Multilayer Planar Resonators with Superconductive Patch on PBG Substrate

Humberto César Chaves Fernandes and George Dennes Fernandes Alves
Federal University of Rio Grande do Norte, DEE, P.O. Box 1583, Lagoa Nova, 59078-970-Natal, RN, Brazil

Abstract — The rectangular discretized multilayer microstrip resonators, analyzed using the full wave TTL — Transverse Transmission Line method in the spectral domain, jointly with the moment method are presented. The homogenization theory is used to calculate the equivalent permittivity for the $s$ and $p$ polarizations of the structure composed of PBG (Photonic Band Gap) material. Numerical results of the resonance frequency for the rectangular multilayer, on isotropic and anisotropic PBG material substrate resonators are presented.

DOI: 10.2529/PIERS060908155123

1. INTRODUCTION
The microstrip antennas have received more attention due to the characteristics and advantages comparison with the conventional antennas. These antennas present small volume, reduced weight, planar configuration, compatibility with integrated hybrid circuits and the possibility of acting with dual frequency [1]. They can be used in several systems, such as: radars, wireless, mobile telephony and communication by satellites.

However, they present disadvantages with relationship to the narrow band, losses due to the low gain and losses by irradiation due to the surface waves. These disadvantages can be eliminated, by using a screening to avoid losses by irradiation [2] and antennas with PBG substrate (Photonic Bandgap) [3]. Some techniques to increase the band width are used among the: thicker substrate [4], multilayer antennas or stacked patches [5, 6].

Antennas with dielectric multilayer have advantages, such as, the flexibility in the operation frequency band, a smaller physical size, etc.

The PBG materials are a new type of periodic substrates structure in which the propagation in certain bands of frequencies is prohibited [7]. In this work the rectangular multilayer resonators on PBG substrate are analyzed, as illustrated in the Fig. 1.

![Figure 1: Rectangular multilayer microstrip resonator.](image)

The TTL — Transverse Transmission Line method [8–10] is used in the determination of the electromagnetic fields components in the Fourier transform domain (FTD), for the three regions of the structures. The moment method is applied and adequate basis functions are used to expand the current densities in the metallic strip.

2. THEORY
The TTL method in the Fourier transform domain, uses a component of propagation in the $y$ direction, treating the general equations of electric and magnetic field as functions of $\vec{E}_y$ and $\vec{H}_y$.

Starting from the Maxwell equations, and after several algebraic manipulations, the equations that represent the electromagnetic fields in the $x$ and $z$ directions are obtained as a function of the
electromagnetic fields the in the $y$ direction,

$$\begin{align*}
\{ \vec{E}_{Ti}, \vec{H}_{Ti} \} &= \frac{1}{k_i^2 + \gamma_i^2} \left[ j\omega \nabla_T \times \{ -\mu \vec{H}_{yi} \} + \frac{\partial}{\partial y} \nabla_T \{ \vec{E}_{yi} \} \right] \\
\end{align*}$$  \hspace{1cm} (1)$$

After then, the two dimensional Fourier transforms are applied. The electromagnetic fields for $i$-th dielectric region are then obtained:

$$\begin{align*}
\vec{E}_{xi} &= \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \vec{E}_{yi} + \omega \mu \beta_k \vec{H}_{yi} \right] \\
\vec{E}_{zi} &= \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\beta_k \frac{\partial}{\partial y} \vec{E}_{yi} - \omega \epsilon \alpha_n \vec{H}_{yi} \right] \\
\vec{H}_{xi} &= \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\alpha_n \frac{\partial}{\partial y} \vec{H}_{yi} - \omega \epsilon \beta_k \vec{E}_{yi} \right] \\
\vec{H}_{zi} &= \frac{1}{\gamma_i^2 + k_i^2} \left[ -j\beta_k \frac{\partial}{\partial y} \vec{H}_{yi} - \omega \epsilon \alpha_n \vec{E}_{yi} \right] \\
\end{align*}$$  \hspace{1cm} (2)–(5)

where $i = 1, 2, 3 \ldots$ are the dielectric regions, $\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2$ where $\gamma_i$ is the propagation constant in $y$ direction, $\alpha_n$ is the spectral variable in $x$ direction, $\beta_k$ is the spectral variable in $z$ direction, $k_i^2 = \omega^2 \epsilon \mu = k_0^2 \epsilon_r^i$ and $\epsilon_r^i = \epsilon_r - j \frac{\sigma_i}{\omega \epsilon_0}$ is the relative dielectric permittivity of the material, $k$ is the wave number, $\omega$ is the angular frequency, $\epsilon_0$ is the dielectric permittivity in the free space, and $\sigma_i$ is the conductivity of the $i$-th layer. The analytical development, using the LTT method, for the multilayer resonators studied in this work is given in [11].

The electromagnetic field and the currents are related with the dyadic Green matrix $[Z]$, [11, 13],

$$\left[ \begin{array}{cc}
Z_{xx} & Z_{xz} \\
Z_{zx} & Z_{zz}
\end{array} \right] \cdot \left[ \begin{array}{c}
\vec{J}_z \\
\vec{J}_x
\end{array} \right] = \left[ \begin{array}{c}
\vec{E}_x \\
\vec{E}_z
\end{array} \right]$$  \hspace{1cm} (6)

Applying the Galerkin technique, the electric fields out of the metallic strip are eliminated. The current densities are expanded in appropriate basis functions and (6) becomes a homogeneous complex matrix as shown in (7).

$$\left[ \begin{array}{cc}
K_{xx} & K_{xz} \\
K_{zx} & K_{zz}
\end{array} \right] \cdot \left[ \begin{array}{c}
a_x \\
\sigma_i
\end{array} \right] = \left[ \begin{array}{c}
0 \\
0
\end{array} \right]$$  \hspace{1cm} (7)

Each element of the $[K]$ characteristic matrix is shown in (8)–(9) for examples:

$$\begin{align*}
K_{xx} &= \sum_{-\infty}^{\infty} \tilde{f}_x(x, z) Z_{xx} \tilde{f}_x^* (x, z) \\
K_{xz} &= \sum_{-\infty}^{\infty} \tilde{f}_z(x, z) Z_{xz} \tilde{f}_x^* (x, z)
\end{align*}$$  \hspace{1cm} (8)–(9)

The non-trivial solution generates the characteristic equation, in which the roots allow the obtaining of the resonance frequency of the structure.

1) Rectangular Resonator

After obtaining (7), appropriate basis functions are used to approximate the values of the current densities to the form the real function, as presented in (10) and (11):

$$\begin{align*}
\tilde{J}_{zh}(x, z) &= \sum_{i=1}^{N_i} a_{zi} f_{zi}(x, z) \\
\tilde{J}_{zh}(x, z) &= \sum_{i=1}^{N_i} a_{zi} f_{zi}(x, z)
\end{align*}$$  \hspace{1cm} (10)–(11)

The basis functions are used in the characteristic matrix $[K]$ for the expansion of the current densities [15].
3. NUMERICAL RESULTS

As a comparison, Fig. 2 presents the results obtained for the multilayer microstrip rectangular resonator with superconductive patch, on isotropic and PBG material, and the second comparison results of the microstrip rectangular resonator of one layer, without superconductive patch. Another comparison using the Cavity model [19] is also made.

For the simulation of the three-layer resonator, the two-layer under patch were considered as a single-layer, whose height of each layer was considered as 0.635 mm (see Fig. 1). In this way, the height of the added two-layers \( h_1 + h_2 = h \) will result in a height \( h = 1.27 \) mm. The substrate was RT Duroid 5880 of relative electric permittivities \( \varepsilon_r_1 = 2.2 \) and \( \varepsilon_r_2 = 2.2 \), the three layer was considered as being the air \( \varepsilon_r_3 = 1.0 \), the width of the patch is of \( w = 15.0 \) mm.

The resonator of one layer was simulated for \( h = 1.27 \) mm, with relative electric permittivities \( \varepsilon_r_1 = 2.2 \) and \( \varepsilon_r_2 = 1.0 \), width of the patch \( w = 15.0 \) mm. The Fig. 3 has shown the result of the variation of the resonance frequency in function of several length of the patch resonator.

Observing the Fig. 2, one notices that the dielectric three-layer resonator with superconductive patch works correctly when it simulates a resonator of one layer without superconductive patch.

![Figure 2: Resonance frequency as function of the length of the patch for \( \varepsilon_r = 2.2 \).](image1)

![Figure 3: Resonance frequency as function of the length of the patch for the cases 1 and 2.](image2)

After the verification that the multilayer resonator is working correctly, simulations were accomplished with PBG 2D material substrate, where the effective permittivity of the dielectric depends on the polarization of the electric field \( s \) (parallel to the axis \( z \)) and \( p \) (perpendicular to the axis \( z \)). Being the equivalent permittivity of the structure in the substrate of Silicon (Si) for the \( p \) polarization \( \varepsilon_r = 8.7209 \) and for the \( s \) polarization \( \varepsilon_r = 10.233 \), the width of the patch is \( w = 15.0 \) mm.

The Fig. 3 has shown the curves for the simulated cases: Case 1: Dielectric layer 1 composed of RT Duroid 5880 with \( \varepsilon_{r1} = 2.2 \), dielectric layer 2 composed of PBG 2D material considering the incident wave to the \( s \) polarization and the dielectric layer 3 is the air with \( \varepsilon_r = 1.0 \); Case 2: Dielectric layer 1 composed of RT Duroid 5880 with \( \varepsilon_{r2} = 2.2 \), dielectric layer 2 composed of PBG 2D material considering the incident wave to the \( p \) polarization and the dielectric layer 3 is the air with \( \varepsilon_r = 1.0 \).

4. CONCLUSION

A theoretical study about the rectangular multilayer dielectric microstrip resonators was presented using the Transverse Transmission Line method in the spectral domain. The theory of the Homogenization was applied for the determination of the equivalent permittivity of the structure composed of PBG.

The rectangular geometry is made in agreement with the desired application, because the triangular geometry tends to occupy a smaller physical space than the rectangular, being able to be used in applications where the miniaturization of the antenna or resonator is necessary. The authors thank the Brazilian agencies CNPQ, CAPES and UFRN for supporting this work.
REFERENCES