Numerical Simulation and Analysis on Mode Property of Photonic Crystal Fiber with High Birefringence by Fast Multipole Method

Wei Song, Yuanyuan Zhao, Yu Bao, Shuguang Li, Zhiyuan Zhang, and Tianfu Xu
Physics Department, College of Science, Yanshan University, Qinhuangdao 066004, China

Abstract — In this paper the mode property of a photonic crystal fiber with triangle-lattice array in a silica matrix has been simulated by fast multipole method. The PCFs studied in this paper have a silica core, obtained by introducing a defect, that is by removing three holes in the center of the fiber transverse section. The model fields, effective index and confinement loss about the fundamental mode in the fibre are analysed and compared. It is demonstrated that lower confinement loss and higher birefringence can be realized in the condition of fewer rings of air holes. At the wavelength of 1.55 \( \mu \)m the confinement loss is \( 7.3 \times 10^{-6} \) dB/m and the birefringence is \( 1.65 \times 10^{-3} \) in this fiber. The simulation results show that birefringence of this triangle-lattice PCF is dominated by inner-ring air holes in the fibre effectively. The simulation results in this paper have important meaning for instructing the fabrication of birefringent photonic crystal fiber.

DOI: 10.2529/PIERS060910211527

1. INTRODUCTION

In recent year, great interest has been focused on the study of photonic crystal fiber (PCF). By using the structure adjustability of the PCF, zero dispersive wavelength moving towards short-wave spectroscopy, chromatic dispersion compensating, dispersion truncating, high nonlinearity and birefringence can be realized [1–15]. The mode birefringence of PCF mainly comes from the geometric structure of fiber and the usual method to cause birefringence is employing double-core or multi-core structure, changing the fiber core or the shape of the air holes and altering the distribution of the air holes. High birefringence PCF can be used in optical fiber sensor, interferometer and polarizer, etc. In addition, by designing the high birefringence and intensive nonlineality PCF required, we can make a fiber not only high birefringence, but also intensive nonlineality. We can also make the large mode property polarization maintaining PCF [15]. This can make the property of PCF integrate, so it can be used in the Raman magnification, ex-continous spectum of the polarization, four-wave frequency mixing and the crossing phase modulation. The development of the birefringence PCF will promote the study and appliance of the new photoelectric device. In the progress of making the PCF fiber, we find that maintaining the ideal structure of the fiber and the number of air holes layers of the PCF cladding is a contradiction [2, 4, 17]. Generally speaking, the more layers the PCF cladding have, the smaller the confinement loss will be, but at the same time increasing the layers will make the distortion of the air holes a big problem when the fibers are produced. On the fundament of the traditional triangular arrayed air holes, this article proposes a birefringence structure PCF which has \( C_{2v} \) symmetry. And by using the fast multipole method, its modal property is numerically simulated. It is found that in this PCF, there will be several conductive modes, and its fundamental mode has very low confinement loss and high birefringence.

2. BASIC THEORY

The ordinary solution about multipole theory at the complex boundary electrostatic field has been deduced and developed by Zheng Qinzhong. Nie Zaiping [22] analyzed the three dimensional vector scattering of complicated object by using the fast multipole method. These examples show that the multipole method is an effective way to analyze electromagnetic field theory. Using the multipole method to simulate the dispersion and the loss property of microstructured fiber is originally proposed by T. P. White and B. T. Kuhlmy [23–25] in Sydney University. Zhao Mingzhu [26] also simulated the PCF by using multipole method. This method is adopted when the air holes of microstructured fiber is cylindrical. By using this method, we can get real part and imaginary part of the effective refractive index of the fiber and
the attenuation along the fiber: $E_Z$ and $H_Z$. The other is sectional part: $E_z$ and $H_z$. In fact, when magnetic field multiple a parameter, it will have the same Maxwell equation form to elastic field, that is $K = ZH$, where $Z$ is impedance of free space. Electromagnetic field can be demonstrated as follows:

$$
\vec{E} = \vec{E}(r, \phi) \exp[i(\beta z - \omega t)]
$$

(1)

$$
\vec{K} = \vec{K}(r, \phi) \exp[i(\beta z - \omega t)]
$$

(2)

where $\omega$ is angular frequency, propagation constant $b$ is a plural. The imaginary part of $b$ denotes the attenuation along the $Z$-axis.

The longitudinal part of electromagnetic field ($V = E_z$ or $V = K$) meet the Helmholtz equation

$$(\nabla^2 + (k_\perp)^2)V = 0 : (3)$$

$k_\perp$ is shown by $k_\perp^l$ and $k_\perp^e$ respectively in the air holes and base material. They could be expanded to cylindrical function. In the $l$th air hole, its vertical electric field $E_Z$ can be expanded in the cylindrical coordinate:

$$
E_z = \sum_{m=-\infty}^{\infty} a_m^{(l)} J_m(k_\perp^l r_l) \exp(i m \phi_l) \exp(i \beta z)
$$

(4)

In the medium next to the $l$th air hole, the vertical electric field can be shown as follow:

$$
E_z = \sum_{m=-\infty}^{\infty} \left[ b_m^{(l)} J_m(k_\perp^l r_l) + c_m^{(l)} H_m^1(k_\perp^l r_l) \right] \times \exp(i m \phi_l) \exp(i \beta z)
$$

(5)

$k_T = k_\perp^l = (k_0^2 n_i^2 - \beta^2)^{1/2}$, $k_e^l = (k_0^2 n_e^2 - \beta^2)^{1/2}$, the refractive index of air $n_i = 1$, $n_e$ is the refractive index of quartz material, $k_0$ is the wavevector in free space, $r_l$ and $j_l$ are the coordinate of regional coordinate system $\overrightarrow{\tau_l}(r_l, \phi_l) = \overrightarrow{r} - \overrightarrow{\tau}$, $\overrightarrow{\tau}$ is the center of the air hole. The expression of magnetic field part $K_z$ is similar to the electric field. Be careful when compute the quadratic root of the plural number, as to the mode which is attenuated traveling along the $+Z$ axis, the real part and imaginary part of the propagation constant must meet this rules: $\Re(\beta) > 0$, $\Im(\beta) > 0$.

By using the boundary condition of electromagnetic field on the interface of the air hole, we can get the expression of $a_m^{(l)}$, $b_m^{(l)}$ and $c_m^{(l)}$. In the real computation by choosing the proper cutoff value corresponding to the air hole diameter of the optical fiber cladding and the wavelength ratio $\lambda$, we can optimize speed and precision of the calculation. Further more, taking in the concept of fast multipole method to programme can increase the computation speed. The effective refractive index of the mode $n_{eff}$ can be obtained by propagation constant $b$.

Then, the imaginary part of the $n_{eff}$ can be used to get the fiber confinement loss (the unit is dB/m).

$$
L = \frac{20}{\ln(10)} \frac{2\pi \Im(n_{eff})}{\lambda} \times 10^6
$$

(6)

The unit of $l$ is micrometer (mm), the dispersion coefficient can be got from the real part

$$
D = -\frac{\lambda}{c} \frac{d^2 \Re(n_{eff})}{d\lambda^2}
$$

(7)

The birefringence index $B$ [9–11] is:

$$
B = n_{eff}^b - n_{eff}^f
$$

(8)
$n_{eff}^s$ and $n_{eff}^f$ are the two cross polarization of the fundamental mode which is corresponding to the slow axis and fast axis. The polarization beat length between the two cross polarization is:

$$L_B = \frac{\lambda}{B}$$

(9)

![Figure 1](image1.png)

Figure 1: (a) The section of birefringence PCF and (b) minimal sector.

3. SIMULATION AND ANALYSIS

The section of birefringence PCF simulated in this paper is shown in Figure 1. PCF cladding is composed of regular triangular arrayed air holes. Three holes along the center of $X$ axis are removed to form the fiber core. The pitch $\Lambda = 2.3 \mu m$, air-holes diameter $d = 1.6 \mu m$. In the most

![Figure 2](image2.png)

Figure 2: The fundamental mode distribution of the birefringence PCF at the wavelength ($\lambda = 1.55 \mu m$), where figures (a) and (b) and (c) respectively stand for their slow axis mode field $|E_z|$ and $|H_z|$ and $|S_z|$, (d) and (e) and (f) stand for their fast axis mode field $|E_z|$ and $|H_z|$ and $|S_z|$ respectively.
inner layer, there are 6 more bigger air holes, their diameter is \(d_0 = 2.0\, \mu m\). As far as the structure of the fiber is concerned, on one hand, because the fiber core can generate birefringence for its asymmetrical in the \(X\) axis and \(Y\) axis direction; on the other hand according to the theory of group, this structure has the \(C_{2v}\) symmetry. Taking advantage of these symmetries can simplify the calculation and enhance the precision and speed of computation. The minimal sector is \(\phi = 0 \sim \pi/2\) when simulated.

Figure 2 shows the fundamental mode distribution of normalized mode field of the PCF shown in Figure 1, where figures (a), (b) and (c) respectively stand for their slow axis mode field \(|E_z|\) and \(|H_z|\) and \(|S_z|\), (d), (e) and (f) stand for their fast axis mode field distribution \(|E_z|\) and \(|H_z|\) and \(|S_z|\) respectively. From these pictures, it can be seen that every mode’s \(|E_z|\), \(|H_z|\), \(|S_z|\) distribution has the \(C_{2v}\) symmetry. The \(|S_z|\) of the fundamental mode has a peak center. The modal field of the fundamental mode distribution shows that \(|E_z|\) of slow axis and \(|H_z|\) of fast axis have similar mode field distribution and the \(|H_z|\) of the slow axis and \(|E_z|\) of fast axis have the similar mode field distribution too. The \(Z\) part of Poynting vector of the slow axis and fast axis has similar distribution of \(|S_z|\).

Figure 3 gives the effective refractive index and Fundamental Space Mode [5,6] (FSM) of the fundamental mode of PCF. In the figure, curve 1 and 2 represent effective refractive index of slow axis and fast axis of fundamental mode. FSM represents the effective refractive index of cladding fundamental space mode. It can be seen from this figure that the effective refractive index of the fundamental mode of slow axis is bigger than the fast axis.

Figure 4 shows the changing rule of the confinement loss \(L\) corresponding to the wavelength of the PCF fundamental mode. In the figure, curve 1 and 2 represent the confinement loss of the slow axis and fast axis. It can be seen in this figure that in the low loss window where=1.55 \(\mu m\) of optical communication, the slow axis mode and the fast axis mode are \(7.3 \times 10^{-6}\) and \(7.2 \times 10^{-6}\) \(dB/m\). This loss is very small. The confinement loss of fundamental mode at 1.55 \(\mu m\) is far below the traditional loss 0.2 \(dB/km\) in optical communication. We can also see that the fiber’s fundamental mode of short infrared and middle infrared band (0.8 \(\mu m\)~3.0 \(\mu m\)) we simulate have confinement loss bellow 0.06 \(dB/m\). If there is another layer of air hole, its loss can be lower. So this design of structure is certainly a type of PCF with low confinement loss.

Figure 5(a) shows the changing rule of fundamental mode birefringence \(B\) and beat length \(L_B\) of PCF as functions of wavelength. Figure 5(b) gives the dispersion coefficient \(D\) of slow axis and fast axis as functions of wavelength. At 1.55 \(\mu m\), birefringence of fundamental mode is \(B = 1.65 \times 10^{-3}\). This belongs to high birefringence fiber. Between infrared band, the birefringence is increased with the increasing of wavelength. Correspondingly, the beat length decreases with the increase of wavelength. It can be seen from the Figure 5(a) that the dispersion coefficient of fast axis module and slow axis corresponding to the fundamental mode has difference. The zero dispersion coefficient of fast axis and slow axis are 0.95 \(\mu m\) and 0.975 \(\mu m\). So there is modal dispersion between fast axis and slow axis.
4. CONCLUSION

Based on traditional structure of triangular arrayed air holes, this paper promotes a type of birefringence PCF which has $C_{2v}$ symmetry property. By using the fast multipole method, the fundamental mode distribution, effective refractive index and the confinement loss of this birefringence PCF are simulated. It is found that the structure of PCF only with several layers of the cladding air-holes can realize very low confinement loss. The result has instructive significance for the manufacture of the birefringence PCF. It can promote the progress of the manufacture of infrared band photoelectric device appliance by using PCF.

REFERENCES