constraint Equation (5), and high SIR is achieved.
Based on the discussion above, the conclusion can be expressed as follows: the novel beamformer can achieve the optimal output gain and high SINR through configuring different impact factors flexibly under different circumstances.

3. SIMULATION RESULTS AND ANALYSIS

In the computer simulation of this paper, we utilize a four-element Uniform Linear Array (ULA) with the elements separated by half-wavelength interval and assume that the interested signal comes from the direction of 0 degree, and an interference signal comes from the direction of 15 degrees with 20 dB SNR and 0 dB SIR, $c = 1$ in QSP beamformer.

Simulation 1: In this simulation, we focus on the beampatterns of the beamformers. Assume that the estimation of DOA is accurate. 1024 samples are used to estimate the auto covariance matrix of received signals in Minimum Variance Distortionless Response (MVDR) beamformer.

Figure 1: Beampatterns of the beamformers.

Figure 2: Beampatterns of the beamformers.

Figure 2 is a part of Fig. 1. According to Figs. 1 & 2, we can get: QSP beamformer can not aim at the exact angle of the interrupt signal when $q_1$ is 0.1 corresponding to curve QSP 01 compared with the curve MVDR or curve QSP 001 which is the output when $q_1$ is 0.01. That means when the value of $q_1$ decreases, the performance of QSP will approach to that of MVDR. The result shows the consistency with the analysis above.

With the same environment, the output SINR of the beamformers is presented and compared.

Figure 3: The output SINR of three beamformers.

According to the Fig. 3, as the impact factor of the interested user decreases, the output SINR of QSP beamformer rises gradually. At last its performance surpasses the performance of the MVDR. The result seems hard to accept, but considering the difference between the ideal MVDR outputs with the real MVDR output using 1024 samples, we think the result is acceptable.
4. CONCLUSION

A novel beamformer named QSP beamformer is presented in this paper through introducing the idea of Quantum Signal Processing into the field of beamforming. It can achieve better output performance through adjusting impact factors flexibly under different circumstances when noise’s effect is unknown. Compared with the fixed working pattern of conventional beamformers, the new beamformer has more flexibility and stable performance.

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